INTRODUCTION

I didn't become a mathematician because mathematics was so full of beautiful and difficult problems that one might waste one's power in pursuing them without finding the central problem.
—Albert Einstein

Mathematics is trivial, but I can't do my work without it.
—Richard Feynman

These two views, from two of the most famous physicists of the last century, on the relationship between physics and mathematics, are quite different. Both won the Nobel Prize (Einstein in 1921 and Feynman in 1965), and their words, however different, deserve some thought. Einstein's are ones without sting, carefully crafted to perhaps even bring a surge of pride to the practitioners of the rejected mathematics. Feynman's, on the other hand, are just what we have come to expect from Feynman—brash, outrageous, almost over-the-top. Feynman's comment really goes too far, in fact, and I think it was uttered as a joke, simply to get attention.[1] Physicist Feynman was also a highly skilled mathematician, and not for a moment do I believe he really thought mathematics to be "trivial." (Mathematicians shouldn't take such "Feynman put-downs" too seriously; I'm an electrical engineer and while Feynman had some jibes for EEs, too, I have never let them influence my appreciation for his genius.[2])

The mathematician Peter Lax, in his 2007 Gibbs Lecture to the American Mathematical Society, gave a good, concise summary of the interplay between mathematics and physics.[3] His talk opened with these words: "Mathematics and physics are different enterprises: physics is looking for laws of nature, mathematics is trying to invent the structures and prove the theorems of mathematics. *Of course these structures are not invented out of thin air but are linked, among other things, to physics*" (my emphasis). A few years earlier a physicist, in his own Gibbs Lecture, gave a specific illustration of this connection:[4] "The influence of general relativity in twentieth-century mathematics has been clear enough. Learning that Riemannian geometry is so central in physics gave a big boost to its growth as a mathematical subject; it developed into one of the most fruitful branches of mathematics, with applications in many other areas." The influence can flow in the other direction, too: it was the *mathematician* Kurt Gödel (Einstein's friend at the Institute for Advanced Study at Princeton) who in 1949 discovered that the equations of general relativity allow for the possibility of time travel into the past, a totally unexpected result—previously thought to be a childish science fiction fantasy—that quite literally left the *physicist* Einstein nearly speechless.[5] I have also long marveled at the intimate connections between physics and mathematics, and this book is the result of that fascination. Indeed, *Number-Crunching* is a continuation of my 2009 book, *Mrs. Perkins's Electric Quilt*, inspired by the same fascination.

As you read through the chapters of this book you'll see how important I find the role of the modern high-speed electronic computer to the study of both mathematics and physics. The relatively new subject of computer science can legitimately be viewed, in fact, as a bridge connecting those two much older subjects (whose pure practitioners have sometimes appeared to be at extreme odds). As one pioneer in the development of electronic computers once bluntly wrote,

> The relationship . . . between physics and mathematics may best be described as an unsuccessful marriage, with no possibility of divorce. Physicists internalize whatever mathematics they require, and eventually claim priority for whatever mathematical theory they become acquainted with. Mathematicians see to it that every physical theory, sooner or later, is freed from all shackles of reality to fly in the thin air of pure reason.[6]

The author of that rather harsh assessment had a background that included all three subjects: he held a PhD in physics, made important contributions to applied mathematics, and was the guiding spirit behind one of the earliest electronic computing machines (the famous MANIAC-I at the Los Alamos Scientific Laboratory), and so I think his words are worth consideration. I personally don't believe, however, that the schism between mathematics and physics is nearly as wide as he asserted, and whatever their actual divide may have been when he wrote, the development of the electronic computer has certainly narrowed rather than widened it. With electronic computers and the development of powerful software by computer scientists to "power" these machines, mathematicians can now perform physics "experiments" via simulation, and physicists can perform mathematical feats that once defeated even such genius as that of Isaac Newton. In this book you'll see examples of both such uses of computers.

