Welcome to reading the fifth anthology in our series of recent writings on mathematics. Almost all the pieces included here were first published during 2013 in periodicals, as book chapters, or online.

Much is written about mathematics these days, and much of it is good, amid a fair amount of uninspired writing. What does and what doesn’t qualify as good is sometimes difficult to decide. I have learned that strictly normative criteria for what “good writing” on mathematics should be are of little utility, and in fact, might be counterproductive. Good writing comes in many styles. I know it when I see it, and I might see it differently from the way you see it. Making a selection called “the best” is inevitably subjective and inevitably doubles into an elimination procedure—a combination of successive positive and negative evaluations, always comparative, guided by the preferences, competence, and probity of the people involved, as well as by publishing and time constraints. I have described elsewhere how we reach the table of contents each year, but I find it useful to retell briefly the main steps of the process.

Driven by curiosity and by personal interest, I survey an enormous amount of academic and popular literature. I have done so for decades, even when access to bibliographic sources was much more difficult than it is today. I enjoy pondering what people say, making up my mind about what I read, and making up my mind independently of what I read. When pieces have a chance to be included in our anthology I take note and I put them aside, coming up with many titles each year (see the Notable Writings section at the end of the volume). From this broad collection I leave out pieces too long for the economy of our book or unaffordable because of copyright or reprinting issues. At this stage I usually look at a few dozen articles and I advance them to the publisher.
Princeton University Press asks independent reviewers to read and rate the articles, a procedure meant to guide us further. The reviewers always agree on some pieces but sharply disagree on others. When I consider the reports I also have in mind some goals I envisioned for the series well before I even found a publisher for this enterprise.

I want accessible but nontrivial content that presents for mathematicians and for the general public a wide assortment of informed and insightful perspectives on pure and applied mathematics, on the historical and philosophical issues related to mathematics, on topics related to the learning and the teaching of mathematics, on the practice and practicality of mathematics, on the social and institutional aspects in which mathematics grows and thrives, or on other themes related to mathematics. I aim to offer each volume as an instrument for gaining nondogmatic views of mathematics, impervious to indoctrination—as much as possible inclusive, panoramic, comprehensive, and reflecting a multitude of viewpoints that see through the apodictic nature of mathematics. No doubt some people dread this diversity of viewpoints when thinking about mathematics and adopt a defensive stance, retreating into overspecialization, frightened by opinions that upset their views on mathematics or by the specter of (what they consider to be) dilettantism. For what it is worth, *The Best Writing on Mathematics* series is meant as an antidote to the contagious power that emanates from such fears.

The final content of each book in the series is dependent on the literature available during the latest calendar year; therefore, each volume reflects to some extent the vagaries of fashion and, on the other hand, it is deprived of the topics temporarily out of favor with writers on mathematics. Thus each volume should properly be viewed in conjunction with the other volumes in the series.

Books are more important for what they make us think than for what we read in them. If we manage to destroy humanity but to leave all the books intact, there would be no consolation (except for the surviving worms and ants). Books are chances for the potential unknown of our imagination; their worth consists not only in their literal content but also in what they make us reflect—immediately or later, in accord with or in opposition to what we read. Books on mathematics, and mathematical books proper, have a special place in history. Over the past several centuries mathematical ideas, in conjunction with the enterprising élan of explorers, innovators, tinkerers, and the common folk,
have served as catalysts for uncountable discoveries (e.g., geographic, technological, scientific, military, and domestic) and for the expansion of life possibilities, thus contributing to human phenotypic diversity.

At first thought opining on mathematics seems benign, having only an upside; yet it is neither trivial nor free of danger. Writing about mathematics is a form of interpreting mathematics; and interpreting mathematics, whether “elementary” or “advanced” mathematics, is not trifling, even for seasoned mathematicians. Interpreting mathematics leaves room for genuine differences of opinion; it allows you to stand your ground even if everybody else disagrees with you—and it’s all about mathematics, a thinking domain on which people are supposed to agree, not to dissent. Private interpretation of mathematics is different from doing mathematics; it is a mark of personal worldview. In that sense it can indeed be dangerous and unsettling on occasion, but it is always rewarding as a full-mind activity free of the trappings that plague most institutionalized mathematics instruction (e.g., rote memorizing, repetitive learning, following strict rules, inside-the-box thinking, and dumb standardized assessment). Interpreting mathematics allows individuals to build niches in the social milieu by using unique thinking and acting features. For such a purpose mathematics is an inexhaustible resource, similar to but more encompassing than art. The freedom to interpret mathematics as I please compensates for the constraints inherent in the conventional content accepted by the prevailing communitarian view of mathematics. By interpreting, I recoup the range of imaginative possibilities that I gave up when submitting to the compelling rigor enforced by a chain of mathematical arguments. Interpreting mathematics supports my confidence to act not only on my supposed knowledge but also on my ignorance. It enables me to appropriate an epistemological payoff arrangement better than any other I can imagine, since my ignorance will always dwarf my hopelessly limited knowledge. Thus interpreting mathematics breaks paths toward opportunities but also opens doors to peril.

