Introduction

> Glory is fleeting, but obscurity is forever.
>
> Napoleon Bonaparte

On 16 February 2012 the British prime minister, David Cameron, gave a speech in Scotland's capital of Edinburgh on a theme that would have had resonance with John Napier:[1] Scottish independence. That is, independence from England, with whom there has long been martial and political conflict and most particularly since the Scots signed the *Auld Alliance* with France in 1295 to the chagrin of the English king Edward I, the *Hammer of the Scots*. The sword has been replaced by the pen, the speeches conciliatory and, in the year of this book's publication, the decision regarding independence from the United Kingdom will have been made in a referendum of the Scottish people: it is also the year in which we celebrate the 400th anniversary of Napier's publication of world significance: *Descriptio*. With the litany of battles,[2] sieges, alliances and intrigues that have absorbed the country, the prime minister was on safe ground with his opening sentence:

> The air in Scotland hangs heavy with history.

His second sentence

> Edinburgh's cityscape is studded with monuments to memories.

brings us, though, to a motivation for this book: a monument of significance is missing. The speechwriters had combed through that long history to find the names of Scotsmen whose contribution in whatever field has been of significance. This was not a difficult task and, as we comb through their speech, we find, in order of mention: Walter Scott, Robert Louis Stevenson, John Knox, Captain Scott, Adam Smith, David Hume, James Maxton, Keir Hardie, John Reith, Lord Lovat, Robert Dunsire, Liam Tasker, James Watt, Robert Owen, Sir Bill Gammell, Ian

[1] We will usually refer to John Napier as Napier, unless there is danger of ambiguity.

[2] The year also marks the 700th anniversary of the Battle of Bannockburn.

Wood, Henry Campbell-Bannerman, Joe Grimond, Iain Macleod, George Younger, Donald Dewar and John Smith. The name of John Napier is missing, and it is particularly ironic that David Hume, a man more literary than scientific, judged him as "the person to whom the title of a great man is more justly due than to any other whom this country ever produced."[3] Had the speechwriters required convenient reference, they might have consulted the magnificent frieze which adorns the entrance hall of the Scottish National Portrait Gallery in Edinburgh: it displays images of significant Scots extending from Thomas Carlyle into prehistory, with Napier peeking through between the shoulders of George Buchanan, historian, poet and tutor to James VI, and James Stewart, Earl of Moray and one of the regents of Scotland during the minority of James VI. Perusal of the frieze perhaps allows Napier's omission from the speech to be forgiven, with the incipient difficulties of choice nicely summarized with a quotation from John Amyatt of modest exaggeration:[4]

> Here I stand at what is called the Cross of Edinburgh, and can, in a few minutes, take 50 men of genius and learning by the hand.

In short, Scottish history is enviably replete with names of significance, but that of the Edinburgh born John Napier is in danger, if not of disappearing from the scientific landscape, then of fading into its shadows. His name is now seldom attached to the logarithms he (in essence) discovered, with the modern nomenclature of *Natural* replacing *Naperian* logarithm. In an online poll conducted by the National Library of Scotland between December 2005 and October 2006, the public were invited to vote for a favourite Scottish scientist out of 24 nominees, Napier was one of them and appeared last of the subsequently published top 10;[5] the 2010 Britannica publication, *The 100 Most Influential Scientists of All Time*, makes no mention of Napier. So, the justification for this book is simple: Napier's name deserves to be remembered in the panoply of great scientists and mathematicians, not just of Scotland but of the world, for the single reason that the calculative device he contrived, constructed and promulgated, which he later called logarithms, was to change the world in which he lived and the world long after his death. It was the first significant mathematical discovery in Great Britain, let

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[3] *The History of England*, 1688, Chapter LVIII. We should acknowledge that the comment was made before many of the others on the list were born.

[4] John Amyatt in 1750; William Smellie, *Literary and Characteristical Lives*, 1800.

[5] See appendix B.

alone Scotland, as he was the first significant mathematician in Great Britain, let alone Scotland. In making these judgments we do not ignore the wandering Scottish polymath Dr. Duncan Liddel, who achieved distinction (most particularly in Germany) in mathematics, philosophy and medicine, or the Welshman Robert Recorde, who gave us the equals sign and who was a successful mathematical expositor,[6] or any other prior intellectual, it is simply that Napier was in modern terms a research mathematician of exceptional power, although one working with crude tools. In Scotland he was succeeded by more of Amyatt's *men of genius*, beginning with James Gregory, James Sterling and Colin Maclaurin; in England the long list of such begins with Henry Briggs, his irreplaceable collaborator in the development of logarithms.

