Introduction

This book, unlike most books on mathematics, is about mathematicians, their extraordinary passion for mathematics and their full complexity of being. We emphasize the social and emotional sides of mathematical life.

In the great and famous works of Euclid and Newton, we find axioms and theorems. The mathematics seems to speak for itself. No first person speaks, no second person is addressed: "Here is Truth, here is Proof, no more need be said." Going back to the reports of Plato and Descartes, mathematical thinking has been seen as pure reason–a perfect and eternal faculty. The thoughts, feelings, and tribulations of the mathematician are not included.

But it doesn't take deep reflection to realize that this perfection is a human creation. Mathematics is an artifact created by thinking creatures of flesh and blood. Indeed, this fact has always been known to poets, novelists, and dramatists. Mathematicians, like all people, think socially and emotionally in the categories of their time and place and culture. In any great endeavor, such as the structuring of mathematical knowledge, we bring all of our humanity to the work. Along with reasoning, the work includes the joy of discovery, the struggle with uncertainty, and many other emotions. And it is shaped by social realities, including war, political oppression, sexism, and racism, as they have affected society and mathematicians in different eras.

Today the connection between thought and emotion is a major active field of scientific research. Recently the neuroscientist Antonio Damasio and a collaborator wrote, "Modern biology reveals humans to be fundamentally emotional and social creatures. And yet those of us in the field of education often fail to consider that the high-level cognitive skills taught in schools, including reasoning, decision making, and processes related to language, reading, and mathematics, do not function as rational, disembodied systems, somehow influenced by but detached from emotion and the body . . . hidden emotional processes underlie our apparently rational real-world decision making and learning."[1] While purely rational decision making is possible in highly structured situations, they point out that emotionally empowered thinking is needed for recruiting "these skills and knowledge usefully outside the structured context of school and laboratory."[2]

Indeed, mathematical experience advances between the twin poles of exaltation and despair. Granted, despair is more familiar to novices, and exaltation is more associated with the great discoverers. But these opposite emotions lie in waiting, during every mathematical struggle, at every level. The deep emotional connections between mathematical experience in earliest childhood, in maturity, and in ripe old age are an important theme in this book.

To unveil these different aspects of mathematical life, we have read many biographies and autobiographies. We quote a great number of cherished life stories of mathematicians who both delighted in and laughed at the quirks of living with numbers and abstraction. We often let famous mathematicians speak for themselves.

One of our major goals is to overcome some distorted, stereotypic images of the field and its practitioners.[3]

First is the myth that mathematicians are different from other people, lacking emotional complexity.

There is a common belief that in order to engage in complex abstract reasoning, a researcher must exclude emotion from his or her thinking. Our first four chapters refute this belief. A mathematician, like anyone else, has an emotional life, which is sustained by nurturance in childhood and youth and by companionship and mutual support in later years.

It's a challenge for everyone to achieve balance in one's emotional life. It's a particularly severe challenge for those working in mathematics, where the pursuit of certainty, without a clearly identified path, can sometimes lead to despair. The mathematicians' absorption in their special, separate world of thought is central to their accomplishments and their joy in doing mathematics. Yet all creative work requires support. Enmeshed in a mental world far removed from the understanding of those with whom he or she is closest, a mathematician risks becoming psychologically isolated. We tell of some mathematicians for whom such isolation became extreme and overwhelmed their existence. We also tell of others for whom the clarity and beauty of mathematics provided an emotional haven against persecution or the tragic effects of war. The common thread is loving and hating mathematics.

We start in chapter 1 by asking, How does a child first begin to become a mathematician? We explore the thrill of discovery and the power of engagement that some young people can experience. We listen to young contestants in international mathematical olympiads and to their parents. We follow young mathematicians through their graduate school years. Among the emotions that accompany mathematical activity, we find affinity and doubt, frustration and elation, camaraderie and rivalry, and friendship and jealousy.

