
Contents

Preface to the Paperback Edition	xiii
What This Book Is About, What You Need to Know to Read It, and WHY You Should Read It	xxv

Preface

“When Did Math Become Sexy?”	xxix
------------------------------	------

Introduction 1

- concept of mathematical beauty
- equations, identities, and theorems
- mathematical ugliness
- beauty redux

Chapter 1. Complex Numbers

(an assortment of essays beyond the elementary involving complex numbers)

1.1 The “mystery” of $\sqrt{-1}$	13
1.2 The Cayley-Hamilton and De Moivre theorems	19
1.3 Ramanujan sums a series	27
1.4 Rotating vectors and negative frequencies	33
1.5 The Cauchy-Schwarz inequality and falling rocks	38
1.6 Regular n -gons and primes	43
1.7 Fermat’s last theorem, and factoring complex numbers	53
1.8 Dirichlet’s discontinuous integral	63

Chapter 2. Vector Trips

(some complex plane problems in which direction matters)

2.1 The generalized harmonic walk	68
2.2 Birds flying in the wind	71
2.3 Parallel races	74
2.4 Cat-and-mouse pursuit	84
2.5 Solution to the running dog problem	89

Chapter 3. The Irrationality of π^2

(“higher” math at the sophomore level)

3.1 The irrationality of π	92
3.2 The $R(x) = B(x)e^x + A(x)$ equation, D-operators, inverse operators, and operator commutativity	95
3.3 Solving for $A(x)$ and $B(x)$	102
3.4 The value of $R(\pi i)$	106
3.5 The last step (at last!)	112

Chapter 4. Fourier Series

*(named after Fourier but Euler was there first—but he was, alas,
partially WRONG!)*

4.1 Functions, vibrating strings, and the wave equation	114
4.2 Periodic functions and Euler’s sum	128
4.3 Fourier’s theorem for periodic functions and Parseval’s theorem	139
4.4 Discontinuous functions, the Gibbs phenomenon, and Henry Wilbraham	163
4.5 Dirichlet’s evaluation of Gauss’s quadratic sum	173
4.6 Hurwitz and the isoperimetric inequality	181

Chapter 5. Fourier Integrals

(what happens as the period of a periodic function becomes infinite, and other neat stuff)

5.1 Dirac's impulse "function"	188
5.2 Fourier's integral theorem	200
5.3 Rayleigh's energy formula, convolution, and the autocorrelation function	206
5.4 Some curious spectra	226
5.5 Poisson summation	246
5.6 Reciprocal spreading and the uncertainty principle	253
5.7 Hardy and Schuster, and their optical integral	263

Chapter 6. Electronics and $\sqrt{-1}$

(technological applications of complex numbers that Euler, who was a practical fellow himself, would have loved)

6.1 Why this chapter is in this book	275
6.2 Linear, time-invariant systems, convolution (again), transfer functions, and causality	275
6.3 The modulation theorem, synchronous radio receivers, and how to make a speech scrambler	289
6.4 The sampling theorem, and multiplying by sampling and filtering	302
6.5 More neat tricks with Fourier transforms and filters	305
6.6 Single-sided transforms, the analytic signal, and single-sideband radio	309

<i>Euler: The Man and the Mathematical Physicist</i>	324
<i>Notes</i>	347
<i>Acknowledgments</i>	375
<i>Index</i>	377