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General Introduction

Medieval Europe, from around 800 to 1450, was a meeting place of three civilizations: the Latin/Christian civilization that was forming on the foundation of the defunct Western Roman Empire; the Jewish/Hebrew civilization, which witnessed great scholarly activity in every location where Jews resided; and the Islamic/Arabic civilization, whose European center was in Spain, but which had a close relationship with the Islamic civilization of North Africa. Although these three civilizations clashed at numerous times in the medieval period—with the Christians gradually pushing the Muslims out of Spain and with both Muslims and Christians attacking Jews at various times—a fertile intellectual exchange took place at this time that is reflected in the developments in mathematics. Thus, the aim of this sourcebook is not just to present original sources in mathematics from the three civilizations, but also to present sources that reflect this interchange. Thus, the reader will frequently find similar problems or methods discussed in texts originally in Latin, Arabic, or Hebrew, and we have provided internal references to similar texts in the various sections.

The original design of this book was to include sections on mathematics originally written in Europe in Latin, in Hebrew, and in Arabic. However, as the book developed, it became clear that confining the geographical locus to European soil was too restrictive. After all, for much of this period, the Islamic rulers of Spain also ruled in North Africa (the Maghrib); and whether the rulers of both areas were from the same dynasty or not, there was much interchange of ideas and people between the two areas. Furthermore, although most of the medieval Hebrew work in mathematics was accomplished in Spain and France, when Jews had to leave those areas, many migrated east to Constantinople, so it seemed important to include a mathematical treatise from the Byzantine and later Ottoman capital. And since we did include such a work, the question came up as to whether we should also include original sources from the Byzantine Empire itself. Unfortunately, precious few such works have been edited to date, so we made a decision to include just a limited number of excerpts of Byzantine work, translated from the Greek, in Appendix 1.

The editorial team also realized that it is nearly impossible to make a sharp distinction between Latin, Hebrew, and Arabic works. For example, some mathematical texts were originally written in Arabic, but no Arabic manuscript is extant. Fortunately, at some point during the medieval period, these manuscripts were translated into Latin or Hebrew. Similarly, there are Hebrew manuscripts that today only exist in a Latin translation made during that © Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

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period. Thus, it is not always clear into which of the three chapters of this book to place a given piece of mathematics. In general, our decision was based on whether the original author was known. If he was, then we placed the work under the language in which the document was originally written. But if not, then it is placed under the language in which the extant manuscript is written. In any case, it is clear that mathematicians in the three cultures—the Latin, the Arabic, and the Hebrew—were aware of what was being written in the other cultures and somehow made use of these materials.

Although medieval texts that involve groundbreaking mathematics are rare, nevertheless, we decided that the excerpts presented would not, in general, be snippets but would be long enough for readers to get a solid understanding of the mathematical ideas the author was expressing. Thus, many of our sources are quite long and develop a mathematical idea in some detail. We hope that readers will come to appreciate the mathematical struggles of our medieval ancestors and the answers they found to the problems they posed. As will be clear from the frequent references to Euclid and other Greek authors, mathematics, mostly in translation, and were trying both to understand that mathematics and, often, to develop it further. However, they also often had practical goals and thus ignored the Greeks when they needed to solve problems that were not part of the Greek mathematical enterprise. In both of these situations, however, these medieval mathematicians prepared the ground for the great advances in mathematics that were made in the Renaissance and later.

The study of the Latin mathematics of medieval Europe has a long history, reflected today in the large number of texts that have been translated from that language into English. By 1974, Edward Grant had edited A Source Book in Medieval Science, which included more than 100 pages of mathematics in its total of over 800 pages. We have, in fact, reused a few of the translations from that book, but it is interesting to note that many of the works that were excerpted there now have complete English editions available, including especially works of Nicole Oresme, Jordanus of Nemore, and Leonardo of Pisa (Fibonacci). Thus, in this book we have been able to use the work of the numerous scholars who contributed to this translation effort, supplementing the original translations by the two editors of our Latin section. We decided to include pieces of mathematics in this section that were originally written in Greek or Arabic but whose influence in Europe only occurred after they had been translated into Latin. For example, we have included excerpts from both the arithmetic and algebra of al-Khwārizmī, both of which appeared in longer excerpts in the earlier Sourcebook. And at the end of the Latin section are some sources originally in Italian that presage the development of Renaissance mathematics and yet are still intimately related to medieval material. In addition, the trigonometry section includes a substantial excerpt from Regiomontanus's On Triangles, which could be considered either the last medieval trigonometry text or the first one of the early modern period, but whose author evidently learned many of his ideas from his medieval ancestors.

Scholars have only been studying medieval Hebrew mathematics in relatively recent times. Such a study needs to begin with editions collated from extant manuscripts and then translated into modern languages, and major efforts in this direction are only a product of the late twentieth century. In particular, very few medieval Hebrew works have previously been translated into English, although some were translated into German early in the twentieth century and others have been translated into French more recently. Thus, the editor and several © Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

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other young scholars contributed numerous original translations to the Hebrew section. And even when a previous translation into German or French existed, we retranslated the material into English, always checking with the Hebrew original when that was available.

The situation for Arabic was somewhat different from that of the other two languages. Although numerous Arabic works from the Muslim East were translated into German or French in the nineteenth century, it is only in recent years that medieval manuscripts from the Maghrib or from Spain (al-Andalus) have been studied, edited, and translated. Early in the twentieth century, some Arabic manuscripts from Spain were translated into Spanish, while the last decades of the twentieth century witnessed a steady flow of translations of North African manuscripts into French. We were able to make use of some of this material in the present work, again checking translations from French or Spanish against the original Arabic. And with the help of several other scholars, the editor of Chapter 3 on Arabic mathematics also translated some material into English directly from Arabic, material that has only recently become available.

One criticism of the earlier Sourcebook was that no attempt had been made either to compare the mathematics of the five civilizations whose sources were included or to speculate about any methods of transmission from one of these civilizations to another. At the time, too little information was available to even attempt the latter task, so each of the five sections stood independently. But in this Sourcebook, it is clear that the Latin, Hebrew, and Arabic mathematical cultures influenced one another. The reader can easily compare, for example, the works on measurement in the three cultures or the problem of men buying a horse or even some of the trigonometrical treatises. In some of these instances, it appears that one author simply borrowed from another, given that the numerical values and even the lettering of the diagrams are identical. In most cases, however, we can only speculate as to whether one author of an included work was actually familiar with the work of another. It is always possible that both borrowed from works that are no longer extant or even that each developed the particular idea or example independently. To help trace the flow of ideas, we have also included a few problems from Diophantus (see Appendix 2) and from Mahāvīra (Appendix 3) that are very similar to those considered in the main text. And as a further help, we have included a time line that includes all authors whose works we have used in the Sourcebook. We therefore hope that historians may be able to take our *Sourcebook* as a starting point for further research in the study of the flow of mathematical ideas during the medieval period.

NOTE TO THE READER

We have used different fonts to make clear to the reader the source of any given paragraph. Namely, the original sources are generally in Arial, while in cases where we have intermingled a source and a later commentary on the source, we have used Arial in the first case and **boldfaced Arial** in the second. Comments by the editors are generally in Times Roman. The numbering for the figures begins anew in each part of each chapter of the book. Thus, the figures have a four-digit identifier in the Latin chapter (Chapter 1) and a three-digit identifier in the other two chapters.