Maria Agnesi and Her "Witch"

Even today, women make up only about 10 percent of the total number of mathematicians in the United States;[1] worldwide their number is much smaller. But in past generations, social prejudices made it almost impossible for a woman to pursue a scientific career, and the total number of women mathematicians up to our century can be counted on two hands. Three names come to mind: Sonia Kovalevsky (1850–1891) of Russia, Emmy Noether (1882–1935), who was born in Germany but emigrated to the U.S., and Maria Agnesi of Italy.[2]

Maria Gaetana Agnesi (pronounced "Anyesi") was born in Milan in 1718, where she spent most of her life.[3] Her father, Pietro, a wealthy professor of mathematics at the University of Bologna, encouraged her to study the sciences. To further her education he founded at their home a kind of "cultural salon" where guests would come from all over Europe, many of them scholars in various fields. Before these guests, young Maria displayed her intellectual talents by presenting theses on a variety of subjects and then defending them in disputation. The subjects included logic, philosophy, mechanics, chemistry, botany, zoology and mineralogy. During intermission her sister, Maria Teresa, who was a composer and harpsichordist, entertained the guests with her music. The scene is reminiscent of Leopold Mozart showing off young Amadeus's musical talents in the salons of the well-to-do of Salzburg, with Mozart's sister Nannerl playing in the background. Maria Gaetana was also versed in languages: at the age of five she was already fluent in French, and when she was nine she translated into Latin and published a long speech advocating higher education for women. Soon she mastered Greek, German, Spanish, and Hebrew and would defend her theses in her guests' native languages. She later collected 190 of these theses and published them in a book, *Proportiones philosophicae* (1738); unfortunately, none of her mathematical thoughts are included in this work.

By the time Agnesi was fourteen she was already solving difficult problems in analytic geometry and physics. At seventeen she began shaping her critical commentary on Guillaume L'Hospital's work, *Traité analytique des sections coniques*; unfortunately, this commentary was never published. About that time she had had enough with the public displays of her talents; she withdrew from social life to devote herself entirely to mathematics. She spent the next ten years writing her major work, *Instituzioni analytiche ad uso della gioventu italiana* (Analytic institutions for the use of young Italians). This work was published in 1748 in two very large volumes, the first dealing with algebra and the second with analysis (that is, infinite processes). Her goal was to give a complete and integrated presentation of these subjects as they were then known (we must remember that in the middle of the eighteenth century the calculus was still in a developmental stage, and new procedures and theorems were constantly being added to its existing core). Agnesi wrote her book in Italian rather than Latin, the scholarly language of the time, in order to make it accessible to as many "young Italians" as possible.

The *Instituzioni* brought Agnesi immediate recognition and was translated into several languages. John Colson (d. 1760), Lucasian professor at Cambridge University, who in 1736 published the first full exposition of Newton's *Method of Fluxions and Infinite Series* (his differential calculus), translated Agnesi's book into English. This he did when he was already advanced in age, learning Italian expressly for this task "so that the British Youth might have the benefit of it as well as the Youth of Italy." His translation was published in London in 1801.

In recognition of her accomplishments, Pope Benedict XIV in 1750 appointed Agnesi professor of mathematics at the University of Bologna. But she never actually taught there, viewing her position merely as an honorary one. After her father's death in 1752 she gradually withdrew from scientific activity, devoting her remaining years to religious and social work. She also raised her father's twenty-one children (from three marriages) and directed their education, while at the same time helping the poor in her parish. She died in Milan in 1799 at the age of 81.

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It is ironic that Agnesi's name is mainly remembered today for a curve she has investigated but was not the first to study: the *witch of Agnesi*. Consider a circle of radius a and center at $(0, a)$ (fig. 50). A line through $(0, 0)$ cuts the circle at point A and is extended until it meets the horizontal line $y = 2a$ at point B. Draw a horizontal line through A and a vertical line through B, and let these lines meet at P. The witch is the locus of P as the line OA assumes all possible positions.

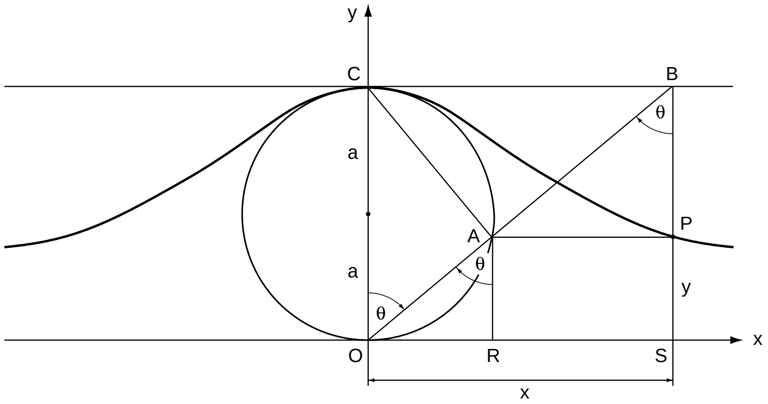

FIG. 50. The Witch of Agnesi.

