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**Dean Corbae, Maxwell B. Stinchcombe & Juraj Zeman: An Introduction to Mathematical Analysis for Economic Theory and Econometrics**

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