Introduction

"It's a crying shame that Weyl is leaving Zurich. He is a great master."[1] Thus Albert Einstein described Hermann Weyl (1885–1955), who remains a legendary figure, "one of the greatest mathematicians of the first half of the twentieth century. . . . No other mathematician could claim to have initiated more of the theories that are now being explored," as Michael Atiyah put it.[2] Weyl deserves far wider renown not only for his importance in mathematics and physics but also because of his deep philosophical concerns and thoughtful writing. To that end, this anthology gathers together some of Weyl's most important general writings, especially those that have become unavailable, have not previously been translated into English, or were unpublished. Together, they form a portrait of a complex and fascinating man, poetic and insightful, whose "vision has stood the test of time."[3]

This vision has deeply affected contemporary physics, though Weyl always considered himself a mathematician, not a physicist.[4] The present volume emphasizes his treatment of philosophy and physics, but another complete anthology could be made of Weyl's general writings oriented more directly toward mathematics. Here, I have chosen those writings that most accessibly show how Weyl synthesized philosophy, physics, and mathematics.

Weyl's philosophical reflections began in early youth. He recollects vividly the worn copy of a book about Immanuel Kant's *Critique of Pure Reason* he found in the family attic and read avidly at age fifteen. "Kant's teaching on the 'ideality of space and time' immediately took powerful hold of me; with a jolt I was awakened from my 'dogmatic slumber,' and the mind of the boy found the world being questioned in radical fashion." At the same time, he was drinking deep of great mathematical works. As "a country lad of eighteen," he fell under the spell of David Hilbert, whom he memorably described as a Pied Piper "seducing so many rats to follow him into the deep river of mathematics"; the following summer found Weyl poring over Hilbert's *Report on the Theory of Algebraic Numbers* during "the happiest months of my life."[5] As these stories reveal, Weyl was a very serious man; Princeton students called him "holy Hermann" among themselves, mocking a kind of earnestness probably more common in Hilbert's Göttingen. There, under brilliant teachers who also included Felix Klein and Hermann Minkowski, Weyl completed his mathematical apprenticeship. Forty years later, at the Princeton Bicentennial in 1946, Weyl gave a personal overview of this period and of the first discoveries that led him to find a place of distinction at Hilbert's side. This address, never before published, may be a good place to begin if you want to encounter the man and hear directly what struck him most. Do not worry if you find the mathematical references unfamiliar; Weyl's tone and angle of vision express his feelings about the mathematics (and mathematicians) he cared for in unique and evocative ways; he describes "Koebe the rustic and Brouwer the mystic" and the "peculiar gesture of his hands" Koebe used to define Riemann surfaces, for which Weyl sought "a more dignified definition."

In this address, Weyl also vividly recollects how Einstein's theory of general relativity affected him after the physical and spiritual desolation he experienced during the Great War. "In 1916 I had been discharged from the German army and returned to my job in Switzerland. My mathematical mind was as blank as any veteran's and I did not know what to do. I began to study algebraic surfaces; but before I had gotten far, Einstein's memoir came into my hands and set me afire."[6] Both Weyl and Einstein had lived in Zurich and taught at its Eidgenössische Technische Hochschule (ETH) during the very period Einstein was struggling to find his generalized theory, for which he needed mathematical help.[7] This was a golden period for both men, who valued the freer spirit they found in Switzerland, compared to Germany. Einstein adopted Swiss citizenship, completed his formal education in his new country, and then worked at its patent office. Among the first to realize the full import of Einstein's work, especially its new, more general, phase, Weyl gave lectures on it at the ETH in 1917, published in his eloquent book *Space-Time-Matter* (1918). Not only the first (and perhaps the greatest) extended account of Einstein's general relativity, Weyl's book was immensely influential because of its profound sense of perspective, great expository clarity, and indications of directions to carry Einstein's work further. Einstein himself praised the book as a "symphonic masterpiece."[8]

As the first edition of *Space-Time-Matter* went to press, Weyl reconsidered Einstein's ideas from his own mathematical perspective and came upon a new and intriguing possibility, which Einstein immediately called "*a first-class stroke of genius*."[9] Weyl describes this new idea in his essay "Electricity and Gravitation" (1921), much later recollecting some interesting personal details in his Princeton address. There, Weyl recalls explaining to a student, Willy Scherrer, "that vectors when carried around by parallel displacement may return to their starting point in changed direction. And he asked 'Also with changed length?' Of course I gave him the orthodox answer [no] at that moment, but in my bosom gnawed the doubt." To be sure, Weyl wrote this remembrance thirty years later, which thus may or may not be a perfectly faithful record of the events; nevertheless, it represents Weyl's own self-understanding of the course of his thinking, even if long after the fact. Though Weyl does not mention it, this conversation was surrounded by a complex web of relationships: Weyl's wife Helene was deeply involved with Willy's brother Paul, while Weyl himself was the lover of Erwin Schrödinger's wife, Anny. These personal details are significant here because Weyl himself was sensitive to the erotic aspects of scientific creativity in others, as we will see in his commentary on Schrödinger, suggesting that Weyl's own life and works were similarly intertwined.[10]

