
Contents

Chapter 1	Introduction	1
1.1	Newton Polyhedra Associated with ϕ , Adapted Coordinates, and Uniform Estimates for Oscillatory Integrals with Phase ϕ	5
1.2	Fourier Restriction in the Presence of a Linear Coordinate System That Is Adapted to ϕ	10
1.3	Fourier Restriction When No Linear Coordinate System Is Adapted to ϕ —the Analytic Case	11
1.4	Smooth Hypersurfaces of Finite Type, Condition (R), and the General Restriction Theorem	17
1.5	An Invariant Description of the Notion of r -Height.	23
1.6	Organization of the Monograph and Strategy of Proof	24
Chapter 2	Auxiliary Results	29
2.1	Van der Corput–Type Estimates	30
2.2	Airy-Type Integrals	31
2.3	Integral Estimates of van der Corput Type	34
2.4	Fourier Restriction via Real Interpolation	38
2.5	Uniform Estimates for Families of Oscillatory Sums	40
2.6	Normal Forms of ϕ under Linear Coordinate Changes When $h_{\text{in}}(\phi) < 2$	46
Chapter 3	Reduction to Restriction Estimates near the Principal Root Jet	50
Chapter 4	Restriction for Surfaces with Linear Height below 2	57
4.1	Preliminary Reductions by Means of Littlewood-Paley Decompositions	58
4.2	Restriction Estimates for Normalized Rescaled Measures When $2^{2j}\delta_3 \lesssim 1$	64
Chapter 5	Improved Estimates by Means of Airy-Type Analysis	75
5.1	Airy-Type Decompositions Required for Proposition 4.2(c)	76
5.2	The Endpoint in Proposition 4.2(c): Complex Interpolation	85
5.3	Proof of Proposition 4.2(a), (b): Complex Interpolation	96

Chapter 6	The Case When $h_{\text{lin}}(\phi) \geq 2$: Preparatory Results	105
6.1	The First Domain Decomposition	107
6.2	Restriction Estimates in the Transition Domains E_l When $h_{\text{lin}}(\phi) \geq 2$	109
6.3	Restriction Estimates in the Domains D_l , $l < l_{\text{pr}}$, When $h_{\text{lin}}(\phi) \geq 2$	115
6.4	Restriction Estimates in the Domain D_{pr} When $h_{\text{lin}}(\phi) \geq 5$	123
6.5	Refined Domain Decomposition of D_{pr} : The Stopping-Time Algorithm	125
Chapter 7	How to Go beyond the Case $h_{\text{lin}}(\phi) \geq 5$	131
7.1	The Case When $h_{\text{lin}}(\phi) \geq 2$: Reminder of the Open Cases	132
7.2	Restriction Estimates for the Domains $D'_{(1)}$: Reduction to Normalized Measures ν_δ	135
7.3	Removal of the Term $y_2^{B-1} b_{B-1}(y_1)$ in (7.7)	138
7.4	Lower Bounds for $h^r(\phi)$	141
7.5	Spectral Localization to Frequency Boxes Where $ \xi_j \sim \lambda_j$: The Case Where Not All λ_j s Are Comparable	143
7.6	Interpolation Arguments for the Open Cases Where $m = 2$ and $B = 2$ or $B = 3$	151
7.7	The case where $\lambda_1 \sim \lambda_2 \sim \lambda_3$	169
7.8	The case where $B = 5$	178
7.9	Collecting the Remaining Cases	178
7.10	Restriction Estimates for the Domains $D'_{(l)}$, $l \geq 2$	179
Chapter 8	The Remaining Cases Where $m = 2$ and $B = 3$ or $B = 4$	181
8.1	Preliminaries	183
8.2	Refined Airy-Type Analysis	185
8.3	The Case Where $\lambda_\rho(\tilde{\delta}) \lesssim 1$	189
8.4	The Case Where $m = 2$, $B = 4$, and $A = 1$	197
8.5	The Case Where $m = 2$, $B = 4$, and $A = 0$	204
8.6	The Case Where $m = 2$, $B = 3$, and $A = 0$: What Still Needs to Be Done	212
8.7	Proof of Proposition 8.12(a): Complex Interpolation	217
8.8	Proof of Proposition 8.12(b): Complex Interpolation	233
Chapter 9	Proofs of Propositions 1.7 and 1.17	244
9.1	Appendix A: Proof of Proposition 1.7 on the Characterization of Linearly Adapted Coordinates	244
9.2	Appendix B: A Direct Proof of Proposition 1.17 on an Invariant Description of the Notion of r -Height	245
Bibliography		251
Index		257