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Introduction

The late 1980s were an exciting time. The Nintendo Entertainment System (better known as the NES) was just a few years old in America, and *Super Mario Bros.* was already a hit. In the summer of 1987, Nintendo released two more gaming blockbusters: *The Legend of Zelda* and *Metroid*. In so doing, it quickly cemented itself as a leader in the nascent video game industry. Though I didn't yet own an NES, I quickly befriended classmates who did.

As in *Super Mario Bros.*, the plot of the original *Legend of Zelda* involved a guy in funny clothes running around in order to rescue a princess. But beyond that, the two games shared few similarities. In *Super Mario Bros.*, players progressed from left to right through a series of essentially linear stages. Players could easily measure their progress by watching the digits increase as they moved from Level 1–1 to Level 8–4. In contrast, *The Legend of Zelda* simply dropped players in the middle of a forest with little indication of where to go. Players were encouraged to explore a world in which progression was nonlinear, restricted only by the abilities of the main character, a precocious boy named Link. The world itself was open, but Link was initially not experienced enough to fully navigate it. To break through a cracked wall, he needed bombs; to cross a river, he needed a raft. As you became more familiar with the game, so too did Link discover the tools needed to navigate his surroundings.

A similar level design was used in *Metroid*. In this game, the player took control of an intergalactic bounty hunter named Samus. Her mission: run-and-gun her way through a colony of space pirates in order to stop them from creating biological weapons on the planet Zebes. Though the side-scrolling perspective made the game look like a sort of *Super Mario Bros.* in outer space, the soul of the game was much closer to that of *Zelda*. Instead of bombs, Samus used missiles; instead of a raft, she enhanced her space suit to access new areas. In both games, upgrading your character was part and parcel of exploring the world.

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The Legend of Zelda series is now one of the most successful video game franchises in the world. Nintendo has produced more than fifteen games in the series, which collectively have sold tens of millions of copies worldwide. *Metroid*'s numbers, while not quite as large, are in the same ballpark. Clearly, these games resonate with players. But why? And what does any of this have to do with mathematics?

To address this question, let's jump ahead a few years to the very end of the twentieth century. In 1994, Princeton professor Andrew Wiles solved one of the most famous open problems in all of mathematics. Known as Fermat's Last Theorem, the problem asserts that the equation $x^n + y^n = z^n$ has no solutions when x, y, z, and n are whole numbers with n greater than 2. This is in sharp contrast to the case $n = 2$. In this case, examples of solutions to $x^2 + y^2 = z^2$ for integer x, y, and z are familiar to geometry students. Such solutions are referred to as Pythagorean triples, and there are infinitely many of them: $x = 3$, $y = 4$, $z = 5$ is one example ($3^2 + 4^2 = 5^2$), as is $x = 5$, $y = 12$, $z = 13$, ($5^2 + 12^2 = 13^2$).

Though the problem is simple enough to state, its solution eluded mathematicians for hundreds of years. Wiles' proof of the theorem (completed with the help of Richard Taylor) garnered him instant recognition, even outside of the mathematical community. Contributing to his newfound fame was a 1997 *NOVA* episode on the problem and Wiles' solution. The episode opens with Wiles describing on what it feels like to do mathematics:

> Perhaps I could best describe my experience of doing mathematics in terms of entering a dark mansion. One goes into the first room, and it's dark, completely dark. One stumbles around bumping into the furniture, and gradually, you learn where each piece of furniture is, and finally, after six months or so, you find the light switch. You turn it on, and suddenly, it's all illuminated. You can see exactly where you were.

The word choice here is particularly apt. If mathematics is a house, then certainly it must be a large one. The mansion of mathematics consists of a vast collection of rooms, and the light from one sometimes only barely illuminates the room across the hall. If your goal is to make a floor plan of the entire house, you've got quite a tall task ahead of you.

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The longer you fumble around through the mansion, though, the clearer things will become. Some rooms will eventually become fully illuminated. If you're diligent, entire wings of the mansion will open up to you. You'll also begin to find connections between different areas of the house that would have been completely surprising to you in the dark. Maybe you discover that the small hallway off of the kitchen provides a more direct route to the sitting room. Perhaps you'll be able to return to a previously discovered locked door and unlock it from the other side.

While we should be careful about stretching this metaphor beyond its natural limits, it does provide a helpful way to think about how some mathematicians work. The sometimes inelegant initial search for truth; the building of connections between seemingly disjointed areas; the development of tools to aid in the solution to a problem—these are all hallmarks of the mathematician's quest for understanding.

