Introduction

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For the seventh consecutive year, we offer an anthology of recent writings on mathematics, easily accessible to general readers but intriguing enough to interest professional mathematicians curious about the reach of mathematics in the broader concert of ideas, disciplines, society, and history. With one exception (which escaped my radar two years ago), these pieces were first published during the calendar year 2015 in various venues, including professional journals, mass media, and online. In the introductions to the previous volumes and in several interviews, I detailed the motivations and the procedures that underlie the making of the books in this series; before I turn to the content of the current volume, I remind you of some of those tenets, then I point out a few constant and a few changing features that will help you, the reader, consider this enterprise in long-term perspective. I see each year’s edition as the product of a convention imposed by our inevitable subservience to calendar strictures; each volume should be considered together with the other volumes, an integral part of the series. It seems to me that the reviewers of all the volumes published so far ignored this aspect. To rectify somehow this prevailing perception of discontinuity, in the near future we will make available a combined index of the pieces published so far.

The main message of the series is that there is a lot more to mathematics than formulas and learning by rote—a lot more than the stringency of proof and the rigor usually associated with mathematics (and held so dear by mathematicians). Mathematics has interpretive sides with endless possibilities, many made manifest by writing in natural language. By now this kind of literature has produced veteran practitioners who are easily recognized by the public but also talented newcomers. It is a new genre that has spun a vast literature but remains
completely ignored in educational settings. I hope that *The Best Writing on Mathematics* series helps educators and perhaps policy makers see that it is worth broadening students’ understanding of mathematics. Mathematics is an extraordinarily diverse phenomenon molded by the human mind but anchored in a realm both abstract and concrete at the same time. That is not easy to comprehend and to master if we restrict the learning of mathematics to the use of conventional mathematical symbols and notations. I am happy to see that in the books of this series we can bring together constituencies that are sometimes uncomfortable with each other, such as mathematicians and mathematics educators, “purists” and mavericks, entrepreneurs and artists. This series can contribute to a change in the perception of mathematics, toward a broad view that includes the anchoring of mathematics in social, cultural, historical, and intellectual phenomena that hold huge stakes in the working of contemporary humanity. For what it is worth, with its good qualities and its faults, this series undoubtedly reflects my vision, even though other people also add opinions along the way. I propose the initial selection of pieces. I choose most of the pieces myself, but occasionally I receive suggestions from other people (including their own productions). I always welcome suggestions, and I try to remain blind to my correspondents’ eventual self-interest and self-promotion. I start from the premise that people are well-intended. I consider everything that comes to my attention, regardless of the manner in which it reaches me. The nontechnical literature on mathematics is immense and it is impossible for me to survey all of it; if people come forward to point out something new to me, I am grateful. A few of the pieces brought to my attention by their authors rightly made it into volumes; most of them did not.

Seizing an opportunity mediated by Steven Strogatz and accepted by Vickie Kearn, I did the first volume in this series (2010) quickly, in just two months, pulling all-nighters in the library at Cornell University; it had a slightly different structure than subsequent volumes. Starting with the second volume (2011) we settled on a template and on certain routines, schedules, and processes that work smoothly—with occasional bumps in the road, to keep us on our toes! This is a fast-paced series; there is not much room for delays and second-guessing.

For the final selection of texts, we take guidance from several professional mathematicians who grade each of about sixty to seventy pieces I
initially propose for consideration; we also have to consider constraints related to length, copyright, and diversity of topics and authors. Thus, the content of the books is dependent on the changing literature available but the format is fairly stable. Yet changes in format do occur, gradually. For instance, starting with the second volume, I added the long list of also-rans, supplemented later by a list of special journal issues and now by two new lists, one of remarkable book reviews and another of interviews. The profile of the volumes in *The Best Writing on Mathematics* has shifted from anthology-only to anthology and reference work for additional sources about mathematics. By now, in each book we offer not only a collection of writings but also a fairly detailed list of supplementary resources, worth the attention of researchers who might study on their own the complex phenomenon of mathematics and its reverberations in contemporary life. The section containing notable writings, which I present toward the end of each volume, takes me a lot more time to prepare than the rest of the book. Many of the sources I list there are difficult to find for most readers of our series; I hope the list will stimulate you to seek at least a few other readings, in addition to those included here. In this volume another new feature is the printing of figures in full color within the pieces where they belong (not in a separate insert of the book).