In *Space-Time-Matter*, Weyl used the implications of parallel transport of vectors to illuminate the inner structure of the theory Einstein had originally phrased in purely *metric* terms, meaning the measurement of distances between points, on the model of the Pythagorean theorem.[11] Weyl questioned the implicit assumption that behind this metrical structure is a fixed, given distance scale, or "gauge." What if the direction as well as the length of meter sticks (and also the standard second given by clocks) were to vary at different places in space-time, just as railway gauge varies from country to country? Perhaps Weyl's concept began with this kind of homely observation about the "gauge relativity" in the technology of rail travel, well-known to travelers in those days, who often had to change trains at frontiers between nations having different, incompatible railway gauges.[12] By considering this new kind of relativity, Weyl stepped even beyond the general coordinate transformations Einstein allowed in his general theory so as to incorporate what Weyl called *relativity of magnitude*.

In what Weyl called an "affinely connected space," a vector could be displaced parallel to itself, at least to an infinitely nearby point. As he realized after talking with Willy Scherrer, in such a space a vector transported around a closed curve might return to its starting point with changed direction *and* length (which he called "non-integrability," as measured mathematically by the "affine connection"). Peculiar as this changed length might seem, Weyl was struck by the mathematical generality of this possibility, which he explored in what he called his "purely infinitesimal geometry," which emphasized infinitesimal displacement as the foundation in terms of which any finite displacement needed to be understood.[13] As he emphasized the centrality of the infinite in mathematics, Weyl also placed the infinitesimal before the finite.

Weyl also realized that his generalized theory gave him what seemed a natural way to incorporate electromagnetism into the structure of space-time, a goal that had eluded Einstein, whose theory treated electromagnetism along with matter as mass-energy *sources* that caused space-time curvature but remained separate from space-time itself. Here Weyl used the literal "gauging" of distances as the basis of a mathematical analogy; his reinterpretation of these equations led naturally to a *gauge field* he could then apply to electromagnetism, from which Maxwell's equations now emerged as intrinsic to the structure of space-time. Though Einstein at first hailed this "stroke of genius," soon he found what he considered a devastating objection: because of the non-integrability of Weyl's gauge field, atoms would not produce the constant, universal spectral lines we actually observe: atoms of hydrogen on Earth give the same spectrum as hydrogen observed telescopically in distant stars. Weyl's 1918 paper announcing his new theory appeared with an unusual postscript by Einstein, detailing his objection, along with Weyl's reply that the actual behavior of atoms in turbulent fields, not to speak of measuring rods and clocks, was not yet fully understood. Weyl noted that his theory used light signals as a fundamental standard, rather than relying on supposedly rigid measuring rods and idealized clocks, whose atomic structure was in some complex state of accommodation to ambient fields.[14]

In fact, the atomic scale was the arena in which quantum theory was then emerging. Here began the curious migration of Weyl's idea from literally regauging length and time to describing some other realm beyond space-time. Theodor Kaluza (1922) and Oskar Klein (1926) proposed a generalization of general relativity using a fifth dimension to accommodate electromagnetism. In their theory, Weyl's gauge factor turns into a *phase* factor, just as the relative phase of traveling waves depends on the varying dispersive properties of the medium they traverse. If so, Weyl's gauge would no longer be immediately observable (as Einstein's objection asserted) because the gauge affects only the *phase*, not the observable *frequency*, of atomic spectra.[15]

At first, Weyl speculated that his 1918 theory gave support to the radical possibility that "matter" is only a form of curved, empty space (a view John Wheeler championed forty years later). Here Weyl doubtless remembered the radical opinions of Michael Faraday and James Clerk Maxwell, who went so far as to consider so-called matter to be a nexus of immaterial lines of force.[16]

Weyl then weighed these mathematical speculations against the complexities of physical experience. Though he still believed in his fundamental insight that gauge invariance was crucial, by 1922 Weyl realized that it needed to be reconsidered in light of the emergent quantum theory. Already in 1922, Schrödinger pointed out that Weyl's idea could lead to a new way to understand quantization. In 1927, Fritz London argued that the gravitational scale factor implied by Weyl's 1918 theory, which Einstein had argued was unphysical, actually makes sense as the complex phase factor essential to quantum theory.[17]

As Schrödinger struggled to formulate his wave equation, at many points he relied on Weyl for mathematical help. In their liberated circles, Weyl remained a valued friend and colleague even while being Anny Schrödinger's lover. From that intimate vantage point, Weyl observed that Erwin "did his great work during a late erotic outburst in his life," an intense love affair simultaneous with Schrödinger's struggle to find a quantum wave equation. But then, as Weyl inscribed his 1933 Christmas gift to Anny and Erwin (a set of erotic illustrations to Shakespeare's *Venus and Adonis*), "The sea has bounds but deep desire has none."[18]

