Introduction

The following considerations are grouped around the preceding list of questions and suggestions entitled “How to Solve It.” Any question or suggestion quoted from it will be printed in italics, and the whole list will be referred to simply as “the list” or as “our list.”

The following pages will discuss the purpose of the list, illustrate its practical use by examples, and explain the underlying notions and mental operations. By way of preliminary explanation, this much may be said: If, using them properly, you address these questions and suggestions to yourself, they may help you to solve your problem. If, using them properly, you address the same questions and suggestions to one of your students, you may help him to solve his problem.

The book is divided into four parts.

The title of the first part is “In the Classroom.” It contains twenty sections. Each section will be quoted by its number in heavy type as, for instance, “section 7.” Sections 1 to 5 discuss the “Purpose” of our list in general terms. Sections 6 to 17 explain what are the “Main Divisions, Main Questions” of the list, and discuss a first practical example. Sections 18, 19, 20 add “More Examples.”

The title of the very short second part is “How to Solve It.” It is written in dialogue; a somewhat idealized teacher answers short questions of a somewhat idealized student.

The third and most extensive part is a “Short Dictionary of Heuristic”; we shall refer to it as the “Dictionary.”
It contains sixty-seven articles arranged alphabetically. For example, the meaning of the term HEURISTIC (set in small capitals) is explained in an article with this title on page 112. When the title of such an article is referred to within the text it will be set in small capitals. Certain paragraphs of a few articles are more technical; they are enclosed in square brackets. Some articles are fairly closely connected with the first part to which they add further illustrations and more specific comments. Other articles go somewhat beyond the aim of the first part of which they explain the background. There is a key-article on MODERN HEURISTIC. It explains the connection of the main articles and the plan underlying the Dictionary; it contains also directions how to find information about particular items of the list. It must be emphasized that there is a common plan and a certain unity, because the articles of the Dictionary show the greatest outward variety. There are a few longer articles devoted to the systematic though condensed discussion of some general theme; others contain more specific comments, still others cross-references, or historical data, or quotations, or aphorisms, or even jokes.

The Dictionary should not be read too quickly; its text is often condensed, and now and then somewhat subtle. The reader may refer to the Dictionary for information about particular points. If these points come from his experience with his own problems or his own students, the reading has a much better chance to be profitable.

The title of the fourth part is “Problems, Hints, Solutions.” It proposes a few problems to the more ambitious reader. Each problem is followed (in proper distance) by a “hint” that may reveal a way to the result which is explained in the “solution.”

We have mentioned repeatedly the “student” and the “teacher” and we shall refer to them again and again. It
may be good to observe that the “student” may be a high school student, or a college student, or anyone else who is studying mathematics. Also the “teacher” may be a high school teacher, or a college instructor, or anyone interested in the technique of teaching mathematics. The author looks at the situation sometimes from the point of view of the student and sometimes from that of the teacher (the latter case is preponderant in the first part). Yet most of the time (especially in the third part) the point of view is that of a person who is neither teacher nor student but anxious to solve the problem before him.