The culture of mathematics is the subject of chapter 2. Socialization into a community implies the sharing of values, cognitive approaches, and social beliefs and practices. It also involves ways of handling internal tensions that can rupture much needed human and professional ties. We report on three recent episodes in the history of mathematics that made headlines: the proof of Fermat's Last Theorem; the recognition of the group of phenomena in dynamics known as "chaos"; and the proof of Thurston's program in four-dimensional topology, including the Poincaré conjecture. This involved two great geometers, Grisha Perelman and S.-T. Yau, and shows us the cost of single-minded commitment and the mathematician's ambivalence toward fame. We also report the struggles of the University of California professor Jenny Harrison. (A biographical directory of mathematicians and scholars appears at the end of the book.)

In chapter 3, we present stories of solace in one's discipline as an escape from oppression and imprisonment. We start with Jean-Victor Poncelet, a captured officer in Napoleon's defeated army, who made great discoveries in geometry while a prisoner in Siberia, and we include José Luis Massera of Uruguay, who became a teacher to his fellow inmates in prison, giving them hope and determination to survive.

The potentially addictive nature of mathematics is examined in chapter 4. What drives a person to live only for mathematics? If that pursuit becomes an obsession, what are the possible psychic repercussions? We look at the story of Alexandre Grothendieck, one of the supreme mathematicians of the 20th century, who, after giving his life to his profession, became a hermit, repudiating his colleagues and the prevalent values of the 20th century. André Bloch engaged in disciplined daily mathematical work (and mastered univalent functions) in a psychiatric asylum at Charenton, France. The best known mad mathematician is Ted Kaczynski, "the Unabomber," whose psychopathology seems to be a murderous parody of mathematical reasoning. The tragic end of the great logician Kurt Gödel shows paranoia coexisting with genius.

Second on our list of four myths is the notion that *mathematics is a solitary pursuit*. This is refuted in chapters 5 and 6.

Intense and sustained thought requires a quiet environment and a highly focused mind-set. But discipline, discovery, and intellectual and emotional renewal thrive with support from caring connections. The popular notion of the solitary and eccentric mathematician is a distortion; it leaves out the rich social life of men and women in this field, including mentorship and collaboration.

In chapter 5, we explore friendships and partnerships in the lives of mathematicians. We look at the pleasures of thinking together that are vividly evoked by David Hilbert and Hermann Minkowski. We examine the complex collaborations of the eminent British mathematicians G. H. Hardy and John Littlewood, and the Indian mathematician Srinivasa Ramanujan. Their relationship is a powerful and bitter story of intellectual partnership and cultural isolation. We look at the professional and intimate connection between the bachelor professor Karl Weierstrass and the young Russian student Sonia Kovalevskaya, whose meeting gave rise to powerful emotional bonds and tragic loss. The singular mathematical marriage of Grace Chisholm and William Young presents surprising and shifting gender roles during their long partnership. The story of Chisholm and Young is particularly interesting, as mathematical marriages became quite frequent later in the 20th century.

In chapter 6, we examine the unique nature and culture of mathematical communities, and the support that they provide to both aspiring and established mathematicians. There are sometimes "golden" periods in the life of a cohesive group or department when inspired leadership, strong personal relationships, and institutional support result in a period of great productivity. This was achieved at Göttingen in the early 20th century. The legacy of Göttingen later inspired Richard Courant to establish a new mathematical community at New York University. (One of the authors, R.H., was lucky enough to spend several years there, first as a graduate student and later as a visiting postdoc.) Another famous mathematical community was the Bourbaki group in the French cities of Nancy and Paris. Forty years ago, Russian mathematicians experienced a "golden" decade of mutual encouragement and inspiration at the Department of Mechanics and Mathematics (Mekh-Mat) at Moscow University—and the impact of those years is still felt by the larger, international community. We also write of women who are struggling for equity, respect, and acceptance through the Association for Women in Mathematics, an organization that provides much needed support for its members.