Weyl's insight into the nature of quantum theory comes forward in a pair of letters he and Einstein wrote in 1922, here reprinted and translated for the first time, responding to a journalist's question about the significance of the new physics. Einstein dismisses the question: for him in 1922, relativity theory changes nothing fundamental in our view of the world, and that is that. Weyl takes the question more seriously, finding a radically new insight not so much in relativity theory as in the emergent quantum theory, which Weyl already understood as asserting that "the entire physics of matter is statistical in nature," showing how clearly he understood this decisive point several years before the formulation of the new quantum theory in 1925–1926 by Max Born, Werner

Heisenberg, Pascual Jordal, P.A.M. Dirac, and Schrödinger. In his final lines, Weyl also alludes to his view of matter as agent (*agens*), in which he ascribed to matter an innate activity that may have helped him understand and accept the spontaneity and indeterminacy emerging in quantum theory. This view led Weyl to reconsider the significance of the concept of a field. As he wrote to Wolfgang Pauli in 1919, "field physics, I feel, really plays only the role of 'world geometry'; in matter there resides still something different, [and] real, that cannot be grasped causally, but that perhaps should be thought of in the image of 'independent decisions,' and that we account for in physics by statistics."[19]

In the years around 1920, Weyl continued to work out the consequences of this new approach. His conviction about the centrality of consciousness as intuition and activity deeply influenced his view of matter. As the ego drives the whole world known to consciousness, he argued that "matter is analogous to the ego, the effects of which, despite the ego itself being non-spatial, originate via its body at a given point of the world continuum. Whatever the nature of this *agens*, which excites the field, might be—perhaps life and will—in physics we only look at the field effects caused by it." This took him in a direction very different from the vision of matter reduced to pure geometry he had entertained in 1918. Writing to Felix Klein in 1920, Weyl noted that "field physics no longer seems to me to be the key to reality; but the field, the ether, is to me only a totally powerless *transmitter* by itself of the action, but matter rests beyond the field and is the reality that *causes* its states." Weyl described his new view in 1923 using an even more striking image: "Reality does not move into space as into a right-angled tenement house along which all its changing play of forces glide past without leaving any trace; but rather as the snail, matter itself builds and shapes this house of its own." For Weyl now, fields were "totally powerless *transmitters*" that are not really existent or effectual in their own right, but only a way of talking about states of *matter* that are the locus of fundamental reality. Though Weyl still retained fields to communicate interactions, his emphasis that the reality of "matter rests beyond the field" may have influenced Richard Feynman and Wheeler two decades later in their own attempt to remove "fields" as independent beings. Weyl also raised the question of whether matter has some significant topological structure on the subatomic scale, as if topology were a kind of activation that brings static geometry to life, analogous to the activation the ego infuses into its world. Such topological aspects of matter only emerged as an important frontier of contemporary investigation fifty years later.[20]

Looking back from 1955 at his original 1918 paper, Weyl noted that he "had no doubt" that the correct context of his vision of gauge theory was "not, as I believed in 1918, in the intertwining of electromagnetism and gravity" but in "the Schrödinger-Dirac potential ψ of the electron-positron field. . . . The strongest argument for my theory seems to be this, that gauge-invariance corresponds to the conservation of electric charge in the same way that coordinate-invariance corresponds to the conservation of energy and momentum," the insight that Emmy Noether's famous theorem put at the foundations of quantum field theory.[21] Nor did Weyl himself stop working on his idea; in 1929 he published an important paper reformulating his idea in the language of what today are called gauge fields; these considerations also led him to consider fundamental physical symmetries long before the discovery of the violation of parity in the 1950s. The "Weyl two-component neutrino field" remains a standard description of neutrinos, all of which are "left-handed" (spin always opposed to direction of motion), as all antineutrinos are "right-handed." In 1954 (a year before Weyl's death, but apparently not known to him), C. N. Yang, R. Mills, and others took the next steps in developing gauge fields, which ultimately became the crucial element in the modern "standard model" of particle physics that triumphed in the 1970s, unifying strong, weak, and electromagnetic interactions in ways that realized Weyl's distant hopes quite beyond his initial expectations.[22]

In the years that Weyl continued to try to find a way to make his idea work, he and Einstein underwent a curious exchange of positions. Originally, Einstein thought Weyl was not paying enough attention to physical measuring rods and clocks because Weyl used immaterial light beams to measure space-time. As Weyl recalled in a letter of 1952,

> I thought to be able to answer his concrete objections, but in the end he said: "Well, Weyl, let us stop this. For what I actually have against your theory is: 'It is impossible to do physics like this (i.e., in such a speculative fashion, without a guiding intuitive physical principle)!' " Today we have probably changed our viewpoints in this respect: E. believes that in this domain the chasm between ideas and experience is so large, that only mathematical speculation (whose consequences, of course, have to be developed and confronted with facts) gives promise of success, while my confidence in pure speculation has diminished and a closer connection with quantum physical experience

> seems necessary, especially as in my view it is not sufficient to blend gravitation and electricity to one unity, but that the wave fields of the electron (and whatever there may still be of nonreducible elementary particles) must be included.[23]

Ironically, Weyl the mathematician finally sided with the complex realities of physics, whereas Einstein the physicist sought refuge in unified field theories that were essentially mathematical. Here is much food for thought about the philosophic reflections each must have undertaken in his respective soul searching and that remain important now, faced with the possibilities and problems of string theory, loop quantum gravity, and other theoretical directions for which sufficient experimental evidence may long remain unavailable.

