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*Preface*

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- concept of mathematical beauty
- equations, identities, and theorems
- mathematical ugliness
- beauty redux

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*(an assortment of essays beyond the elementary involving complex numbers)*

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