The laws of physics are quite compact. In their natural language—mathematics—these laws are easy to write down on paper. All of the known fundamental laws can in fact be completely written out on just a couple of pages. And for many analysts it is that compactness that is the real puzzle, not the laws themselves, as much of the observed behavior of the real world appears far too complex to be explainable by a mere few lines of symbols. Einstein famously gave life to this puzzle in a 1936 essay when he declared, "The fact that it [the world] is comprehensible is a miracle." Feynman included a particularly poetic statement of the wonderful entanglement of mathematics and physics in his *Lectures on Physics* when he ended one presentation with these words: "The next great era of awakening of human intellect may well produce a method of understanding the *qualitative* content of equations. Today we cannot. Today we cannot see that the water flow equations contain such things as the . . . structure of turbulence.. . . Today we cannot see whether Schrödinger's equation [the probability wave equation of quantum mechanics] contains frogs, musical composers, or morality—or whether it does not. We cannot say whether something beyond it like God is needed, or not. And so we can all hold strong opinions either way."[7]

The ancients had an effective way of avoiding a need for mathematical physics; everything they found perplexing was "explained" (in the form of a good story usually involving revenge, envy, sex, and death—see any good book on mythology) by just saying "the gods make it happen." And, of course, it *is* a lot easier to make up good stories around the campfire at night than it is to discover good physics, which is why the god myths are thousands of years old and the origins of good physics are considerably more recent.

The theme here remains the same as in *Mrs. Perkins*. The theme of *Mrs. Perkins* and of *Number-Crunching* is to show by example how the fundamental laws of mathematical physics, combined with the tremendous computational power of modern computers and their software, can explain extremely complicated behaviors, behaviors often so counterintuitive as to bring gasps of astonishment even from experienced, professional analysts. If you have studied the first two years of college physics and math and understand the basic ideas behind the writing of computer codes in a high-level language (I use MATLAB), then you should be able to read this book. Each chapter ends with at least one challenge problem (solutions are provided at the end of the book) to let you enjoy your own gasps of astonishment. (Here's a quick example of what I mean by a surprise: a straight, precisely one-mile-long stretch of continuous railroad track is laid during a cold night, with the two ends firmly fixed in the ground. The next morning the hot sun causes the track to expand by exactly one foot, and so the track buckles upward. Now, quickly, off the top of your head, is the midpoint of the track raised above the ground by (1) several inches, (2) several feet, or (3) several yards? The answer is at the end of this Introduction.)

As a second, far more serious example of both the sort of analyses in this book and the minimum level of the mathematics and physics you should feel comfortable with to read them, let me now describe a situation that at first glance may seem to present an impossible task. To set the stage for this problem, consider first the following recent extreme weather event that I personally experienced. In December 2008 a huge ice storm devastated the northeastern United States, and my home state of New Hampshire in particular. As the storm developed, vast numbers of trees, covered in thick layers of ice, collapsed, and as they fell they tore down nearby power lines that hadn't already had their wires snapped by the weight of the heavy ice that coated them. The damage was enormous, with lines down everywhere, resulting in widespread power outages that shut down hundreds of thousands of home heating systems and so drove large numbers of people from their freezing houses and into hotels and community shelters.

It took up to two weeks to restore electrical power in some areas of the state, but the repair crews could at least *see* the lines that were down.

Just imagine how much more difficult it would have been if the repair crews couldn't have seen the damaged wires. Now, that might seem to be an artificial complication, but it really isn't. It occurs in a natural way, for example, in even more extreme fashion, if there is a break in an undersea communication cable. Locating such "invisible" breaks became a problem right from the beginning of the laying of such cables in the middle of the nineteenth century. One could, of course, imagine a repair ship sailing along the known path of the submerged cable, periodically grabbling for the cable, and pulling it up to the surface until the break (or *fault*) is found. Considering that a cable might be hundreds of miles long, in hundreds or even in thousands of feet of water, however, should make it clear that that could easily be a lengthy and most expensive approach. Can we do better in finding the location of a fault, even if we can't see the cable? Yes. What I'll next show you is a technique called the *Blavier method*, invented by the French telegraph engineer Edouard Ernst Blavier (1826–1887).

