Introduction

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I

Hermann Weyl (1885–1955) was, according to Fields medalist Sir Michael Atiyah, "one of the greatest mathematicians of the first half of the twentieth century." Every great mathematician is great in his or her own way, but Weyl's way was special. Most modern scientists choose one or a few narrow areas to explore, and look neither sideways nor back. Weyl was different; he surveyed the whole world.

A few words of biography: Weyl was a student of David Hilbert at Göttingen, and thus stood in the line of intellectual descent from Gauss, Riemann, and Dirichlet. Upon Hilbert's retirement, Weyl was invited to take up his chair, but conditions in 1930s Germany plus an attractive offer from the new Institute for Advanced Study combined to bring him to Princeton, where he stayed.

Together with Albert Einstein and John von Neumann, Weyl made the trinity of refugee stars who brought the new Institute matchless scientific luster. More than the rebellious Einstein or the protean von Neumann, who both grew up in it, Weyl embodied the grand German literary and pan-European cultural tradition that was rocked and then shattered by the two World Wars.

Weyl's most characteristic work is *Philosophy of Mathematics and Natural Science*. No other book I know is like it. No one else could have written it. The main body of text was written in German in 1926, as an article for R. Oldenburg's *Handbuch der Philosophie*. In 1947, for the English translation, Weyl altered many details and added six appendices, comprising almost a hundred pages, which center on relevant scientific events in the intervening years (the first of these, on Gödel's theorem, and the third, on quantum physics and causality, are especially brilliant); but the core had its genesis in the vanished Handbuch tradition of magisterial reviews in natural philosophy.

In his preface Weyl says, "I was also bound, though less consciously, by the German literary and philosophical tradition in which I had grown up" (xv). It was in fact a cosmopolitan tradition, of which *Philosophy of Mathematics and Natural Science* might be the last great expression. Descartes, Leibniz, Hume, and Kant are taken as familiar friends and interlocutors. Weyl's erudition is, implicitly, a touching affirmation of a community of mind and inquiry stretching across time and space, and progressing through experience, reflection, and open dialogue. Between

1926 and 1947, of course, the specifically German literary-philosophical tradition experienced a traumatic discontinuity. In his reflective conclusion, however, Weyl reaffirms the universal:

> The more I look into the philosophical literature the more I am impressed with the general agreement regarding the most essential insights of natural philosophy as it is found among all those who approach the problems seriously and with a free and independent mind. (216)

Apart from its lasting historical, philosophic, and scientific value, *Philosophy of Mathematics and Natural Science* contains passages of poetic eloquence. This, I think, ranks among the most beautiful and profound passages in all of literature:

> The objective world simply *is*, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time. (116)

As does this:

> Leibniz . . . believed that he had resolved the conflict of human freedom and divine predestination by letting God (for sufficient reasons) assign existence to certain of the infinitely many possibilities, for instance to the beings Judas and Peter, whose substantial nature determines their entire fate. This solution may objectively be sufficient, but it is shattered by the desperate outcry of Judas, "Why did *I* have to be Judas?" (124–125)

I've consulted *Philosophy of Mathematics and Natural Science* many times, and each time I've come away enriched. By now, of course, many of the topics Weyl addressed there look quite different. Some of his questions have been answered; some of his assumptions have even been disproved. (In the following section, I mention several important examples.) Even in these cases, *Philosophy of Mathematics and Natural Science* instructs and inspires. For by showing us how a great thinker in command of the best thought of his era could see the world quite differently, it both invites us to stretch our minds in empathy and reminds us how far we've come. But many of *Philosophy of Mathematics and Natural Science*'s questions are still with us, very much alive. Near the end, for example, after a penetrating critique of the concept of causality, Weyl turns to what he calls the body-soul problem, what today is called the problem of consciousness:

> It is an altogether too mechanical conception of causality which views the mutual effects of body and soul as being so paradoxical

> that one would rather resort, like Descartes, to the occasionalisitic intervention of God or, like Leibniz, to a harmony instituted at the beginning of time.
>
> The real riddle, if I am not mistaken, lies in the double position of the ego: it is not merely an existing individual which carries out real psychic acts but also 'vision,' a self-penetrating light (sense-giving consciousness, knowledge, image, or however you may call it); as an individual capable of positing reality, its vision open to reason; "a force into which an eye has been put," as Fichte says. (215–216)

Here I think science has yet to catch up with, and bring to fruition, Weyl's visionary intuition.

II

Much has happened in mathematics and natural science since Weyl updated his *Philosophy of Mathematics and Natural Science*. This classic speaks to fundamental issues, and considers them profoundly. Even in an overview from such lofty heights, however, we shouldn't fail to notice big changes in the landscape.