Both here and throughout his life, Weyl used philosophical reflection to guide his theoretical work, preferring "to approach a question through a deep analysis of the concepts it involves rather than by blind computations," as Jean Dieudonné put it. Though others of his friends, such as Einstein and Schrödinger, shared his broad humanistic education and philosophical bent, Weyl tended to go even further in this direction. As a young student in Göttingen, Weyl had studied with Edmund Husserl (who had been a mathematician before turning to philosophy), with whom Helene Weyl had also studied.[24]

Weyl's continuing interest in phenomenological philosophy marks many of his works, such as his 1927 essay on "Time Relations in the Cosmos, Proper Time, Lived Time, and Metaphysical Time," here reprinted and translated for the first time. The essay's title indicates its scope, beginning with his interpretation of the four-dimensional space-time Hermann Minkowski introduced in 1908, which Weyl then connects with human time consciousness (also a deep interest of Husserl's). Weyl treats a world point not merely as a mathematical abstraction but as situating a "point-eye," a living symbol of consciousness peering along its world line. Counterintuitively, that point-eye associates the objective with the relative, the subjective with the absolute.

Weyl uses this striking image to carry forward a mathematical insight that had emerged earlier in his considerations about the nature of the continuum. During the early 1920s, Weyl was deeply drawn to L.E.J. Brouwer's advocacy of intuition as the critical touchstone for modern mathematics. Thus, Brouwer rejected Cantor's transfinite numbers as not intuitable, despite Hilbert's claim that "no one will drive us from the paradise which Cantor created for us." Hilbert argued that mathematics should be considered purely formal, a great game in which terms like "points" or "lines" could be replaced with arbitrary words like "beer mugs" or "tables" or with pure symbols, so long as the axiomatic relationships between the respective terms do not change. Was this, then, the "deep river of mathematics" into which Weyl thought this Pied Piper had lured him and so many other clever young rats?[25]

By the mid-1920s, Weyl was no longer an advocate of Brouwer's views (though still reaffirming his own 1918 work on the continuum). In his magisterial *Philosophy of Mathematics and Natural Science* (written in 1927 but extensively revised in 1949), Weyl noted that "mathematics with Brouwer gains its highest intuitive clarity. . . . It cannot be denied, however, that in advancing to higher and more general theories the inapplicability of the simple laws of classical logic eventually results in an almost unbearable awkwardness. And the mathematician watches with pain the larger part of his towering edifice which he believed to be built of concrete blocks dissolve into a mist before his eyes."[26]

Even so, Weyl remained convinced that we should not consider a continuum (such as the real numbers between 0 and 1) as an actually completed and infinite set but only as capable of endless subdivision. This understanding of the "potential infinite" recalls Aristotle's critique of the "actual infinite." In his 1927 essay on "Time Relations," Weyl applied this view to time as a continuum. Because an infinitely small point could be generated from a finite interval only through actually completing an infinite process of shrinkage, Weyl applies the same argument to the presumption that the present instant is a "point in time." He concludes that "there is no pointlike Now and also no exact earlier and later."[27] Weyl's arguments about the continuum have the further implication that the past is *never completely determined*, any more than a finite, continuous interval is ever exhaustively filled; both are potentially infinite because always further divisible. If so, the past is not fixed or unchangeable and continues to change, a luxuriant, ever-proliferating tangle of "world tubes," as Weyl called them, "open into the future and again and again a fragment of it is lived through." This intriguing idea is psychologically plausible: A person's past seems to keep changing and ramifying as life unfolds; the past today seems different than it did yesterday. As a character in Faulkner put it, "the past is never dead. It's not even past."[28]

In Weyl's view, a field is intrinsically continuous, endlessly subdividable, and hence an abyss in which we never come to an ultimate *point* where a decision can be made: To be or not to be? Conversely, pointlike, discrete matter is a locus of decisive spontaneity because it is not predictable through continuous field laws, only observable statistically. As Weyl wrote to Pauli in 1919, "I am firmly convinced that statistics is in principle something independent from causality, the 'law'; because it is in general absurd to imagine a continuum as something like a finished being." Because of this independence, Weyl continued in 1920,

> the future will act on and upon the present and it will determine the present more and more precisely; the past is not finished. Thus, the fixed pressure of natural causality disappears and there remains, irrespective of the validity of the natural laws, *a space for autonomous and causally absolutely independent decisions*; I consider the elementary quanta of matter to be the place of these decisions.[29]

"Lived time," in Weyl's interpretation, keeps evoking the past into further life, even as it calls the future into being. Weyl's deep thoughts may still repay the further exploration they have not received so far.