I should, however, start with some words on how a fault might occur in an undersea cable. You won't be surprised, I think, if I begin by saying that an ice storm generally is not the cause! Much more likely is that shifting water currents would move the cable back and forth across a rough sea bottom and thereby cause its outer shielding to be abraded. Another, perhaps less obvious cause of a cable fault was a fish bite! As one observer noted in an 1881 letter to the English trade journal *The Electrician*, a cable laid in 1874 was soon after found to have suffered "at least four indubitable fish-bite faults . . . where the iron sheathing had been forcibly crushed up and distorted from the core as if by the powerful jaws of some marine animal." That writer presented convincing evidence pointing to *Plagyodus ferox* ("one of the most formidable of deep-sea fishes") as the culprit. In any case, before the cable broke completely it would first have developed a so-called *electrical leakage fault* from its still physically intact internal wires to the surrounding water.

Well, no matter the origin of the fault; we'll mathematically model it and the cable as shown in Figure I.1. Let's assume the cable has a fixed, known electrical resistance per unit length, and that we write the total resistance of the cable from one end to the other as a ohms, the resistance from the left end of the cable to the fault as x ohms, the resistance from the fault to the right end of the cable as $a - x$ ohms,

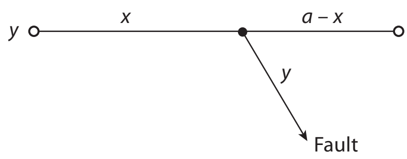

FIGURE I.1. Locating the fault

and the resistance from the fault to the sea water (which we'll take as equivalent to a good earth ground) as y ohms. If we can determine the value of x, then we can use the resistance per unit length of the cable to calculate how far the fault is from either end.

To start, let's write

$$b = x + y.$$

We can measure the value of b from the left end of the cable by open circuiting the right end, applying a battery of known voltage between the left end and the earth, and measuring the resulting current. Ohm's law then gives us b. Next, we'll ground the right end of the cable, which puts the $a - x$ portion of the cable in parallel with the fault resistance. If we call c the resulting resistance now seen from the left end of the cable, then

$$c = x + \frac{y(a-x)}{y+a-x}.$$

We now have two equations in two unknowns, x and y, so we can eliminate the fault resistance y and solve for x. Since $y = b - x$ from the first equation, substituting this into the second equation gives, with just a little algebra, the quadratic equation

$$x^2 - 2cx + c(a+b) - ab = 0,$$

which is easily solved (with the famous quadratic formula) to give us two *real* values for x:

$$x = c \pm \sqrt{(a-c)(b-c)}.$$

(Can you see why both $(a - c) > 0$ *and* $(b - c) > 0$, and so we have the square root of a positive quantity, and thus x is *guaranteed* to be real?)

The fault is located somewhere in *one* place, of course, not two, and so we can't have two values for x. So, which sign do we use in front of the square root, the plus or the minus? The answer is the minus sign, because (by definition) $x < a$ (and so $a - x > 0$), and the fault resistance must obviously be positive ($y > 0$). This says

$$x = c - \frac{y(a-x)}{y+a-x} < c.$$

Thus, the unique solution for x is

$$x = c - \sqrt{(a-c)(b-c)}.$$

Some years ago, while researching a biography of the English mathematical electrophysicist Oliver Heaviside (1850–1925), I discovered one of his notebooks in which Heaviside had recorded an application of this formula that he had made while working as a telegraph operator in Denmark.[8] Observing that an electrical fault had occurred somewhere along the 360-mile undersea cable between Sondervig in Denmark (the right end) and Newbiggin-by-the-Sea in England (the left end), the opened and shorted resistance measurements gave the values of $b = 1,040$ ohms and $c = 970$ ohms, respectively. Knowing that the cable had a resistance of 6 ohms per mile, he had $a = (6)(360) = 2,160$ ohms. Therefore,

$$x = 970 - \sqrt{(2,160-970)(1,040-970)} = 682 \text{ ohms}.$$

Thus, the fault was located $682/6 = 113\frac{2}{3}$ miles from the left end (the English end). Heaviside was clearly quite pleased with this pretty calculation: his notebook entry (dated January 16, 1871) recording his work ends with the happy words, "All over. Dined roast beef, apple tart and rabbit pie with claret and enjoyed ourselves." As you work your way through this book I hope you'll find it isn't all just numbers, theorems, and calculus calculations. It is permissible, indeed *required*, that you have fun, too. If a serious fellow like Heaviside can enjoy solving a quadratic equation, so can you.