The third of our four myths is often stated as a quote from G. H. Hardy, *that mathematics is a young man's game*. In chapter 7, we look at the issues of maturity, aging, and gender in the lives of mathematicians. We find that men and women continue to produce in their senior years, creating many ways to stay connected to their lifelong pursuits. We explore the seldom discussed experiences of mathematicians who are over 50—and in some cases even 70—years of age. This myth also assumes that mathematics is a male endeavor. It is refuted by the new reality of a steadily increasing number of women joining departments of mathematics and research institutions. While sexist attitudes linger and cause psychological damage, new female leadership has emerged. We recall some of the early female pioneers (Sophie Germain, Sonia Kovalevskaya, and Emmy Noether) and the accounts of contemporaries who discuss their commitment to a mathematical life (Mary Rudin, Joan Birman, Lenore Blum, Karen Uhlenbeck, et al.).

The last of the four myths is the notion that *mathematics is an effective filter for higher education*. Our last two chapters, in different ways, are about mathematics education, from the elementary up to the collegiate level. This topic cannot be avoided; mathematics education is a central part of mathematical life. There is an interesting paradox in how mathematics is perceived by the public at large. On the one hand, it is thought of as a world apart from practical human existence, and often it is taught as pure abstraction, which reinforces this perception. On the other hand, it is supposed to be the most useful qualification for students preparing for high-prestige professions such as engineering and architecture. In discussing this myth, we consider the role of mathematics in broader social contexts including elitist versus democratic perspectives on who should become a mathematician.

Some of the most emotional issues in the mathematical world are about the right and wrong ways to impart mathematics to children. In our last two chapters we confront these controversies frankly and argue for a realistic and humane approach to the difficult problem of improving math education in the United States.

In chapter 8, we review two opposite, yet intertwined, American educational traditions: the elite "Moore method" in Austin, Texas, and the universalist "Potsdam model" in upstate New York. These two narratives force us to face the issue of racial segregation in the history of mathematics in the United States.

This brings us in chapter 9 to the big question: How should mathematics be taught? Can we better serve those who say they like it? Must we disregard those who say they hate it? We address the never-ending call for improving mathematical achievement in the United States by offering some nonstandard, but realistic and humane, proposals that take into account the emotional aspects of learning.

In the book's conclusion, we envision a joyous and balanced view of the mathematical life of reason, emotion, and learning.

This is not a math education book of the kind that contains curriculum recommendations or statistical analyses of classroom experiments. But we do deliver the pain and pleasure of teaching, learning, and schooling. Neither is it the kind of math book that teaches some particular branch of mathematics. For example, when we refer to Fritz John's "functions of bounded mean oscillation" or Teiji Takagi's "class field theory," we only mention by name some of their mathematical contributions. For deeper exploration, one can turn to the plentiful literature of treatises and textbooks. However, we do attempt an overview of the most important work of Alexandre Grothendieck as part of understanding his emotional life story.

This book benefited from contributions to understanding mathematical life by nonmathematicians—psychologists, neuroscientists, anthropologists, and sociologists. We are grateful for these contributions and look forward to more such studies.

The writing of this book is a collaboration between a mathematician and a psychologist. Our disciplinary training and interests differ greatly, but we both view the life of the mind as a deeply humanistic activity. One eloquent spokesman for humanist philosophy was the physicist Jacob Bronowski. He wrote that the society of scientists must be governed by "independence and originality, dissent and freedom and tolerance: such are the first needs of science; and these are the values which of itself it demands and forms. The society of scientists must be a democracy. It can keep alive and grow only by a constant tension between dissent and respect; between independence from the views of others, and tolerance for them. The crux of the ethical problem is to fuse these, the private and the public needs."[4]

We hope that this joint examination will increase awareness of the richness of mathematical life in its many tensions between isolation and community, logic and playful exploration, and dissent and respect.

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