Weyl also contributed notably to the application of general relativity to cosmology. He found new solutions to Einstein's equation and already in 1923 calculated a value for the radius of the universe of roughly one billion light years, six years before Edwin Hubble's systematic measurements provided what became regarded as conclusive evidence that our galaxy is only one among many. Weyl also reached a seminal insight, derived from both his mathematical and his philosophical considerations, that the topology of the universe is "the first and most important question in all speculations on the world as a whole." This prescient insight was taken up only in the 1970s and remains today at the forefront of cosmology, still unsolved and as important as Weyl thought. He also noted that relativistic cosmology indeed "left the door open for possibilities of every kind." The mysteries of dark energy and dark matter remind us of how much still lies beyond that door. Then too, we still face the questions Weyl raised regarding the strange recurrence throughout cosmology of the "large numbers" like 10^{20} and 10^{40} (seemingly as ratios between cosmic and atomic scales), later rediscovered by Dirac.[30]

Other of Weyl's ideas long ago entered and transformed the mainstream of physics, characteristically bridging the mathematical and physical through the philosophical. He considered his greatest mathematical work the classification of the semisimple groups of continuous symmetries (Lie groups), which he later surveyed in *The Classical Groups: Their Invariants and Representations* (1938).

In the introduction to this first book he wrote in English, Weyl noted that "the gods have imposed upon my writing the yoke of a foreign tongue that was not sung at my cradle." But even in his adopted tongue he does not hesitate to critique the "too thorough technicalization of mathematical research" in America that has led to "a mode of writing which must give the reader the impression of being shut up in a brightly illuminated cell where every detail sticks out with the same dazzling clarity, but without relief. I prefer the open landscape under a clear sky with its depth of perspective, where the wealth of sharply defined nearby details gradually fades away toward the horizon." Such writing exemplifies Weyl's uniquely eloquent style.

Soon after quantum theory had first been formulated, Weyl used his deep mathematical perspective to shape *The Theory of Groups and Quantum Mechanics* (1928). It is hard to overstate the importance of his marriage of the mathematical theory of symmetry to quantum theory, which has proved ever more fruitful, with no end in sight. At first, as eminent and hardheaded a physicist as John Slater resisted the "group-pest" as if it were a plague of abstractness. But Weyl, along with Eugene Wigner, prevailed because the use of group theory gave access to the symmetries essential for formulating all kinds of physical theories, from crystal lattices to multiplets of fundamental particles. It was this depth and generality that moved Julian Schwinger to "read and re-read that book, each time progressing a little farther, but I cannot say that I ever—not even to this day—fully mastered it." Thus, Schwinger considered Weyl "one of my gods," not merely an outstanding teacher, because "the ways of gods are mysterious, inscrutable, and beyond the comprehension of ordinary mortals."[31] This from someone regarded as rather godlike by many physicists because of his own inscrutable powers. Weyl's insights about the fundamental mathematical symmetries led Schwinger and others decades later to formulate the *TCP* theorem, which expresses the fundamental identicality between particles and antiparticles under the combined symmetries of time reversal (T), charge conjugation (reversal of the sign of the charge, C), and parity (mirror) reversal (P).

In one of his most powerful interventions in physics, Weyl used such symmetry principles to argue that Dirac's newly proposed (and as-yet unobserved) "holes" (antielectrons) could not be (as Dirac had suggested) protons, which are almost two thousand times heavier than electrons. Weyl showed mathematically that antielectrons had to have the same mass as electrons, though having opposite charge; this was later confirmed by cosmic ray observations.

Weyl's purely mathematical argument struck Dirac, who drew from this experience his often-cited principle that "it is more important to have beauty in one's equations than to have them fit experiment," a principle that continues to be an important touchstone for many physicists. Even though Weyl's mathematics moved Dirac to this radical declaration, Weyl's own turn away from mathematical speculation about physics raises the question whether in the end to prefer beautiful mathematics to the troubling complexities of experience.[32]