I learned firsthand, at great expense, that voicing opinions about mathematics is risky. I have seen it; I lived it. Long ago—a recent emigrant, poor and naive but buoyantly optimistic—I voiced some of my thoughts on the use and misuse of mathematics during classes at a leading business school. The displeased reaction to what I considered common-sense observations puzzled and befuddled me. To my dismay
I saw that complying, obeying, and blindly turning in assignments—that mattered, that was expected and appreciated, not hard thinking about the issues at hand. The more I spoke, the stranger the atmosphere became. For a while I lingered there surrounded by bristling silence, polite condescension, and impatient tolerance. The denouement came when a faculty member asked me why I went to that school if I thought I was so smart. The message, one of the many ensconced in that interpolation, was unmistakable: If you happen to have some ideas (and made the dumb error of coming here by borrowing a lot of money), better be careful how you handle them. The next morning I was out of there, at a staggering financial loss.

That episode had long-lasting consequences for me, highlighting the fickle rhetoric of the slogans encouraging free inquiry and open dialogue of ideas. When recurring effects and escalating misadventures added up over the ensuing years, I became a lot more cautious with what I said—yet matters kept slipping out of control, from terrible to worse, to desperate, until I chose not to say much anymore. If speaking up on mathematics proved to be such a disaster, fortunately I accomplished the next to “best” thing for me—the chance to give exposure to other people’s diverse views on mathematics, in the volumes of The Best Writing on Mathematics series.

In conclusion to this preamble, I intend to include here pieces mostly about mathematics, not necessarily mathematical writing, although a bit of mathematical exposition makes its way into each volume. Expository mathematical writing is scrutinized, recognized, and rewarded by publication in the mathematics journals, consideration for professional awards, republication in extended versions as monographs, and circulation among the mathematical community. But writings about mathematics are spread wide and thin, in venues less frequented by mathematicians; my goal is to find what is notable in that literature and to make it easy for people to read it or at least to find it on their own, quickly and conveniently. In the books of this series you will meet a deliberate medley of styles, sources, and perspectives. With this choice for the content we succeed in placing in bookstores volumes that include authors who previously were known only inside the mathematical community and in circulating among mathematicians the names of interesting authors who care about mathematics from the outside.
Contents of the Volume

In the opening piece of the selection, Stephen Pollard reviews John Dewey’s conception of human experience and notes that, in the philosopher’s views, mathematics and its practice offer an integrative function that transcends the utilitarian goals commonly bestowed on mathematics by enhancing the minds and the lives of the people who study it.

Kenneth Cukier and Viktor Mayer-Schönberger discuss some of the consequences of the recent explosion in digital information, pointing out the qualitative shift it brings to data use and analysis, and concluding that these trends are radically changing our lives, work, and thinking about the world.

Tanya Khovanova poses and solves an ingenious puzzle invented decades ago by John Conway.

In the next piece John Conway observes that some true arithmetical statements are not provable and gives unsophisticated examples that support his assertion.

Brian Hayes traces the history of space-filling curves and offers surprising applications of the counterintuitive process that leads to their construction.

Lav Varshney and John Sun observe that nonlinear, logarithmic scaling is natural to our perception (but distorted into linear scaling by institutionalized education) and contend that, by reducing estimative errors, perceiving logarithmically confers evolutionary advantages.

Keith Devlin formulates negative and positive criteria for evaluating the instructive or noninstructive qualities of games that purport to help children learn mathematics and concludes that most games currently available do not deliver on the educational side.

Bahman Kalantari and Bruce Torrence describe certain features of the “graph” of a polynomial with a complex variable; then they use this intuitive geometrical guidance to draw the main lines in the chain of arguments that proves that any polynomial can be factored into a number of linear factors equal to its degree.

Nicole Lazar reviews the virtues and the pitfalls of using statistical methods to analyze and to create art.

Carlo Séquin catalogs varieties of geometrical shapes similar to a Klein bottle (a geometrical surface with a single face) and briefly indicates the mathematical elements that can help classify them rigorously.
Also starting with a Klein bottle surface, sarah-marie belcastro tackles issues related to the knitting of mathematical objects and illustrates her comments with photographs of needlework.

Marshall Gordon details the activities he developed for teaching eleventh graders the engineering applications of the mathematical study of quadratic quantities and relates the response these activities elicited from the students, stressing that his “multiple-centered” instruction reaches all students according to their levels of interest.

Penelope Dunham brings food into the classroom, not only to facilitate the intuitive understanding of the mathematical concepts encountered in a variety of undergraduate courses she devised but also to stimulate and reward the students.