Let me give you an example of what I mean by having fun. More than thirty years ago (1978) the publisher of *Penthouse* started a new magazine called *Omni*. *Omni* lasted not quite two decades, ceasing publication in 1995, but while it existed *Omni* was a bright star in a part of the literary sky that is all too often ignored. It was devoted to the reporting of cutting-edge scientific breakthroughs, the use of beautiful science-based art, and the publication of short science fiction stories. Other than *Playboy*, *Omni* was the only glossy print magazine publishing first-rate science fiction *and* paying *Playboy*-scale money to fiction writers. One of *Omni*'s regular features was its interview essay, similar to a regular feature in *Playboy*. Rather than interviewing rock music stars, Hollywood celebrities, or politicians, however, *Omni*'s interviews focused on visionary thinkers and world-class scientists; Feynman's friend, the mathematical physicist Freeman Dyson of the Institute for Advanced Study in Princeton, was the subject of the very first interview. A few months later, Feynman himself was the featured personality in the magazine.

Another regular feature in *Omni* was the one-page humor essay that always appeared as the final page of each issue, and so was appropriately called "Last Word." In 1982 I wrote the following "Last Word," which I've reproduced below with no editing other than adding some elaborative endnotes for which there was no room in the magazine.[9]

Just about everyone who learns to read eventually comes across the story of the monkeys and the typewriters. The idea is elegantly simple. Merely put a bunch of monkeys in front of an equal number of typewriters (and a *lot* of paper) and let them bang away on the keys. They'll produce mostly gibberish, of course, but the immutable laws of probability predict they will also reproduce verbatim *everything* that has ever been written.[10] Like this issue of *Omni*, this essay included.

And they'll produce everything else that will and even could be written. All the books of the future will be included in the monkeys' output: masterpieces of science fiction, fascinating volumes of "history" describing and analyzing events yet to occur and a lot that won't, and treatises on marvelous scientific breakthroughs.

This cosmic collection of written words is called the Universal Library, a term coined by Kurd Lasswitz, a German philosopher and mathematics professor. According to Willy Ley, the original concept of such a collection can be traced back to the medieval Spanish philosopher and mystic Ramon Lully.[11]

Whatever you want to call it, this incredible storehouse of knowledge would lead to tantalizing revelations. If a time machine could ever be built, somewhere in the megatons of paper spewing from the typewriters will be contained the directions on how to put one together. So would the recipe for a faster-than-light spaceship drive, a cure for cancer, and the secret of longevity.

But the rub, of course, is that there would be a multitudinous quantity of chaff with the wheat. How could we search through this literary equivalent of the Augean stables to find such gems? Until now, we couldn't, and the idea has remained just a vague speculation, good for a quick chuckle, but soon discarded so we can attend to more practical and more pressing matters.

No more. Using high-speed digital computers, coupled with the latest innovations in high-resolution imaging, instant developing, and reusable microfilm, we could begin right now realizing the ancient dream of the Universal Library. Better yet, we can do something more impressive: put together the Universal Photo Album (TUPA).

To understand TUPA, it is necessary to grasp just a few elementary concepts. First, any black-and-white photograph is nothing more than a composite of small flecks of brightness levels varying from black through various shades of gray to white.[12] Second, a computer can easily be programmed to generate sets of flecks in a fine mesh grid. The programmer can also instruct the fleck-generating machine never to repeat itself. The computer would toil, like a team of monkeys, to produce all possible combinations and permutations of brightness levels. Instead of generating a mountain of typed gibberish, it would create a mountain of visual gibberish, countless random images flying out from this tireless image factory.

Then, to examine the enormous number of images, we could take advantage of recent advances in machine vision, artificial intelligence, and work in pattern recognition. Special computers would be used to process the first batch of photos, rejecting the ones that are clearly nonsensical and referring the interesting ones to humans for interpretation. (Since this would require just about every piece of film ever made, we could save on film costs by recycling the rejected photos and using the film over again.)

What would we get from all of this? Simply all possible black-and-white photographs: all that have ever been taken, all now being taken, and all that could ever be taken. We get TUPA.

