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Vladimir G. Berkovich: Integration of One-forms on P-adic Analytic Spaces

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Introduction

One of the basic facts of complex analysis is the exactness of the de Rham complex of *sheaves* of analytic differential forms on a smooth complex analytic space. In its turn, its proof is based on the fact that every point of such a space has an open neighborhood isomorphic to an open polydisc, which reduces the verification of the exactness to the classical Poincaré lemma. The latter states that the de Rham complex of *spaces* of analytic differential forms on an open polydisc is exact. Its proof actually works over any non-Archimedean field k of characteristic zero as well, and so it implies also that the de Rham complex of sheaves of analytic differential forms on a smooth k -analytic space (as introduced in [Ber1] and [Ber2]) is exact at every point that admits a fundamental system of étale neighborhoods isomorphic to an open polydisc. One can show (Corollary 2.3.3) that a point x of a smooth k -analytic space possesses the above property if and only if the non-Archimedean field $\mathcal{H}(x)$, associated with the point x , possesses the property that its residue field $\widetilde{\mathcal{H}(x)}$ is algebraic over \widetilde{k} and the group $|\mathcal{H}(x)^*|/|k^*|$ is torsion.

It is a distinctive feature of non-Archimedean analytic spaces that the subset X_{st} of points with the latter property does not coincide with the whole space X . Notice that X_{st} contains the set $X_0 = \{x \in X \mid |\mathcal{H}(x) : k| < \infty\}$ (the underlying space of X in rigid analytic geometry) and, in particular, the set of k -rational points $X(k) = \{x \in X \mid \mathcal{H}(x) = k\}$. Although X is locally arcwise connected, the topology induced on X_{st} is totally disconnected and, if the valuation on k is nontrivial, X_{st} is dense in X . Moreover, if X is smooth, X_{st} is precisely the set of points at which the de Rham complex is exact and, in fact, for every point $x \notin X_{\text{st}}$ there is a closed one-form, defined in an open neighborhood of x , that has no primitive at any étale neighborhood of x .

We now recall that a locally analytic function is a map $f : X(k) \rightarrow k$ such that, for every point $x \in X(k)$, there is an analytic function g defined on an open neighborhood U of x with $f(y) = g(y)$ for all $y \in U(k)$. It is clear that the local behavior of such a function does not determine its global behavior. For example, if its differential is zero, the function is not necessarily constant. On the other hand, for a long time number theorists have been using very natural locally analytic functions possessing certain properties that make them look like analytic ones. An example of such a function (for $X = \mathbf{G}_m = \mathbf{A}^1 \setminus \{0\}$) is a homomorphism $k^* \rightarrow k$ from the multiplicative to the additive group of k which extends the homomorphism $a \mapsto \log(a)$ on the subgroup $k^1 = \{a \in k^* \mid |a - 1| < 1\}$, where $\log(T)$ is the usual logarithm defined by the power series $-\sum_{i=1}^{\infty} \frac{(1-T)^i}{i}$ (convergent on k^1).

Let us assume (till the end of the introduction) that k is a closed subfield of \mathbf{C}_p , the completion of the algebraic closure $\overline{\mathbf{Q}_p}$ of the field of p -adic numbers \mathbf{Q}_p .

