
Contents

1	Introduction	1
2	Preliminaries	8
2.1	Definable sets	8
2.2	Pro-definable and ind-definable sets	9
2.3	Definable types	14
2.4	Stable embeddedness	17
2.5	Orthogonality to a definable set	17
2.6	Stable domination	20
2.7	Review of ACVF	24
2.8	Γ -internal sets	26
2.9	Orthogonality to Γ	29
2.10	\widehat{V} for stable definable V	31
2.11	Decomposition of definable types	32
2.12	Pseudo-Galois coverings	35
3	The space \widehat{V} of stably dominated types	37
3.1	\widehat{V} as a pro-definable set	37
3.2	Some examples	39
3.3	The notion of a definable topological space	41
3.4	\widehat{V} as a topological space	42
3.5	The affine case	42
3.6	Simple points	46
3.7	v -open and g -open subsets, $v+g$ -continuity	47
3.8	Canonical extensions	50
3.9	Paths and homotopies	52
3.10	Good metrics	54
3.11	Zariski topology	55
3.12	Schematic distance	56
4	Definable compactness	57
4.1	Definition of definable compactness	57
4.2	Characterization of definable compactness	57

5	A closer look at the stable completion	70
5.1	$\widehat{\mathbb{A}^n}$ and spaces of semi-lattices	70
5.2	A representation of $\widehat{\mathbb{P}^n}$	73
5.3	Relative compactness	74
6	Γ-internal spaces	76
6.1	Preliminary remarks	76
6.2	Topological structure of Γ -internal subsets	78
6.3	Guessing definable maps by regular algebraic maps	83
6.4	Relatively Γ -internal subsets	86
7	Curves	92
7.1	Definability of \widehat{C} for a curve C	92
7.2	Definable types on curves	94
7.3	Lifting paths	95
7.4	Branching points	98
7.5	Construction of a deformation retraction	101
8	Strongly stably dominated points	104
8.1	Strongly stably dominated points	104
8.2	A Bertini theorem	107
8.3	Γ -internal sets and strongly stably dominated points	113
8.4	Topological properties of $V^\#$	115
9	Specializations and $\text{ACV}^2\mathbf{F}$	119
9.1	g-topology and specialization	119
9.2	v-topology and specialization	122
9.3	$\text{ACV}^2\mathbf{F}$	126
9.4	The map $R_{21}^{20} : \widehat{V}_{20} \rightarrow \widehat{V}_{21}$	130
9.5	Relative versions	133
9.6	g-continuity criterion	134
9.7	Some applications of the continuity criteria	135
9.8	The v-criterion on \widehat{V}	137
9.9	Definability of v- and g-criteria	140
10	Continuity of homotopies	142
10.1	Preliminaries	142
10.2	Continuity on relative \mathbb{P}^1	145
10.3	The inflation homotopy	147
10.4	Connectedness and the Zariski topology	150
11	The main theorem	154
11.1	Statement	154
11.2	Proof of Theorem 11.1.1: Preparation	156
11.3	Construction of a relative curve homotopy	161

11.4	The base homotopy	164
11.5	The tropical homotopy	165
11.6	End of the proof	171
11.7	Variation in families	174
12	The smooth case	177
12.1	Statement	177
12.2	Proof and remarks	178
13	An equivalence of categories	183
13.1	Statement of the equivalence of categories	183
13.2	Proof of the equivalence of categories	184
13.3	Remarks on homotopies over imaginary base sets	186
14	Applications to the topology of Berkovich spaces	187
14.1	Berkovich spaces	187
14.2	Retractions to skeleta	193
14.3	Finitely many homotopy types	196
14.4	More tame topological properties	200
14.5	The lattice completion	201
14.6	Berkovich points as Galois orbits	203
	Bibliography	207
	Index	211
	List of notations	215