I receive a lot of feedback on this series, most of it informal, most of it good, pleasing, cheery, even exalted. The more formal feedback comes in book reviews. It is fascinating and reassuring to compare the reviews. Almost every aspect criticized by some reviewer is praised by other reviewers. Where some people find faults, others see virtue. The point I am making is that regardless of what *would* be desirable to include in the volumes of this series, we are circumscribed by the material extant out there. I cannot include a certain type of article if I encounter nothing of that sort or if we cannot overcome republishing hurdles.

Inevitably, the final selection of pieces for each volume is slightly tilted each year toward certain themes. This happens despite my contrary tendency, to reach a selection as varied as possible, unbiased, illustrating as many different perspectives on mathematics as the recent literature allows. The attentive reader will notice that in this edition a group of contributions refer to the dynamic tension between the object and the practice of “pure” versus “applied” mathematics. This
topic is intertwined with the millennia-old history of mathematics and has been addressed by many mathematicians, nonmathematicians, and philosophers.

**Overview of the Volume**

I now offer a brief outline of each contribution to the volume, with the caveat that the sophisticated arguments made by our authors and the assumptions that support those arguments are worth reading in detail, comparing, corroborating, and contrasting.

Hyman Bass reveals the learning experiences that shaped his views of mathematics and places them in a broader discussion of the complex rapport between mathematics and its pedagogy.

Daniel Silver presents H. G. Hardy’s blazing ideas from that most famous exposition of a working “pure” mathematician, *A Mathematician’s Apology*—and gives us a potpourri of reactions that followed its publication.

In a piece that can be seen as a genuine continuation on the same theme, Hannah Elizabeth Christenson and Stephan Ramon Garcia detail the fortuitous confluence of mathematics, cricket, and genetics—all of which conspired to make Hardy, at least for once, an *applied* mathematician.

Derek Abbott contends that the applicability of mathematics is considerably overrated, takes the decidedly anti-Platonist position that mathematics is entirely a human construct, and proposes that accepting these unpopular tenets will “accelerate progress.”

Aided by suggestive and clear illustrations, Burkard Polster manages to use no formulas in a plain-language proof of a geometric result concerning circles arranged in a rectangle—and leaves us with a few challenges on similar topics.

Joshua Bowman considers the problem of periodic paths of billiard balls in pools of triangular shape; he gives some known results and mentions a few open problems.

In his second piece included in this volume, Burkard Polster teaches us how to invent our own variants of the card game *Spot It!*

Jennifer Quinn reports on a contest in mathematical virtuosity, with participants so ingenious that the victor cannot be settled on the spot;
all this comes from one of the newly popular, enormous, always sold-out arenas of mathematical-gladiatorial disputes!

In a compelling piece that combines elements of group, string, and number theories, Erica Klarreich weaves together considerations made by several mathematicians and physicists struck by the connections they find among fields of research apparently far apart.

Davide Castelvecchi describes the avatars of the proposed proof for the $abc$ conjecture in numbers theory and relates them to the peculiarities of the proof’s author, Shinichi Mochizuki.

Kevin Hartnett tells the decisive story of a recent breakthrough, Ciprian Manolescu’s proof that some spaces of dimension higher than three cannot be subdivided.

Steven Strogatz presents a brilliantly insightful proof of the Pythagorean theorem from the young Albert Einstein.

In a semiautobiographical recollection, Brian Greene outlines the evolving speculative assumptions of the physicists who developed the mathematical underpinnings of string theory.

Tanya Khovanova, Eric Nie, and Alok Puranik describe step by step a generalization about hexagonal grids of a fractal iteration first proposed by Stanislaw Ulam.