Whether a "god" or no, Weyl seemed to feel that the philosophical enterprise cannot remain on the godlike plane but really requires the occasions of human conversation. The two largest works in this anthology contain the rich harvest of Weyl's long-standing interest in expressing his ideas to a broader audience; both began as lecture series, thus doubly public, both spoken and written. To use the apt phrase of his son Michael, *The Open World* (1932) contains "Hermann's dialogues with God" because here the mathematician confronts his ultimate concerns.[33] These do not fall into the traditional religious traditions but are much closer in spirit to Spinoza's rational analysis of what he called "God or nature," so important for Einstein as well. As Spinoza considered the concept of infinity fundamental to the nature of God, Weyl defines "God as the completed infinite." In Weyl's conception, God is not merely a mathematician but is mathematics itself because "*mathematics is the science of the infinite*," engaged in the paradoxical enterprise of seeking "the symbolic comprehension of the infinite with human, that is finite, means." In the end, Weyl concludes that this God "cannot and will not be comprehended" by the human mind, even though "mind is freedom within the limitations of existence; it is open toward the infinite." Nevertheless, "neither can God penetrate into man by revelation, nor man penetrate to him by mystical perception. The completed infinite we can only represent in symbols." In Weyl's praise of openness, this freedom of the human mind begins to seem even higher than the completed infinity essential to the meaning of God. Does not his argument imply that God, as actual infinite, can never be *actually* complete, just as an infinite time will never have passed, however long one waits? And if God's actuality will never come to pass, in what sense could or does or will God exist at all? Perhaps God, like the continuum or the field, is an infinite abyss that needs completion by the decisive seed of matter, of human choice.

Weyl inscribes this paradox and its possibilities in his praise of the *symbol*, which includes the mathematical no less than the literary, artistic, poetic, thus bridging the presumed chasm between the "two cultures." At every turn in his writing, we encounter a man of rich and broad culture, at home in many domains of human thought and feeling, sensitive to its symbols and capable of expressing himself beautifully. He moves so naturally from quoting the ancients and moderns to talking about space-time diagrams, thus showing us something of his innate turn of mind, his peculiar genius. His quotations and reflections are not mere illustrations but show the very process by which his thought lived and moved. His philosophical turn of mind helped him reach his own finest scientific and mathematical ideas. His self-deprecating disclaimer that he thus "wasted his time" might be read as irony directed to those who misunderstood him, the hardheaded who had no feeling for these exalted ideas and thought his philosophizing idle or merely decorative. Weyl gained perspective, insight, and altitude by thinking back along the ever-unfolding past and studying its great thinkers, whom he used to help him soar, like a bird feeling the air under its wings.

In contrast, Weyl's lectures on *Mind and Nature*, published only two years later (1934), have a less exalted tone. The difference shows his sensitivity to the changing times. Though invited to return to Göttingen in 1918, he preferred to remain in Zurich; finally in 1930, he accepted the call to succeed Hilbert, but almost immediately regretted it. The Germany he returned to had become dangerous for him, his Jewish wife, and his children. Unlike some who were unable to confront those ugly realities, Weyl was capable of political clear-sightedness; by 1933 he was seeking to escape Germany. His depression and uncertainty in the face of these huge decisions shows another side of his humanity; as Richard Courant put it, "Weyl is actually in spite of his enormously broad talents an inwardly insecure person, for whom nothing is more difficult than to make a decision which will have consequences for his whole life, and who mentally is not capable of dealing with the weight of such decisions, but needs a strong support somewhere."[34] That anxiety and inner insecurity gives Weyl's reflections their existential force. As he himself struggled along his own world line through endlessly ramifying doubts, he came to value the spontaneity and decisiveness he saw in the material world.

Weyl's American lectures marked the start of a new life, beginning with a visiting professorship at Princeton (1928–1929), where he revised his book on group theory and quantum mechanics in the course of introducing his insights to this new audience. Where in 1930 Weyl's *Open World* began with God, in 1933 his lectures on *Mind and Nature* start with human subjectivity and sense perception. Here, symbols help us confront a world that "does not exist in itself, but is merely encountered by us as an object in the correlative variance of subject and object." For Weyl, mathematical and poetic symbols may disclose a path through the labyrinth of "mirror land," a world that may seem ever more distorted, unreal on many fronts. Though Weyl discerns "an abyss which no realistic conception of the world can span" between the physical processes of the brain and the perceiving subject, he finds deep meaning in "the enigmatic twofold nature of the ego, namely that I am both: on the one hand a real individual which performs real psychical acts, the dark, striving and erring human being that is cast out into the world and its individual fate; on the other hand light which beholds itself, intuitive vision, in whose consciousness that is pregnant with images and that endows with meaning, the world opens up. Only in this 'meeting' of consciousness and being both exist, the world and I."[35]

Weyl treats relativity and quantum theory as the latest and most suggestive symbolic constructions we make to meet the world. The dynamic character of symbolism endures, even if the particular symbols change; "their truth refers to a connected system that can be confronted with experience only as a whole."[36] Like Einstein, Weyl emphasized that physical concepts as symbols "are constructions within a free realm of possibilities," freely created by the human mind. "Indeed, space and time are nothing in themselves, but only a certain order of the reality existing and happening in them." As he noted in 1947, "it has now become clear that physics needs no such ultimate objective entities as space, time, matter, or 'events,' or the like, for its constructions symbols without meaning handled according to certain rules are enough."[37] In *Mind and Nature*, Weyl notes that "in nature itself, as [quantum] physics constructs it theoretically, the dualism of object and subject, of law and freedom, is already most distinctly predesigned." As Niels Bohr put it, this dualism rests on "the old truth that we are both spectators and actors in the great drama of existence."