With our (not so very) modern view of logarithms it may be difficult at first to appreciate that they needed inventing; for us they are, after all, simply the inverse of the exponential functions, with the problem merely one of notation:

$$\text{If } y = x^2 \text{ then } x = \pm\sqrt{y}, \text{ but if } y = 2^x, \text{ then how do we write } x?$$

We must thank the Silesian mathematician Christoff Rudolff for the radical sign; the answer to the second question, known to all high-school mathematics students, is of course $x = \log_2 y$; the instructor is left with the motivational difficulty of why a lump of wood enters mathematical notation. Yet, the problem is also a calculative one: the solution to the equation $2^x = 4$ is far removed in difficulty from that of $2^x = 3$; this latter equation would have us reaching for the calculator button which provides logarithms to any base or the ones labelled log and ln. Of these last two, the former produces what is now a logarithm of convenience and the latter one of essence and it is in the distinction between them that there lies concealed a nice logarithmic paradox. It is that base 10 logarithm, log, which was to bring to a world desperate for calculative help a mechanism, realized as a table of numbers, which conjured the immensely challenging problem of multiplication to the comparatively simpler one of addition. The younger reader, who rightly takes for granted modern calculative tools, should beware complacency: we do not doubt that the product $742849628465 \times 269355497183$ would, with time and great care, be correctly accomplished by hand but now let this problem be one of a hundred such, a thousand such, …, and we begin to comprehend the immense calculative difficulties faced by

[6] A noble art.

scientists before the advent of base 10 logarithms; and what of division and root extraction? Insert a decimal point at the front of the two numbers[7] and we may imagine ourselves dealing with a trigonometric calculation of great accuracy, as they routinely did; perhaps it arose from a problem in positional astronomy with the use of the sine rule or the cosine rule or one of numerous other such rules, all of which they had. But it is one thing to find a numeric expression for an unknown quantity: it is quite another to evaluate that expression. These then, in tabular form, are the logarithms that were in use for calculation up to the mid 1970s (and, in many places, still later) but, if the single purpose of logarithms was their original purpose, to aid calculation, their place would now be solely the remit of the mathematical historian yet, even though the tables of logarithms have disappeared, logarithms themselves remain an essential current mathematical tool: measurement of sound levels, earthquake intensity, pH levels, entropy, stimulus and sensation response, etc., hardly touches the number of real-world phenomena which exhibit logarithmic behaviour. With these and their like the choice of base is not a critical matter, but it is with base e, or ln, that logarithms are most notably imbued with their immortality: where would calculus be without $\ln x$ and e^x? In fact, Napier's original logarithms were not base 10, neither were they base e, nor any other base: his conception did not involve a base at all and, anyway, he had no exponential notation in which to frame such an idea. If, using the calculus he never had, we choose to attach a base to Napier's original logarithm, that base must be $1/e$, and here is the paradox: the version of logarithms that gives them their central importance and permanency is, in essence, that which Napier had originally conceived but abandoned in favour of those to base 10, the more congenial, but now redundant, servant of decimal calculation.

This book has been written to its title, which has brought about difficult decisions regarding inclusion and omission, and it may justly be characterized as a scientific biography: biography because, in as much as we have record, it describes the man; scientific because it describes his work. There is, though, an appreciable imbalance between what we know of him and of his achievements, with the latter a matter of published record whereas the former went largely unrecorded, or for which the record was lost, and the book's structure necessarily reflects this. The biographical facets are mainly confined to the first chapter, with two short appendixes adding historical perspective, and there follows

[7] Which they studiously avoided doing.