Dov and Rina Zazkis find that the wonder part of mathematics resides in doing mathematics more than in contemplating mathematical results and present examples that link the nature of wonder in mathematics to encountering surprise, counterintuition, and the unexpected.

Francis Su’s blog entry, based on a talk he gave as an awardee, is an impassioned plea for renouncing the infatuation with achievement and performance in education and focusing instead on the irreducible humanity that often goes missing in the interaction between teacher and student.

Uri Leron and Orit Zaslavsky reflect on the strengths, limitations, and educational value of generic proofs—that is, mathematical proofs based on an example that serves as a springboard for the main ideas supporting the generalization for all cases represented by the example.

John Conway and Joseph Shipman distinguish among different proofs of the same mathematical result based on proofs’ dominant features, then exemplify what they call the proof space by comparing six ways of proving that the square root of the number 2 is not a proportion of integers.

Michael Barany shows that peculiar conservative inclinations led Augustin-Louis Cauchy to fuse algebraic and geometric thinking, thus inadvertently igniting the theoretical impulse that led to the rigor of modern mathematics.

Lawrence Brenton tells the long story of the speculation concerning the shape of the universe—and how its many avatars relate to geometrical ideas, old and new.
Mark Braverman explains the structural incompatibility between real numbers, which we represent intuitively as a continuous line, and the discrete nature of calculation achievable by mechanical computations. Then he examines the implications for the study of dynamical systems.

Adilson Motter and David Campbell start off by describing the deterministic mind-set characteristic of the exact sciences until about the middle of the twentieth century and continue by tracing the inroads made over the past half-century by the new chaos paradigm and its role in dynamical systems theory.

Roberto Behar, Pere Grima, and Lluís Marco-Almagro list and comment briefly on analogies they find useful for teaching statistics to students who take an introductory course.

David Gale and Lloyd Shapley’s article is the only one in this volume that was first published long ago—although attention to it rekindled recently. Gale and Shapley found a well-determined and optimal solution to the problem of assigning people to institutions under conditions of uncertainty.

Jordan Ellenberg reports on an important theoretical breakthrough, Yitang Zhang’s proof of the bounded gaps conjecture, and muses on the consequences it might have for the theory of numbers and for our knowledge of randomness.

More Writings on Mathematics

Writing on mathematics for a nonspecialist audience is now a full-time profession for some authors and a pastime for many others. The market is huge and growing. With the caveat that the following list covers only a small part of the publishing output in this area, here are some books that came to my attention over the past year.

I start by noting a volume meant as an overview of the state of mathematics in the United States and as a benchmark for future policy directions, The Mathematical Sciences in 2025 written by a committee of the National Research Council (for accurate and complete references, please see the list of Books Mentioned at the end of the Introduction).

A book on some of the most important problems in mathematics and physics is Ian Stewart’s Visions of Infinity. Other presentations of mathematical topics and/or how they spring up in life can be found in Paul Lockhart’s Measurement, Göran Grimvall’s Quantify!, Yutaka
Nishiyama’s *The Mysterious Number 6174*, Günter Ziegler’s *Do I Count?*,

More technical books with chapters or parts accessible to nonspecialist readers are *The Tower of Hanoi* by Andreas Hinz and his colleagues, *Configurations from a Graphical Viewpoint* by Tomaž Pisanski and Brigitte Servatius, *Magnificent Mistakes in Mathematics* by Alfred Posamentier and Ingmar Lehmann, *The Joy of Factoring* by Samuel Wagstaff, and even Terence Tao’s *Compactness and Contradiction*.


Mathematicians have always been fond of telling their own life stories and the ideas that animated them—or others found that it is worth writing about mathematicians’ lives and their ideas. The autobiographical literature is picking up speed, with the late Martin Gardner’s *Undiluted Hocus-Pocus*, Benoit Mandelbrot’s *The Fractalist*, Edward Frenkel’s *Love and Math*, Larry Baggett’s *In the Dark on the Sunny Side*, and *An Accidental Statistician* by George Box. Key autobiographical elements are also present in Reuben Hersh’s anthology of articles *Experiencing Mathematics*. Phillip Schewe has written Freeman Dyson’s story as *Maverick Genius*. A historical biography on Ada Lovelace is James Essinger’s *A Female Genius*; others are Dirk van Dalen’s *L. E. J. Brouwer* and *Vito Volterra* by Angelo Guerraggio and Giovanni Paoloni. A collection of historical studies is *Robert Recorde*, edited by Gareth Roberts and Fenny Smith. A children’s book on Paul Erdős is *The Boy Who Loved Math* by Deborah Heiligman. Two books on the history of mathematics at Harvard, with emphasis on personalities, are *A History in Sum* by Steve Nadis and...
Shing-Tung Yau, and *Carnap, Tarski, and Quine at Harvard* by Greg Frost-Arnold. And a history of computing through personalities is *Giants of Computing* by Gerard O’Regan.