After Weyl left Germany definitively for Princeton in 1933, he continued to reflect on these matters. In the remaining selections, one notes him retelling some of the same stories, quoting the same passages from great thinkers of the past, repeating an idea he had already said elsewhere. These repetitions posed a difficult problem, for the later essays contain some interesting new points along with the old. Because of this, I decided to include these later essays, for Weyl's repetitions also show him reconsidering. Reiterating a point in a new or larger context may open further dimensions. Then too, we as readers are given another chance to think about Weyl's points and also see where he held to his earlier ideas and where he may have changed. For he *was* capable of changing his mind, more so than Einstein, whose native stubbornness may well have contributed to his unyielding resistance to quantum theory. As noted above, Weyl was far more able to entertain and even embrace quantum views, despite their strangeness, precisely because of his philosophical openness.[38]

Weyl's close reading of the past and his philosophical bent inspired his continued openness. In his hitherto unpublished essay "Man and the Foundations of Science" (written about 1949), Weyl describes "an ocean traveler who distrusts the bottomless sea and therefore clings to the view of the disappearing coast as long as there is in sight no other coast toward which he moves. I shall now try to describe the journey on which the old coast has long since vanished below the horizon. There is no use in staring in that direction any longer."[39] He struggles to find a way to speak about "a new coast [that] seems dimly discernible, to which I can point by dim words only, and maybe it is merely a bank of fog that deceives me." Here symbols might be all we have, for "it becomes evident that now the words 'in reality' must be put between quotation marks; we have a symbolic construction, but nothing which we could seriously pretend to be the true real world." Yet even legerdemain with symbols cannot hide the critical problem of the continuum: "The sin committed by the set-theoretic mathematician is his treatment of the field of possibilities open into infinity as if it were a completed whole all members of which are present and can be overlooked with one glance. For those whose eyes have been opened to the problem of infinity, the majority of his statements carry no meaning. If the true aim of the mathematician is to master the infinite by finite means, he has attained it by fraud only—a gigantic fraud which, one must admit, works as beautifully as paper money." By his reaffirmation of his critique of the actual infinite, we infer that Weyl continued to hold his radical views about "lived time," especially that "we stand at that intersection of bondage and freedom which is the essence of man himself."

Indeed, Weyl notes that he had put forward this relation between being and time years before Martin Heidegger's famous book on that subject appeared. Weyl's account of Heidegger is especially interesting because of the intersection between their concerns, no less than their deep divergences. Yet Weyl seemingly could not bring himself to give a full account of Heidegger or of his own reactions, partly based on philosophical antipathy, partly (one infers) from his profound distaste for Heidegger's involvement with the Nazi regime. Though he does not speak of it, Weyl may well also have known of the way Heidegger abandoned their teacher, Husserl, in those dark days. Most of all, Weyl conveys his annoyance that Heidegger had botched important ideas that were important to Weyl himself and, in the process, that Heidegger had lost sight of the nature of science. Taking up a crucial term they both use, Weyl asserts that "no other ground is left for science to build on than this dark but very solid rock which I once more call the concrete Dasein of man in his world." Weyl grounds this Dasein, man's being-in-the-world, in ordinary language, which is "neither tarnished poetry nor a blurred substitute for mathematical symbolism; on the contrary, neither the one nor the other would and could exist without the nourishing stem of the language of our everyday life, with all it complexity, obscurity, crudeness, and ambiguity." By thus connecting mathematical and poetic symbolism as both growing from the soil of ordinary human language, Weyl implicitly rejects Heidegger's turn away from modern mathematical science.

In his late essay "The Unity of Knowledge" (1954), Weyl reviews this ground and concludes that "the shield of Being is broken beyond repair," but does not take this disunity in a tragic sense because "on the side of Knowing there may be unity. Indeed, mind in the fullness of its experience has unity. Who says 'I' points to it." Here he reaffirms his old conviction that human consciousness is not simply the product of other, more mechanical forces, but is itself the luminous center constituting that reality through its "complex symbolic creations which this lumen built up in the history of mankind." Even though "myth, religion, and alas! also philosophy" fall prey to "man's infinite capacity for self-deception," Weyl implicitly holds out greater hope for the symbolic creations of mathematics and science, though he admits that he is still struggling to find clarity.

