Introduction

Holomorphic dynamics is one of the earliest branches of dynamical systems which is not part of classical mechanics. As a prominent field in its own right, it was founded in the classical work of Fatou and Julia (see [Fa1, Fa2] and [J]) early in the 20th century. For some mysterious reason, it was then almost completely forgotten for 60 years. The situation changed radically in the early 1980s when the field was revived and became one of the most active and exciting branches of mathematics. John Milnor was a key figure in this revival, and his fascination with holomorphic dynamics helped to make it so prominent. Milnor's book *Dynamics in One Complex Variable* [M8], his volumes of collected papers [M10, M11], and the surveys [L1, L5] are exemplary introductions into the richness and variety of Milnor's work in dynamics.

Holomorphic dynamics, in the sense we will use the term here, studies iterates of holomorphic maps on complex manifolds. Classically, it focused on the dynamics of rational maps of the Riemann sphere $\widehat{\mathbb{C}}$. For such a map f, the Riemann sphere is decomposed into two invariant subsets, the *Fatou set* $\mathcal{F}(f)$, where the dynamics is quite tame, and the *Julia set* $\mathcal{J}(f)$, which often has a quite complicated fractal structure and supports chaotic dynamics.

Even in the case of quadratic polynomials $Q_c\colon z \mapsto z^2 + c$, the dynamical picture is extremely intricate and may depend on the parameter c in an explosive way. The corresponding bifurcation diagram in the parameter plane is called the *Mandelbrot set*; its first computer images appeared in the late 1970s, sparking an intense interest in the field [BrMa, Man].

The field of holomorphic dynamics is rich in interactions with many branches of mathematics, such as complex analysis, geometry, topology, number theory, algebraic geometry, combinatorics, and measure theory. The present book is a clear example of such interplay.

The papers **"Arithmetic of Unicritical Polynomial Maps"** and **"Les racines de composantes hyperboliques de M sont des quarts d'entiers algébriques,"** which open this volume,[1] exemplify the interaction of holomorphic dynamics with number theory. In these papers, John Milnor and Thierry Bousch study number-theoretic properties of the family of polynomials $p_c(z) = z^n + c$, whose bifurcation diagram is known as the *Multibrot set*.

In the celebrated Orsay Notes [DH1], Douady and Hubbard undertook a remarkable combinatorial investigation of the Mandelbrot set and the corresponding bifurcations of the Julia sets. In particular, they realized (using important contributions from Thurston's work [T]) that these fractal sets admit an explicit topological model as long as they are locally connected (see [D]). This led to the most famous conjecture in the field, on the local connectivity of the Mandelbrot set, typically

[1] Both these papers were originally written circa 1996 but never published. Milnor's paper is a follow-up to Bousch's note, but it was significantly revised by the author for this volume.

abbreviated as *MLC*. The MLC conjecture is still currently open, but it has led to many important advances, some of which are reflected in this volume.

In his thesis [La], Lavaurs proved the non-local-connectivity of the cubic connectedness locus, highlighting the fact that the degree two case is special in this respect. In attempt to better understand this phenomenon, Milnor came across a curious new object that he called the *tricorn*: the connectedness locus of antiholomorphic quadratic maps $q_c(z) = \bar{z}^2 + c$. In the paper **"Multicorns are not path connected,"** John Hubbard and Dierk Schleicher take a close look at the connectedness locus of its higher degree generalization, defined by $p_c(z) = \bar{z}^n + c$.

The paper by Alexandre Dezotti and Pascale Roesch, **"On (non-)local connectivity of some Julia sets,"** surveys the problem of local connectivity of Julia sets. It collects a variety of results and conjectures on the subject, both "positive" and "negative" (as Julia sets sometimes fail to be locally connected). In particular, in this paper the reader can learn about the work of Yoccoz [H, M7], Kahn and Lyubich [KL], and Kozlovski, Shen, and van Strien [KSvS]; the latter gives a positive answer in the case of "non-renormalizable" polynomials of any degree.

Related to connectivity, an important question that has interested both complex and algebraic dynamicists is that of the irreducibility of the closure of X_n, the set of points $(c, z) \in \mathbb{C}^2$ for which z is periodic under $Q_c(z) = z^2 + c$ with minimal period n. These curves are known as *dynatomic curves*. The irreducibility of such curves was proved by Morton [Mo] using algebraic methods, by Bousch [Bou] using algebraic and analytic (dynamical) methods, and by Lau and Schleicher [LS], using only dynamical methods. In the paper **"The quadratic dynatomic curves are smooth and irreducible,"** Xavier Buff and Tan Lei present a new proof of this result based on the *transversality theory* developed by Adam Epstein [E].