Physical cosmology has come of age. The essential correctness of the Big Bang picture is no longer in doubt. It is supported by a dense web of evidence, including detailed mapping of distant galaxies and their redshifts, allowing reconstruction of the history of cosmic expansion; the concordant ages of the oldest stars; the existence of an accurate blackbody relic background radiation at 2.7° Kelvin; a successful evolutionary account of the relative abundance of different chemical elements, and more. Quantitative understanding of the emergence of structure in the universe, demonstrating how it arises from very small early inhomogeneities (observed in the background radiation!) amplified over time by gravitational instabilities, is a recent triumph. The most popular theoretical explanation for the existence of those initial inhomogeneities traces them to quantum fluctuations, normally confined to the subatomic domain, that get stretched to cosmic dimensions during a period of cosmic inflation.

Physical biology has come of age. Central metabolic processes, and the basis of heredity, are understood at the level of specific chemical reactions and molecules. At this level, the essential unity of life is revealed. Yeast, fruitfly, mouse, and human run on the same molecular principles. Study of changes in genomes, against a common background, allows us to reconstruct the history of biological evolution, with a depth of detail Weyl would have found astonishing.

But what I think would have delighted Weyl most of all is the triumph of the two concepts closest to his own heart and most central to his work: symmetry and symbol.

SYMMETRY

Symmetry has proven a fruitful guide to the fundamentals of physical reality, both when it is observed and when it is not!

The parity symmetry P, which asserts the equivalence of left and right in the basic laws of physics, fascinated Weyl, and in *Philosophy of Mathematics and Natural Science* he emphasized its fundamental importance. In 1956, however, P was discovered to fail—spectacularly—in the so-called weak interaction (which is responsible for beta radioactivity and many elementary particle decays). Indeed, the failure appeared in some sense *maximal.* Consider, for example, the most common decay mode of a muon (μ^-), namely, its decay into an electron, muon neutrino, and electron antineutrino:

$$\mu^- \to e^- \ \nu_\mu \bar{\nu}_e.$$

In this process, the emitted electron is almost always found to be *left-handed*: if you point your *left* thumb in the direction of the electron's motion, your curled-up fingers will point in the direction that it rotates. On the other hand (no pun intended!), if you use your right hand, you'll get the sense of rotation wrong. So there is an objective physical distinction between left and right, contrary to the "law" of parity symmetry.

A slightly more complex symmetry was then proposed as a more accurate refinement of spatial parity (P). This is the combined parity (CP) transformation. It supplements spatial parity with the transformation of particles into their antiparticles, the so-called charge conjugation transformation (C). In other words, CP requires that you both perform spatial inversion and simultaneously change particles into antiparticles. Thus, for example, CP relates our muon decay process to a process of antimuon decay,

$$\mu^+ \to e^+ \ \bar{\nu}_\mu \nu_e,$$

in which the final *anti*-electron is *right-handed.* If CP is a valid symmetry, then these decays must occur at the same rate. But they don't, quite. Although CP is a much more accurate symmetry than P, it too fails. For many years the only observed failure of CP symmetry occurred in K meson decays, where the asymmetry was a small and subtle effect. Recently, principally through the study of B meson decays, the phenomenology of CP violation has become a rich subject, in which very subtle aspects of quantum theory are beautifully deployed.

What P does for space, time-reversal T does for time. Time-reversal symmetry asserts the equivalence of past and future, *in the microscopic laws of physics.* Of course, both the concrete history of the universe and (at a mundane, but more specific and practical level) the laws of thermodynamics distinguish past and future. Yet the laws of Newtonian mechanics and Maxwellian electrodynamics—and indeed, all the basic laws of the microcosmos known in Weyl's day—do not. Very general principles of quantum theory and relativity suffice to prove the *CPT theorem*, which asserts that after supplementing the *CP* transformation with *T*, the overall operation—*CPT*—is an accurate symmetry of physical law. Given this theorem, violation of *CP* is equivalent to violation of *T*.

These violations of symmetry are no mere curiosities; each has powered major advances in fundamental physics, and remains central to big questions.[1]

In particular, though it can be cleanly formulated for fields, chirality—that is, left- or right-handedness—can only be an approximate, observer-dependent characterization of massive particles. Indeed, consider an observer who moves so rapidly as to overtake the particle in question. To that observer, the particle's direction of motion will appear to be reversed, compared to that seen by a stationary observer. Since the particle's rotation retains the same sense, its handedness, as defined above, reverses. Now according to the special theory of relativity, observers moving at constant velocity must find the same laws of physics as stationary ones. Thus there is considerable tension between the idea that only one handedness of particle participates in the weak interactions, and the theory of relativity.

For particles of mass zero, which move at the speed of light *c*, this difficulty does not arise: since *c* is the limiting velocity, such particles cannot be overtaken. For this reason (maximal) parity violation suggests that the most fundamental equations of physics must be formulated in terms of underlying zero-mass particles having definite chirality. Thus, for instance, we must introduce separately left-handed and right-handed electrons, which have quite different properties. Specifically, left-handed electrons participate fully in weak interactions, while right-handed ones do not feel them.