The final essay in this anthology, "Insight and Reflection" (1955), is Weyl's rich *Spätlese*, the intense, sweet wine made from grapes long on the vine. This philosophical memoir discloses his inner world of reflection in ways his other, earlier essays did not reveal quite so directly, perhaps aware of the skepticism and irony that may have met them earlier on. We are reminded of his "point-eye," disclosing his thoughts and feelings while creeping up his own world line. Nearing its end, Weyl seems freer to say what he feels, perhaps no longer caring who might mock. He gives his fullest avowal yet of what Husserl meant to him, but does not hold back his own reservations; Husserl finally does not help with Weyl's own deep question about "the relation between the one pure

I of immanent consciousness and the particular lost human being which I find myself to be in a world full of people like me (for example, during the afternoon rush hour on Fifth Avenue in New York)." Weyl is intrigued by Fichte's mystic strain, but in the end Fichte's program (analyzing everything in terms of I and not-I) strikes him as "preposterous." Weyl calls Meister Eckhart "the deepest of the Occidental mystics . . . a man of high responsibility and incomparably higher nobility than Fichte." Eckhart's soaring theological flight beyond God toward godhead stirred Weyl, along with Eckhart's fervent simplicity of tone. Throughout his account, Weyl interweaves his mathematical work, his periods of soberness after the soaring flights of philosophical imagination, though he presents them as different sides of what seems to his point-eye a unified experience. Near the end, he remembers with particular happiness his book *Symmetry* (1952), which so vividly unites the poetic, the artistic, the mathematical, and the philosophical, a book no reader of Weyl should miss.[40] In quoting T. S. Eliot that "the world becomes stranger, the pattern more complicated," we are aware of Weyl's faithful openness to that strangeness, as well as the ever more complex and beautiful symmetries he discerned in it.

Weyl's book on symmetry shows the fundamental continuity of themes throughout his life and work. Thinking back on the theory of relativity, Weyl describes it not (as many of his contemporaries had) as disturbing or revolutionary but really as "another aspect of symmetry" because "it is the inherent symmetry of the four-dimensional continuum of space and time that relativity deals with." Yet as beautifully as he evokes and illustrates the world of symmetry, Weyl still emphasizes the fundamental difference between perfect symmetry and life, with its spontaneity and unpredictability. "If nature were all lawfulness then every phenomenon would share the full symmetry of the universal laws of nature as formulated by the theory of relativity. The mere fact that this is not so proves that *contingency* is an essential feature of the world." Characteristically, Weyl recalls the scene in Thomas Mann's *Magic Mountain*

> in which his hero, Hans Castorp, nearly perishes when he falls asleep with exhaustion and leaning against a barn dreams his deep dream of death and love. An hour before when Hans sets out on his unwarranted expedition on skis he enjoys the play of the flakes "and among these myriads of enchanting little stars," so he philosophizes, "in their hidden splendor, too small for man's naked eye to see, there was not one like unto another; an endless inventiveness governed the development

> and unthinkable differentiation of one and the same basic scheme, the equilateral, equiangular hexagon. Yet each in itself—this was the uncanny, the antiorganic, the life-denying character of them all—each of them was absolutely symmetrical, icily regular in form. They were too regular, as substance adapted to life never was to this degree—the living principle shuddered at this perfect precision, found it deathly, the very marrow of death—Hans Castorp felt he understood now the reason why the builders of antiquity purposely and secretly introduced minute variation from absolute symmetry in their columnar structures."[41]

Weyl's own life and work no less sensitively traced out this interplay between symmetry and life, field and matter, mathematics and physics, reflection and action.

So rich and manifold are Weyl's writings that I have tried to include everything I could while avoiding excessive repetitiveness. I thank Erhard Scholz and Skúli Sigurdsson for their very helpful advice and for the guidance I gained from their own writings about Weyl; Nils Röller, Thomas Ryckman, Brandon Fogel, and Andrew Ayres were most friendly in sharing their thoughts and findings. I am especially grateful to Philip Bartok for giving me essential help with the translations, for which Norman Sieroka also offered invaluable critical guidance and advice; reading his own work on Weyl and corresponding with him was of great help to me. I thank the John Simon Guggenheim Memorial Foundation for its support, as well as Vickie Kearn and her associates at Princeton University Press for their enthusiastic collaboration. Finally, Michael Weyl and Annemarie Weyl Carr were most generous in sharing their recollections.

Not long after making his epochal contributions to quantum theory, Dirac was invited to visit universities across the United States. When he arrived in Madison, Wisconsin, in 1929, a reporter from the local paper interviewed him and learned from Dirac's laconic replies that his favorite thing in America was potatoes, his favorite sport Chinese chess.[42] Then the reporter wanted to ask him something more.

> "They tell me that you and Einstein are the only two real sure-enough high-brows and the only ones who can understand each other. I won't ask you if this is straight stuff for I know you are too modest to admit it. But I want to know this—Do you ever run across a fellow that even you can't understand?"

> "Yes," says he.
>
> "This will make a great reading for the boys down at the office," says I. "Do you mind releasing to me who he is?"
>
> "Weyl," says he.
>
> The interview came to a sudden end just then, for the doctor pulled out his watch and I dodged and jumped for the door. But he let loose a smile as we parted and I knew that all the time he had been talking to me he was solving some problem that no one else could touch.
>
> But if that fellow Professor Weyl ever lectures in this town again I sure am going to take a try at understanding him. A fellow ought to test his intelligence once in a while.

So should we—and here is Professor Weyl himself, in his own words.

Peter Pesic