But these are also hallmarks of the game design featured in *Zelda, Metroid,* and many other video game franchises. Without a map to guide the way or a linear world that made navigation a simple matter of moving from left to right, exploration of these early game worlds was often rife with missteps, wrong turns, and confusion about where to go or what to do. With enough patience and skill, however, the game world's internal logic began to show itself. A locked door on the planet Zebes might eventually reveal itself to be a shortcut between two formerly distant parts of the planet, but only after Samus obtained the artillery necessary to blast the door open. To reach an entirely new portion of the kingdom of Hyrule, Link might first need to get his hands on a ladder to help him climb up the ominously named Death Mountain. I don't mean to suggest that solving Fermat's Last Theorem is an achievement anywhere near comparable to finding a virtual upgrade that allows you to freeze space pirates with ice (although the latter may make for a more entertaining way to spend an afternoon). I do contend, however, that there are some similarities between doing mathematics and playing games. Moreover, I believe that exploring the intersection of mathematics and video games can help to build a deeper appreciation for both.

Beyond these high-level qualitative comparisons, there are a number of more concrete and quantitative connections between mathematics

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and video games. While sometimes surprising, these links arise organically enough once you've developed an eye for spotting them. Here's just a sampling of some of the questions we'll explore together:

- What do video games have to teach us about our physical world … and more important, what aren't they teaching us?
- Why is it that certain classic games, such as *Jeopardy* and *Pictionary*, make for lackluster video games?
- What's the best way to measure the quality of user-generated content in video games, and what can this teach us about the mathematics of voting?
- How do those pesky red shells in *Mario Kart* work, and what's the best strategy for avoiding them?

Of course, there are many differences between doing mathematics and playing video games. For example, no matter how many hours you spend in the mansion of mathematics, there will always remain rooms cloaked in darkness. The worlds presented in many video games, however, are finite. This is why people can spend their entire career doing mathematics without ever getting bored, while many video games, once finished, sit idly on a shelf. But by looking at a wide assortment of games across time and different platforms, the mansion of video games grows substantially. While it still may not be as large as the mansion of mathematics, between these two domiciles there's more than enough to inspire some fun and interesting questions.

Although we'll be talking about math and video games, this is not a book about game theory. It's also not necessarily about game development, even though there's some juicy math there. And while tablets and Chromebooks have given developers new avenues to explore games for the classroom, I'm not necessarily interested in games whose purpose is to explicitly help students learn mathematics. Mostly this is because, with some exceptions, most educational math games aren't great at being educational or great at being games.

Instead, the primary goal of these chapters is to search for interesting and sometimes unexpected bits of mathematics using video games as a springboard. The moral here is that beautiful mathematics can be found in the unlikeliest of places. And by using video games as the entry point,

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my hope is to make mathematics topics at all levels more accessible to a general audience.

If you are a prolific gamer, I'd like to get you to think about the games you play a little differently, and encourage you to seek out mathematical ideas in those games. If you're a teacher, maybe something in these pages will give you a new idea for a way to inspire your students. If you are a lover of both games and mathematics, then welcome! You should feel right at home here.

If you love video games but hate math, here comes the bad news: we'll be doing some mathematics in this book. You should be able to overcome the mathematical barrier to entry, however, provided you've taken some high school math classes (think geometry, algebra, trigonometry, and the like). I'll do my best to explain what's going on under the mathematical hood in the simplest possible terms. If you're a more advanced reader, technical facts and discussions have generally been relegated to endnotes[1] and appendices. My aim is to reach as large an audience as possible, so if you're one of many people with an ingrained math phobia, please don't fret. There will be plenty here for you to enjoy.

Finally, as with writing about anything in popular culture, there is always a danger that this book will become irrelevant by the time it reaches the reader. Thankfully, beautiful mathematics never goes out of fashion, and so to combat this, one only needs to be careful in the selection of games. To that end, I've adopted the following guidelines when deciding on whether or not to discuss a certain game or franchise. Though it may not garner me much street cred with the hardcore gaming community, the examples I will discuss draw primarily from games that have already established themselves and are reasonably wellknown or are based on other established properties. This may be bad news for obscure titles but should be good news for you, since, with rare exception, I hope you will have heard of the games we'll discuss. While the latest Mario game may fade in popularity, Mario himself does not appear to be going anywhere.

Here's how the book is structured. In the first chapter, we'll take a look at realism (or lack thereof) in games and explore how a game's reality can affect learning. We'll also take a look at a few games that are often touted as exemplars when it comes to the intersection of commercial games and education.

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In the chapters that follow, we'll examine games that don't typically come to mind when thinking about education and highlight examples of rich mathematics that can emerge from these games. The chapters are organized thematically—one focuses on the mathematics of voting, another on computational complexity, and so on—and for the most part these chapters are logically independent. Feel free to peruse them in any order you like.

Once we've explored a wide variety of examples, the last chapter focuses on a simple question: *why games*? After all, aren't games simply a diversion from the more important things we should actually be doing? To close things out, I'll try to convince you (if I haven't already) why games have value.

Though mathematics is undeniably one of humankind's noblest pursuits, for many people it simply isn't as engaging as killing zombies or building empires. But part of this is a marketing problem, one that mathematics has suffered from for far too long. What follows is my attempt to convince you that mathematics can be fun and frequently appears in unexpected places; or, if you need no convincing, to at least give you some interesting examples to motivate mathematical thinking.

Ready? Let's-a go!

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