Unfortunately (for this line of thought), physical electrons have

[1]For experts: Coming to terms with maximal *P* violation gave rise to the $V-A$ theory. That theory, in turn, provided a powerful intimation of quantum field theory, at a time when many physicists had abandoned it, and gave impetus to vector-meson exchange theories, culminating in modern gauge theory. *CP* (or equivalently *T*) violation opened up the possibility of explaining how the universe could come to be dominated by matter, as opposed to antimatter. *T* violation appears only to occur through weak interactions, whereas our modern theory of the strong interaction—quantum chromodynamics (QCD)—could accommodate much larger violation. Why? The most promising answer leads us to predict the existence of a peculiar new particle, the *axion*. Axions, if they exist, could plausibly supply the astronomical dark matter.

non-zero mass. How can we keep the theoretical advantages of massless electrons, while paying reality its due?

The solution is to postulate the existence of a background field that pervades the universe. In the absence of that medium, electrons would be massless. They would obey conceptually simpler equations, which would allow them to have a fixed definite handedness. It is interaction with the medium that slows them down, gives them mass, and complicates the description of their weak interactions. It is as if we are intelligent fish who have, after millennia of taking our environment for granted and taking the ocean as the baseline for emptiness (the "vacuum"), finally realized that we could get a more satisfactory theory of mechanics by taking into account that we move through a medium (water!).

Current theory goes deeper. At the level of equations, we do not postulate the background field directly. Instead, we postulate more perfect equations, with more symmetry than the world exhibits. (Indeed, we postulate an underlying symmetry of the type that Weyl first identified and emphasized, namely, local, or gauge symmetry—see below.) All the *stable solutions* of the equations, however, feature the background field, and the world we experience is described by one of those stable solutions.

As I write these words, in December 2008, we don't know what our strange ocean is made of. No known form of matter is adequate to compose it. A great enterprise of experimental physics at the Large Hadron Collider, due to begin operations in coming months, is (metaphorically) to resolve the atomic building-blocks of our cosmic ocean.

Weyl first introduced gauge symmetry in the context of an abortive theory of electromagnetism. In this theory the symmetry postulated that the laws of physics are unchanged by arbitrary rescalings of the overall size of space and time intervals, independently at each point in space-time. (Hence the name "gauge transformations.") Unfortunately (for this theory!), the phenomena of physics do supply definite length-scales, such as the size of a hydrogen atom, that can be transported over space and time. As quantum theory emerged, Weyl promptly noticed that the mathematical idea of his earlier theory was already realized, in the *conventional* theory of *quantum* electrodynamics. In this context, the "gauge freedom" is not associated to the scale of length, but rather to the phase of the electron field. Modern gauge theories of both the weak and strong interactions generalize this concept, by allowing more complex transformations on the matter fields. The mathematics of these transformations is the mathematics of continuous (Lie) groups—whose deep theory was perhaps Weyl's greatest scientific achievement.

SYMBOL

Hilbert's program of reducing mathematics to rules for manipulating symbols assumed startlingly new significance and urgency, with the rise of powerful electronic (digital) computing machines. These machines deal with nothing other than ideally simple symbols—sequences of 0s and 1s—and they must be programmed explicitly, in full detail. For better or worse, these mathematical "minds" embody Hilbert's concept precisely.

In a practical sense, as tools, computers emphasize the importance of constructive, algorithmic mathematical methods. Conceptually, they force the question: Is that all there is? As yet, computing machines have not supplanted humans as creative mathematicians or scientists. Does this state of affairs reflect a fundamental limitation—or does it challenge us to further evolve, and to better instruct, our electronic colleagues?

Computer science casts new light on concrete foundational issues in mathematics, and poses fundamental new problems. Under the influence of information technology, attention has turned from the issue, famously pioneered by Gödel and Turing, of determining the limits of what is computationally *possible*, to the more down-to-earth problem of determining the limits of what is computationally *practical*. It appears probable (though it hasn't been rigorously proved) that a large class of natural problems—the so-called *NP* complete problems—can be solved only after astronomically long calculations, however cleverly they are approached.

The difficulty (also not rigorously proved!) of factoring large numbers into primes has become, through some clever number theory, the keystone of modern cryptography. Yet this problem can be solved efficiently if we allow ourselves to use machines more powerful than those Turing envisaged, that put quantum mechanics fully to work. Quantum computers are fully consistent with the known laws of physics, but their engineering requirements appear daunting; at present they exist only as conceptual designs.

How Weyl would have loved this bubbling ferment of philosophy, mathematics, physics, and technology!

It seems appropriate, in conclusion, to quote Atiyah once more:

> [T]he last 50 years have seen a remarkable blossoming of just those areas that Weyl initiated. In retrospect one might almost say that he defined the agenda and provided the proper framework for what followed.

REFERENCES

Atiyah, M., "Hermann Weyl, 1885–1955: A Biographical Memoir," in *Biographical Memoirs*, vol. 82 (Washington, D.C.: National Academy of Sciences, 2002), 1–13.