On mathematics education, too many books to mention here are published every year. For now I note first several addressed mainly to teachers and parents: a delightful little book by Stephen Brown titled *Insights into Mathematical Thought*; then *How Math Works* by Arnell Williams, *Success from the Start* by Rob Wieman and Fran Arbaugh, *Captivate, Activate, and Invigorate the Student Brain in Science and Math, Grades 6–12* by John Almarode and Ann Miller, *Math Power* by Patricia Kenschaft, and *One Equals Zero and Other Mathematical Surprises* by Nitsa Movshovitz-Hadar and John Webb. David Tall has published a wide-ranging study on *How Humans Learn to Think Mathematically*. Ed Dubinsky’s APOS theory is presented in detail by a group of authors, including Dubinsky but also Ilana Arnon and others. An excellent anthology pieced from the 75 years of National Council of Teachers of Mathematics yearbooks was edited by Francis Fennell and William Speer, with the title *Defining Mathematics Education*. Other collective volumes are *Mathematics & Mathematics Education* edited by Michael Fried and Tommy Dreyfus, *Third International Handbook of Mathematics Education* edited by Ken Clements and his collaborators, *Reconceptualizing Early Mathematics Learning*.

Accessible books on mathematics and other disciplines or applied to various practical or entertainment endeavors are *Six Sources of Collapse* by Charles Hadlock, *Mathematical Morphology in Geomorphology and GISci* by Daya Sagar, *Mathematical Card Magic* by Colm Mulcahy, and *All the Right Angles* by Joel Levy. An algorithmic fusing of evolutionary, learning, and intelligence perspectives situated in the living environment of organisms is presented by Leslie Valiant in *Probably Approximately Correct*. A decoding and explaining of mathematical motifs in a popular TV series is given by Simon Singh in *The Simpsons and Their Mathematical Secrets*. Readers interested in connections between mathematics and music might find it useful to consult *The Geometry of Musical Rhythm* by Godfried Toussaint and *Mathematics and Music* by James Walker and Gary Don. Those passionate about using mathematics to make origami will find well-illustrated treasure troves of ideas in *Origami Tessellations* by Eric Gjerde and especially the massive *Origami Design Secrets* by the master of the trade, Robert Lang. A more general introduction to the mathematical motifs in the arts is *Manifold Mirrors* by Felipe Cucker. A collection of legal cases and situations that unveil the dangerous mis-haps that occur in the (ab)use of mathematics in courtrooms is presented by Leila Schneps and Coralie Colmez in *Math on Trial*. Finally in this category, harnessing mathematics to the search for extraterrestrial life is the leitmotif in *Science, SETI, and Mathematics* by Carl DeVito.

Every year I include a few online resources, more or less randomly, as I notice them or other people point them out to me.

Philipp Legner is the creator of Mathigon (http://mathigon.org/), an attractive, colorful website where he uses new technologies to build realistic 3-D images of many geometric objects. Another page with good illustrations of solid geometry objects is Paper Models of Polyhedra (http://www.korthalsaltes.com/). A site hosting many images, applets, and videos illustrative of mathematical concepts, from elementary math...
to multivariable calculus, is Math Insight (http://mathinsight.org/about/mathinsight), maintained by a group of faculty from the University of Minnesota. Useful educational materials at the secondary school level can be found on the Basic Mathematics page (http://www.basic-mathematics.com/) and Interactive Math (http://www.mathsteacher.com.au/index.html); more advanced material is on the Art of Mathematics site (http://www.artofmathematics.org). AwesomeMath (https://www.awesomemath.org/) is a summer program at three universities geared toward gifted secondary students who aspire to receive training beyond school curricula and eventually qualify for advanced mathematical competitions. A web page constructed with information taken from Florian Cajori’s book *A History of Mathematical Notations* is Earliest Uses of Various Mathematical Symbols (http://jeff560.tripod.com/mathsymb.html). Nate Silver’s online magazine FiveThirtyEight (http://fivethirtyeight.com/) posts critical articles on journalistic punditry invoking data and on other public uses of statistics. An online store specializing in mathematics manipulables is MathsGear (http://mathsgear.co.uk/).

Lastly, the recently restructured site of the Simmons Foundation, under the name *Quanta Magazine* (https://www.simonsfoundation.org/quanta/), hosts some of the best writing on mathematics; I did not include any of the *Quanta* pieces in this volume solely because they are freely and easily accessible online.

I encourage you to send comments, suggestions, and materials I might consider for future volumes to Mircea Pitici, P.O. Box 4671, Ithaca, NY 14852, or send electronic correspondence to mip7@cornell.edu; my Twitter handle is @MPitici.

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**Books Mentioned**


For general queries, contact webmaster@press.princeton.edu


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