It is easiest to find the equation of the witch in terms of the angle θ between OA and the y-axis. Let the coordinates of P be (x, y). From figure 50 we see that $\angle OAC = 90^\circ$, OC being a diameter of the circle; in the right triangle OAC we thus have $OA = OC \cos\theta = 2a\cos\theta$. Let R and S be the feet of the perpendiculars from A and B to the x-axis, respectively; then in the right triangle OBS we have $OS = x = BS\tan\theta = 2a\tan\theta$, and in the right triangle OAR we have $AR = y = OA\cos\theta = 2a\cos^2\theta$. The parametric equations of the witch are thus

$$x = 2a\tan\theta, y = 2a\cos^2\theta. \tag{1}$$

To find its rectangular equation we must eliminate θ between equations (1). Using the identity $1 + \tan^2\theta = 1/\cos^2\theta$ to express y in terms of x, we get

$$y = \frac{8a^3}{x^2 + 4a^2}. \tag{2}$$

Several conclusions follow from equation (2). First, as $x \to \pm\infty$, $y \to 0$, showing that the x-axis is a horizontal asymptote of the witch. Secondly, using calculus one can show that the area between the witch and its asymptote is $4\pi a^2$, or four times the area of the generating circle.[4] It can also be shown—either directly from equation (2) or from the parametric equations (1)—that the witch has two inflection points (points where the curve changes its concavity), located at $\theta = \pm\pi/6$. The calculation is a bit lengthy but otherwise straightforward, and we will omit it.[5]

As already mentioned, the “witch” did not originate with Agnesi; it was already known to Pierre Fermat (1601–1665), and Luigi Guido Grandi (1671–1742), a professor of mathematics at the University of Pisa, gave it the name *versiera* (from the

Latin *vertere*, to turn). It so happened, however, that a similar-sounding Italian word, *avversiera* means a female devil or devil's wife. According to D. J. Struik, "Some wit in England once translated it 'witch,' and the silly pun is still lovingly preserved in most of our textbooks in the English language."[6] So Grandi's *versiera* became "Agnesi's witch." It is somewhat of a mystery why this particular curve, which rarely shows up in applications, has interested mathematicians for so long.[7] Its strange name may have something to do with it, or perhaps it was Agnesi's role in making the curve known.

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NOTES AND SOURCES

1. This figure is based on the Annual *AMS-IMS-MAA* Survey, Notices of the American Mathematical Society, Fall 1993.

2. A good source on women scientists is Marilyn Bailey Ogilvie, *Women in Science—Antiquity through the Nineteenth Century: A Biographical Dictionary with Annotated Bibliography* Cambridge, Mass.: MIT Press, 1988). See also Lynn M. Osen, *Women in Mathematics* (1974; rpt. Cambridge, Mass.: MIT Press, 1988), and *Women of Mathematics: A Bibliographical Sourcebook*, ed. Louise S. Grinstein and Paul J. Campbell (New York: Greenwood Press, 1987).

3. The biographical details in this chapter are adapted from the *DSB*, vol. 1, pp. 75–77. See also Ogilvie, *Women in Science*, pp. 26–28.

4. This follows from the formula

$$2\int_0^\infty y\,dx = 8a^2\tan^{-1}(x/2a)\Big|_0^\infty = 4\pi a^2,$$

where $\tan^{-1}$ is the inverse tangent (or arctangent) function; we have used the fact that the witch is symmetric about the y-axis.

5. Some additional properties of the witch can be found in Robert C. Yates, *Curves and their Properties* (Reston, Va.: National Council of Teachers of Mathematics, 1974), pp. 237–238.

6. *A Source Book in Mathematics: 1200–1800* (Cambridge, Mass.: Harvard University Press, 1969), pp. 178–180. According to this source, the first to use the name "witch" in this sense may have been B. Williamson in his *Integral Calculus* (1875). Yates (in *Curves*, p. 237) has a different version of the evolution of the name "witch": "It seems Agnesi confused the old Italian word 'versorio' (the name given to the curve by Grandi), which means 'free to move in any direction', with 'versiera', which means 'goblin', 'bugaboo', 'Devil's wife', etc."

7. The curve does show up in probability theory as the Cauchy distribution $f(x) = 1/\pi(1 + x^2)$, whose equation, apart from the constants, is identical with that of the witch.