Marc Frantz shows us beautiful fractal images and explains how he constructed them.

Joseph Dauben and Marjorie Senechal took strolls through the Metropolitan Museum in New York and discovered a treasure trove of mathematical content—some in plain view, some deftly hidden in details.

Alan Schoenfeld places the contentious Common Core Standards for School Mathematics in perspective, relating them to previous programmatic documents in U.S. education and discussing their rapport with a few important issues in mathematics instruction.

Katharine Beals and Barry Garelick take a decidedly critical stand toward the Common Core, arguing that the standards run against developmental constraints and may impede the learning of mathematics, not promote it.

David Acheson, Peter Turner, Gilbert Strang, and Rachel Levy present four visions of teaching applied mathematics, each of them emphasizing different elements they see as important in such work.
David Richeson details the very early history of one of the most widely recognized mathematical facts, the constancy of the proportion between the circumference and the diameter.

Isabel Serrano and Bogdan Suceavă reconsider the contribution of the medieval thinker Nicholas Oresme to the history of the idea of curvature and conclude that Oresme deserves more credit than he is usually given in accounts of the history of geometrical concepts.

In a similar reconsideration but with an opposite twist than Serrano and Suceavă, Viktor Blåsjö contends that Gottfried Wilhelm Leibniz receives too much credit for “proving” the fundamental theorem of calculus.

Amy Shell-Gellasch examines some connections between geometric curves and mechanical drawing devices, with a focus on the drawing toy Spirograph.

John Stillwell ponders what qualities might account for the “depth” of mathematical results and compares from this viewpoint a number of well-known theorems.

Drawing on their work experience, Marco Puts, Piet Daas, and Ton de Waal propose solutions for the instances in which errors find their way into large sets of data.

Brian Hayes summarizes the problematic current state of achievement in the quest to construct probabilistic (as opposed to deterministic) programming languages.

In a piece that combines psychological insights and mathematical rigor, Jorge Almeida analyzes several common misperceptions of the people who attempt to rationalize the most likely outcome of the lottery process.

Andrew Gelman explains why some research claims based on null hypothesis significance tests are spurious, especially in psychological studies.

Howard Wainer and Richard Feinberg point out that the practice of administering long tests under the uncritical pretense that they result in accurate assessments leads to colossal wastes of time, when aggregated to a social scale.

In the last piece of the volume, Ian Stewart attempts to circumscribe popular mathematics as a genre, affirms its value for the public, and advises those who want to contribute to it.
More Writings on Mathematics

In every edition of this series, I list other writings on mathematics published mainly during the previous year, to stimulate readers in their search for broad views of mathematics. The writings mentioned are either books and online resources (listed in the introduction) or articles (listed in the section of notable writings). Most of these items are not highly technical, but occasionally mathematical virtuosity or even theory does enter the picture; a clear distinction between technical and nontechnical writings on mathematics is not possible. The pretense that these bibliographies are comprehensive would be quixotic in the present state of publishing on mathematics. I list only items I have seen—courtesy of authors and publishers who sent me books (thank you!) or through the valuable services of the Cornell University Library and, in a few instances, of the Z. Smith Reynolds Library at Wake Forest University, where my wife worked for four years. For additional books I refer the reader to the monthly new books section published by The Notices of the American Mathematical Society and to the excellent website for book reviews hosted by the Mathematical Association of America.

Among the books that came to my attention over the past year, three collective volumes stand out: The Princeton Companion to Applied Mathematics edited by Nicholas Higham, The First Sourcebook on Asian Research in Mathematics Education edited by Bharath Sriraman, Jinfa Cai, and Kyeong-Hwa Lee, and the anthology An Historical Introduction to the Philosophy of Mathematics edited by Russell Marcus and Mark McEvoy.