Similarly, in the case of the family of cubic polynomial maps with one marked critical point, parametrized by the equation $F(z) = z^3 - 3a^2z + (2a^3 + v)$, one can study the *period p-curves* $\mathcal{S}_p$ for $p \geq 1$. These curves are the collection of parameter pairs $(a, v) \in \mathbb{C}^2$ for which the marked critical point a has period exactly p; Milnor proved that $\mathcal{S}_p$ is smooth and affine for all $p > 0$ and irreducible for $p \leq 3$ [M9]. The computation of the Euler characteristic for any $p > 0$ and the irreducibility for $p = 4$ were proved by Bonifant, Kiwi and Milnor [BKM]. The computation of the Euler characteristic requires a deep study of the unbounded hyperbolic components of $\mathcal{S}_p$, known as *escape regions*. Important information about the limiting behavior of the periodic critical orbit as the parameter tends to infinity within an escape region is encoded in an associated *leading monomial vector*, which uniquely determines the escape region, as Jan Kiwi shows in **"Leading monomials of escape regions."**

As we have alluded to previously, a locally connected Julia set admits a precise topological model, due to Thurston, by means of a *geodesic lamination* in the unit disk. This model can be efficiently described in terms of the *Hubbard tree*, which is the "core" that encodes the rest of the dynamics. In particular, it captures all the cut-points of the Julia set, which generate the lamination in question. This circle of ideas is described and is carried further to a more general topological setting in the paper by Alexander Blokh, Lex Oversteegen, Ross Ptacek and Vladlen Timorin **"Dynamical cores of topological polynomials."**

The realm of general *rational dynamics* on the Riemann sphere is much less explored than that of polynomial dynamics. There is, however, a beautiful bridge connecting these two fields called *mating*: a surgery introduced by Douady and

Hubbard in the 1980s, in which the filled Julia sets of two polynomials of the same degree are dynamically related via external rays. In many cases this process produces a rational map. It is a difficult problem to decide when this surgery works and which rational maps can be obtained in this way. A recent breakthrough in this direction was achieved by Daniel Meyer, who proved that in the case when f is postcritically finite and the Julia set of f is the whole Riemann sphere, every sufficiently high iterate of the map can be realized as a mating [Me1, Me2]. In the paper **"Unmating of rational maps, sufficient criteria and examples,"** Meyer gives an overview of the current state of the art in this area of research, illustrating it with many examples. He also gives a sufficient condition for realizing rational maps as the mating of two polynomials.

Another way of producing rational maps is by "singular" perturbations of complex polynomials. In the paper **"Limiting behavior of Julia sets of singularly perturbed rational maps,"** Robert Devaney surveys dynamical properties of the families $f_{c,\lambda}(z) = z^n + c + \lambda/z^d$ for $n \geq 2$, $d \geq 1$, with c corresponding to the center of a hyperbolic component of the Multibrot set. These rational maps produce a variety of interesting Julia sets, including *Sierpinski carpets* and *Sierpinski gaskets*, as well as laminations by Jordan curves. In the current article, the author describes a curious "implosion" of the Julia sets as a polynomial $p_c = z^n + c$ is perturbed to a rational map $f_{c,\lambda}$.

There is a remarkable *phase-parameter relation* between the dynamical and parameter planes in holomorphic families of rational maps. It first appeared in the early 1980s in the context of quadratic dynamics in the Orsay notes [DH1] and has become a very fruitful philosophy ever since. In the paper **"Perturbations of weakly expanding critical orbits,"** Genadi Levin establishes a precise form of this relation for rational maps with one critical point satisfying the *summability condition* (certain expansion rate assumption along the critical orbit). This result brings to a natural general form many previously known special cases studied over the years by many people, including the author.

One of the most profound achievements in holomorphic dynamics in the early 1980s was *Thurston's topological characterization of rational maps*, which gives a combinatorial criterion for a postcritically finite branched covering of the sphere to be realizable (in a certain homotopical sense) as a rational map (see [DH2]). A wealth of new powerful ideas from hyperbolic geometry and Teichmüller theory were introduced to the field in this work. The *Thurston Rigidity Theorem*, which gives uniqueness of the realization, although only a small part of the theory, already is a major insight, with many important consequences for the field (some of which are mentioned later).