a chapter having no place in a scientific work, but we feel its omission would have been a fault greater than its inclusion. Napier's analysis of the Book of Revelation, the last book of the Bible, sits uncomfortably in a modern setting and uncomfortably among the work of a scientist of international significance but it was, in his view, his greatest achievement: it was, after all, exposing the details of not only his but the world's salvation. He was a religious extremist, but one in a world of religious extremism, and the work is a window through which we can peer to gain an extra view of the man and the times in which he lived; he had applied his analytical mind to the most profound of problems, he was greatly acclaimed for his efforts, and we have been minded to detail some of them. In doing so we acknowledge that we must test the patience of our readers: the informed with our tentative and abridged analysis, the lay with our detailed and lengthy commentary. The body of what follows deals by chapter with what he contributed by book, and is itself followed by a brief commentary on a portion of his mathematics which remained unpublished until the nineteenth century. The intention has been to provide the readers who choose to consult a full version of one of his works with a framework for its study, and those who do not with a representative synopsis of each of them, together with an analysis and a perspective. Here we meet with *logarithms*, in their original and modified forms, and his other inventions: *Napier's Bones*, which were a popular alternative to logarithms; his *Promptuary*, which developed the idea; his *Local Arithmetic*, which utilized a disguised binary representation of number, and with the formative stages of a mathematical textbook. It all combines to an amalgam of an arithmetic primer and explanations of clever ideas to assist calculation, all from the busy mind of a brilliant man. At the end we discuss his legacy and then move to the appendixes, which have varied but relevant purpose: the last of them gives ear to another just claimant to the invention of tables of numbers designed to simplify arithmetic process; this man, Jost Bürgi, deserves elevation to a major chapter in a book devoted to the history of logarithms but can find none such in one largely devoted to Napier and his works.

The mathematical level is not high; indeed, it is high school, albeit with some of the material off-syllabus in its content and sometimes its approach. Our hope is as much to engage the high-school student of today as it is to engage those who have graduated over the past forty years; for these readers, logarithms hide underneath calculator buttons, subject to several useful laws which must be learnt, their role often intimately linked to calculus. Of course, we also hope to rekindle

the interest of those of more mature years, who will readily appreciate the great debt we owe to an invention 400 years old: the original definition of logarithm will surprise many from both cohorts.

Our desire has been to add to the comparatively small corpus devoted to Napier but to replace none of it, least of all the work which must necessarily remain the definitive biography of him, written by his kinsman Mark Napier. This author enjoyed the inestimable privilege of having access to such private papers that remained after the accidental destruction by fire of a significant archive and his 591-page book of 1834, *Lineage, Life and Times*, should be consulted for detail and analysis; we are bound to say, though, that the author's bias, meandering style and exaggerated language render the work a challenging read. There is another particularly significant work. A century ago, to be precise at the end of July 1914, a congress was held in Edinburgh as the major among several across the world that commemorated the tercentenary of the publication of *Descriptio*, in which logarithms were first announced to the world. The congress was closely followed by a conflict that even in Napier's troubled time would have been of unimaginable horror: a week later came the outbreak of World War I, with the delegates hurriedly dispersing to their home countries before its start. Notwithstanding this, a memorial volume of some 441 pages was produced in 1915, comprising varied contributions which reflected on Napier and his achievements and on calculation and tables in more general terms. We should recall that at that time the role of logarithms as the central means of calculation was undiminished and many of the papers reflect this. It remains an excellent testimonial, though, of international appreciation of a great achievement. A century later, with the quadcentenary of the *Descriptio*, we are provided with an opportunity to reinvigorate its significance, particularly in these days of lightening computation, and so confront the present danger of Napier, *John of Logs*, *Marvelous Merchiston*, and his work moving to popular obscurity.

Appropriately, the 1914 congress held a memorial service in Scotland's mother church of Presbyterianism, St Giles, and equally appropriately, the sermon was preached by the minister of St Cuthbert's church, Edinburgh, where Napier worshiped, where he was an Elder, where the name Napier appears on gravestones and where a plaque dated 1842 informs us[8] that

[8] In translation from Latin.

> Near this spot was laid the body of John Napier of Merchiston, who gained for himself the imperishable memory of future ages by his wonderful discovery of logarithms.

None of those gravestones are his; he has no marked grave. Napier university, which has engulfed Napier's birthplace of Merchiston Tower, is host to a modern bust of him, located at the centre of a car turning circle in front of its Craighouse Campus, and a statue of him stands in line on the Queen's Street aspect of the National Portrait Gallery we earlier mentioned: we argue, though, that his prominent place in history warrants a more prominent memorial to him in the city of his birth and death. Perhaps the quadcentenary provides that opportunity.

The sermon in that memorial service was preached to the text of Psalms 90:12:

> So teach us to number our days that we may apply our hearts unto wisdom.

We cannot help but feel that Napier would have approved.