Some books in which the authors focus on mathematics as it comingles with our lives, in its habitual, entertaining, puzzling, and gaming aspects, are The Proof and the Pudding by Jim Henle, A Numerate Life by John Allen Paulos, Truth or Truthiness by this year’s contributor to our volume Howard Wainer, The Magic Garden of George B and Other Logical Puzzles by the almost centenarian Raymond Smullyan, Problems for Metagrobologists by David Singmaster, as well as the collective volumes The Mathematics of Various Entertaining Subjects edited by Jennifer Beineke and Jason Rosenhouse, Numbers and Nerves edited by Scott and Paul Slovic, and even Digital Games and Mathematics Learning edited by Tom Lowrie and Robyn Jorgensen (the last one with an emphasis on educational

An area enjoying phenomenal growth is the history of mathematics, in which I include the history of mathematical ideas and the histories of mathematical people. Here are some recent titles: *I, Mathematician*, a remarkable collection of pieces by working mathematicians, edited by Peter Casazza, Steven G. Krantz, and Randi Ruden; *The War of Guns and Mathematics* edited by David Aubin and Catherine Goldstein; *Music and the Stars* by Mary Kelly and Charles Doherty; *The Real and the Complex* by Jeremy Gray; and *Taming the Unknown* by Victor Katz and Karen Hunger Parshall. Biographical works on remarkable mathematicians are *Leonhard Euler* by Ronald Calinger, *The Scholar and the State: In Search of Van der Waerden* by Alexander Soifer, *Genius at Play: The Curious Mind of John Horton Conway* by Siobhan Roberts, *Fall of Man in Wilmslow* (a novel that starts with the death of Alan Turing) by David Lagercrantz, and *The Astronomer and the Witch: Johannes Kepler’s Fight for His Mother* by Ulinka Rublack. Two collective volumes in the same biographical category are *Oxford Figures* edited by John Fauvel, Raymond Flood, and Robin Wilson; and *Lipman Bers* edited by Linda Keen, Irwin Kra, and Rubí Rodríguez. *Birth of a Theorem* by Cédric Villani is autobiographical. Collected works and new editions of old books include *The G. H. Hardy Reader* edited by Donald J. Albers, Gerald Alexanderson, and William Dunham; *Birds and Frogs* by Freeman Dyson; *A Guide to Cauchy’s Calculus* by Dennis Cates; and *Tartaglia’s Science of Weights and Mechanics in the Sixteenth Century* by Raffaele Pisano and Danilo Capecchi. Historical in perspective with a strong sociological component (and on an actual topic) is *Inventing the Mathematician* by Sara Hottinger.

Recent books on philosophical aspects of mathematics are *Mathematics, Substance and Surmise* edited by the son-and-father pair Ernest and Philip Davis; *G.W. Leibniz, Interrelations between Mathematics and Philosophy* edited by Norma Goethe, Philip Beeley, and David Rabouin; *Mathematical Knowledge and the Interplay of Practices* by José Ferreirós; and *The Not-Two* by Lorenzo Chiesa. Wide-ranging and difficult to categorize is *Algorithms to Live By*, by Brian Christian and Tom Griffiths.
A great number of books on mathematics education are published every year; it is not feasible for me to mention all that literature. Here are a few recent titles that came to my attention: *Confessions of a 21st Century Math Teacher* and *Math Education in the U.S.* by our contributor Barry Garelick, *What’s Math Got to Do with It?* by Jo Boaler, *Mathematical Mindsets* by Jo Boaler and Carol Dweck, *More Lessons Learned from Research* edited by Edward Silver and Patricia Ann Kenney, *Assessment to Enhance Teaching and Learning* edited by Christine Suurtamm, *How to Make Data Work* by Jenny Grant Rankin, and the refreshingly iconoclastic *Burn Math Class* by Jason Wilkes.


Highly visual is the new volume in *The Best American Infographics* series edited by Gareth Cook.

A work of fiction is *Zombies and Calculus* by Colin Adams.

I hope that you, the reader, will enjoy reading this anthology at least as much as I did while working on it. I encourage you to send comments, suggestions, and materials I might consider for (or mention in) future volumes to Mircea Pitici, P.O. Box 4671, Ithaca, NY 14852 or electronic correspondence to mip7@cornell.edu.
Books Mentioned