Attempts to generalize Thurston's characterization to the transcendental case faces many difficulties. However, in the exponential family $z \mapsto e^{\lambda z}$, they were overcome by Hubbard, Schleicher and Shishikura [HSS]. In the paper **"A framework towards understanding the characterization of holomorphic dynamics,"** Yunping Jiang surveys these and further results, which, in particular, extend the theory to a certain class of postcritically infinite maps. His paper includes an appendix by the author, Tao Chen, and Linda Keen that proposes applications of the ideas developed on the survey to the characterization problem for certain families of quasi-entire and quasi-meromorphic functions.

The field of *real one-dimensional dynamics* emerged from obscurity in the mid-1970s, largely due to the seminal work by Milnor and Thurston [MT], where they laid down foundations of the combinatorial theory of one-dimensional dynamics, called *kneading theory*. To any piecewise monotone interval map f, the authors associated a topological invariant (determined by the ordering of the critical orbits on the line) called the *kneading invariant*, which essentially classifies the maps in question. Another important invariant, the *topological entropy* $h(f)$ (which measures "the complexity" of a dynamical system) can be read off from the kneading invariant. One of the conjectures posed in the preprint version of [MT] was that in the real quadratic family $f_a : x \mapsto ax(1-x)$, $a \in (0, 4]$, the topological entropy depends monotonically on a. This conjecture was proved in the final version using methods of holomorphic dynamics (the Thurston Rigidity Theorem alluded to earlier). This was the first occasion that demonstrated how fruitful complex methods could be in real dynamics. Much more was to come: see, e.g., [L4], a recent survey on this subject.

Later on, Milnor posed the general *monotonicity conjecture* [M6] (compare [DGMTr]) asserting that *in the family of real polynomials of any degree, isentropes are connected* (where an *isentrope* is the set of parameters with the same entropy). This conjecture was proved in the cubic case by Milnor and Tresser [MTr], and in the general case by Bruin and van Strien [BvS]. In the survey **"Milnor's conjecture on monotonicity of topological entropy: Results and questions,"** Sebastian van Strien discusses the history of this conjecture, gives an outline of the proof in the general case, and describes the state of the art in the subject. The proof makes use of an important result by Kozlovski, Shen, and van Strien [KSvS] on the density of hyperbolicity in the space of real polynomial maps, which is a far-reaching generalization of the Thurston Rigidity Theorem. (In the quadratic case, density of hyperbolicity had been proved in [L3, GrSw].) The article concludes with a list of open problems.

The paper **"Entropy in dimension one"** is one of the last papers written by William Thurston and occupies a special place in this volume. Sadly, Bill Thurston passed away in 2012 before finishing this work. In this paper, Thurston studies the topological entropy h of postcritically finite one-dimensional maps and, in particular, the relations between dynamics and arithmetics of e^h, presenting some amazing constructions for maps with given entropy and characterizing what values of entropy can occur for postcritically finite maps. In particular, he proves: *h is the topological entropy of a postcritically finite interval map if and only if $h = \log \lambda$, where $\lambda \geq 1$ is a weak Perron number, i.e., it is an algebraic integer, and $\lambda \geq |\lambda^\sigma|$ for every Galois conjugate $\lambda^\sigma \in \mathbb{C}$.*

The editors received significant help from many people in preparing this paper for publication, to whom we are very grateful. Among them are M. Bestvina, M. Handel, W. Jung, S. Koch, D. Lind, C. McMullen, L. Mosher, and Tan Lei. We are especially grateful to John Milnor, who carefully studied Bill's manuscript, adding a number of notes which clarify many of the points mentioned in the paper.

In the mid-1980s, Milnor wrote a short conceptual article *"On the concept of attractor"* [M5] that made a substantial impact on the field of real one-dimensional dynamics. In this paper Milnor proposed a general notion of *measure-theoretic attractor*, illustrated it with the *Feigenbaum attractor*, and formulated a problem of existence of *wild attractors* in dimension one. Such an attractor would be a Cantor set that attracts almost all orbits of some topologically transitive periodic interval. It turns out that the answer depends on the degree: in the quadratic case, there are no wild attractors [L2], while they can exist for higher degree unicritical maps $x \mapsto x^d + c$ [BKNvS]. This work made use of the idea of *random walk*, which describes transitions between various dynamical scales. In the paper **"Metric stability for random walks (with applications in renormalization theory),"** by Carlos Moreira and Daniel Smania, this idea was carried further to prove a surprising *rigidity* result: the conjugacy between two unimodal maps of the same degree with Feigenbaum or wild attractors is *absolutely continuous*.

One of the central results of classical *local* one-dimensional holomorphic dynamics is the *Leau-Fatou Flower Theorem* describing the local dynamics near a parabolic point (see [M8]). A natural problem is to develop a similar theory in higher dimensions. In the late 1990s important results in this direction were obtained by M. Hakim [Ha1, Ha2], but unfortunately, they are only partially published. In the paper **"On Écalle-Hakim's theorems in holomorphic dynamics,"** Marco Arizzi and Jasmin Raissy give a detailed technical account of these advances. This paper is followed by a note, **"Index theorems for meromorphic self-maps of the projective space,"** by Marco Abate, in which local techniques are used to prove three index theorems for global meromorphic maps of projective space.

The field of *global multivariable holomorphic dynamics* took shape in the early 1990s. It was pioneered by the work of Friedland and Milnor [FrM], Hubbard and Oberste-Vorth [HOV], Bedford and Smillie [BS1], [BS2], [BS3], Bedford, Lyubich, and Smillie [BLS1], [BLS2], Fornæss and Sibony [FS1], [FS2], [FS3], followed by many others. It focused on the dynamics of complex Hénon automorphisms,

$$f: \mathbb{C}^2 \to \mathbb{C}^2, \quad (x, y) \mapsto (x^2 + c + by,\, x)$$

and holomorphic endomorphism of projective spaces. It revealed a beautiful interplay between ideas coming from dynamics (such as *entropy*, *Lyapunov exponents*, *Pesin boxes*, ...), algebraic geometry (such as *dynamical degree*, *Bezout's theorem*, *blowups*, ...), and complex analysis (such as *pluripotential theory of currents*). In time, this theory was extended to a more general setting on complex manifolds.

The paper of Friedland and Milnor [FrM] made clear that in $\mathbb{C}^2$, the only interesting polynomial automorphisms are those that are conjugate to a Hénon automorphism or to compositions of Hénon automorphisms. In more generality, automorphisms of compact complex surfaces X with non-trivial dynamics have been classified by Cantat [C] and Gizatulin [Gi]. In [C], Cantat shows that if X is not Kähler, then the automorphisms of X have no interesting dynamics (in particular, X has topological entropy zero). He classifies the surfaces having *automorphisms with positive entropy* as *complex tori, K3 surfaces* (i.e., simply connected Kähler surfaces with a nonvanishing holomorphic 2-form), *Enriques surfaces* (i.e., quotients of K3 surfaces by a fixed-point free involution) or *non-minimal rational surfaces*. In the survey **"Dynamics of automorphisms of compact complex surfaces,"** Serge Cantat describes these and further developments in the study of the dynamics of automorphisms with positive entropy. In particular, he carries to this general setting the interplay between ideas from ergodic and pluripotential theories.

This interplay turned out to be important even for one-dimensional holomorphic dynamics since the parameter spaces of polynomials of degree > 2 (as well as the spaces of rational maps of any degree) are higher dimensional. In fact, this is one of the reasons why the bifurcations in the quadratic family $Q_c(z) = z^2 + c$ (described by the Mandelbrot set) are much better understood than the bifurcations in the space of cubic polynomials. An efficient approach to the problem appeared in the work of DeMarco who constructed a *bifurcation current* in any holomorphic family of one-dimensional rational maps [De]. (In the quadratic case, this current is just the harmonic measure on the boundary of the Mandelbrot set.) This construction was then generalized to higher-dimensional endomorphisms by Bassanelli and Berteloot [BaBe]. In the survey **"Bifurcation currents and equidistribution in parameter space,"** Romain Dujardin presents an overview of this circle of ideas and then describes his recent work with Bertrand Deroin, where the bifurcation current is constructed for holomorphic families of groups of Möbius transformations. This adds a new interesting line to *Sullivan's dictionary* between rational maps and Kleinian groups.

A *(singular) holomorphic lamination* by holomorphic curves on a complex manifold is called *hyperbolic* if every leaf of the foliation is a *hyperbolic Riemann surface*. Each leaf of such a lamination is endowed with the canonical hyperbolic metric, but there is no a priori knowledge of how this metric depends on the transverse parameter. Starting with the work of Verjovsky [V], this important issue has been extensively studied by many authors, see [Gh], [Ca], [CaGM], [Glu], [Lin] and [FS4]. In the series of two papers **"Entropy for hyperbolic Riemann surface laminations I and II,"** Tien-Cuong Dinh, Viet-Anh Nguyên, and Nessim Sibony estimate the modulus of transverse continuity for these metrics. Then they apply these results to prove finiteness of the geometric entropy for singular foliations with linearizable singularities.

A notion of geometric entropy for regular Riemannian foliations was introduced by Ghys, Langevin, and Walczak [GhLnW]: roughly speaking, it measure the exponential rate at which nearby leaves diverge. They proved finiteness of the entropy and showed that if the entropy vanishes then the foliations admit a transverse measure. In the preceeding articles, Dinh, Nguyên, and Sibony modify the definition to make it suitable for singular laminations, and prove the finiteness result for this difficult setting.

Any real closed k-current on a compact manifold M^n, considered as a linear functional on the space of differential forms, naturally gives rise to a homology class in $H_{n-k}(M, \mathbb{R})$. When we have an oriented lamination $\mathcal{S}$ in M (also called a *solenoid*) endowed with a transverse measure μ, we obtain a *geometric current* by integrating forms along the leaves of $\mathcal{S}$ and then averaging over μ. The homology classes obtained this way are invariants of the solenoid. For flows, they appeared in the work of Schwartzman [Sc] under the name of *asymptotic cycles*. A general notion is due to Ruelle and Sullivan [RS]. (Note that these geometric ideas played an important motivational role in the study of currents in the higher-dimensional holomorphic dynamics discussed earlier.)

In a series of papers [MuPM1, MuPM2], Vincente Muñoz and Ricardo Pérez-Marco undertook a systematic study of this interplay between dynamics and algebraic topology. To this end, they refine a notion of a geometric current: impose a suitable transverse regularity and allow the solenoids to be *immersed.* Their paper, **"Intersection theory for ergodic solenoids,"** in this volume develops the geometric intersection theory for these objects.

A broad range of applications of the idea of asymptotic cycle to topology and physics (including Donaldson, Jones, and Seiberg-Witten invariants for foliations) is discussed in the paper **"Invariants of four-manifolds with flows via cohomological field theory,"** by Hugo García-Compeán, Roberto Santos-Silva and Alberto Verjovsky.

The final part of this volume is devoted to two areas of *geometry* and *algebra* strongly influenced by earlier work of John Milnor. At the early stage they were not directly related to holomorphic dynamics, but some deep connections were discovered more recently.

One of Milnor 's impacts on contemporary mathematics lay in his careful treatment of various topological, combinatorial, and geometric *structures*, discovering striking distinctions and relations between them. His discovery of exotic spheres and his counterexample to Hauptvermutung are celebrated examples of this kind.

Less known to general mathematical public is Milnor's role in developing the theory of flat bundles and affine manifolds. This is the theme of William Goldman's article **"Two papers which changed my life: Milnor's seminal work on flat manifolds and bundles."** Specifically, these are *"On the existence of a connection with curvature zero"* [M1], which deeply influenced the theory of characteristic classes of flat bundles, and *"On fundamental groups of complete affinely flat manifolds"* [M4], which clarified the theory of affine manifolds, setting the stage for its future flourishing. Goldman's article describes the history of the *Milnor-Wood inequality* and the *Auslander Conjecture* and then proceeds to more recent developments, including a description of *Margulis space-times*, a startling example of an affine 3-manifold with free fundamental group.

Our volume concludes with Rostislav Grigorchuk's survey **"Milnor's problem on the growth of groups and its consequences."** The notion of *group growth* first appeared in 1955 in a paper of A. S. Schwarz [Sch], but it remained virtually unnoticed for over a decade. The situation changed after Milnor's papers from 1968 [M2, M3], which sparked significant interest in this area. Particularly influential were two problems raised in these papers: the characterization of groups of polynomial growth (Milnor conjectured that they must be virtually nilpotent) and the question of the existence of groups of intermediate growth (in between polynomial and exponential).

Both problems were solved in the early 1980s: Gromov proved Milnor's Conjecture on groups of polynomial growth [Gro] while Grigorchuk constructed examples of groups of intermediate growth [G1, G2]. These breakthroughs led to exciting advances in the past 30 years that changed the face of *Combinatorial Group Theory*. Grigorchuk's survey presents a broad picture of the area, and suggests a number of further interesting problems and directions of research.

Let us mention one relation between this area and holomorphic dynamics that was discovered recently. To a rational endomorphism f of the Riemann sphere, one can associate an *iterated monodromy group* that describes the covering properties of all the iterates of f. One way to define this group is as the holonomy of the affine lamination constructed in [LMin], a more algebraic approach was proposed by Nekrashevych in [N]. These groups exhibit many very interesting algebraic, combinatorial, and geometric properties (see [BGN]); in particular, it turned out that some of them have intermediate growth [BuPe].

We hope this volume gives a glimpse into one beautiful field of research that was strongly influenced by Milnor's work. Read and enjoy!

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