What a treasure it would be! In TUPA would be included pictures of every human who has ever lived—Moses, Jesus, Henry VIII, you, me—as well as every human who will ever live. There will be images of every creature in the universe, some of them more incredible than we could imagine. You'll see in TUPA all the vacation photos you will ever take, including the ones where you left the lens cap on.[13] It is clear that purveyors of pornography, for example, will be galvanized by the discovery of a hoard of titillating photos[14] in TUPA. Best of all, the Universal Library will be there as well, because TUPA will have a picture of every page in every book ever published.

This entire project could be started immediately with a front-end capitalization of $20 million—give or take a few million—for computers, laser equipment, computer-program development, and film-chemistry engineering. We would also need a sustaining budget of $3 million for computer time, film, and salaries for the photo interpreters. This level of funding is trivial compared to the Defense Department's and is well worth the investment if you consider the potential payoff militarily. TUPA would include pictures of every top-secret Soviet military document, present and future.

Project TUPA could be our next great endeavor, rivaling the Apollo and Manhattan projects. But we must act now because, as the Russians will soon realize, if they haven't already, TUPA contains photos of every top-secret American military document, too.

I was (and still am) enormously intrigued by the concept of TUPA. TUPA is a generalization of the Universal Library that goes far beyond the Universal Library. As I mentioned in my "Last Word" essay, TUPA contains the Universal Library as a subset because in TUPA there is an image of every page of every book in the Universal Library. And not just in English, but in every language that has ever existed, exists now, or will exist, on Earth as well as on *every* other planet in the entire universe that has, does now, or will support intelligent life with a written language. The Great Library of Alexandria, Egypt, the largest collection of books in the ancient world that was destroyed numerous times, from when it was accidently burned in 48 B.C. by Caesar's troops until its final destruction in A.D. 642 by Arab invaders, would be lost no more. TUPA contains all possible illustrated books, too, something not in the original Universal Library. And not only that, TUPA contains itself as a subset, because in TUPA there are images of ever more slightly reduced versions of every image in TUPA.

Some readers of *Omni* were just as excited by TUPA as I was, in fact, and wrote to the magazine to ask just why in heck the U.S. government wasn't going forward with Project TUPA. As much as I love the concept of TUPA, however, those letters caught me by surprise. After all, while it is true that TUPA violates no laws of physics, there is nonetheless a very good *mathematical* reason why it will forever remain out of reach—see Challenge Problem I.4. On the other hand, maybe those readers were simply returning the joke!

Okay, while I take the position that even though a good nighttime campfire story is always fun to hear while roasting marshmallows on a stick, or that conceits like TUPA can stimulate some fun talk, in the end, such amusements, when matters finally turn to serious business, are simply no substitute for disciplined, logical, analytical thinking. Since you're reading this, I take it that you agree with me. So, with no further delay, let's get started. Here are your first four challenge problems, to give you a feeling for the level of the mathematics in this book. The first one, in particular (it's not computationally difficult), reflects, I believe, Feynman's true feelings about the non-trivial nature of mathematics.

Challenge Problems

CP I.1. In a letter dated July 7, 1948, to the physicist Hans Bethe, winner of the Nobel Prize in 1967, Feynman outlined his latest research efforts in quantum electrodynamics. At one point he said it all hinged on the use of what he called a "great identity": $\frac{1}{ab} = \int_0^1 \frac{dx}{[ax + b(1-x)]^2}$, where a and b are constants. Assuming a and b are real but otherwise arbitrary constants, explore the truth of Feynman's identity. In particular, explain the following "puzzle": if a and b have opposite signs, then obviously $\frac{1}{ab} < 0$, but just as obviously, the integral is *never* less than zero for *any* real a and b since the integrand is a real quantity squared, and so is nowhere negative. Clearly, *something* more needs to be said here. What's going on?

CP 1.2. Here's another purely mathematical problem that, even though it is "just" high school algebra and trigonometry, almost always astonishes even professional mathematicians. Just about all (that includes me) who first see this problem initially think it is impossible to do—but that isn't so! In addition, it has a technical application to a famous physics problem first solved by Einstein in 1905 (the reflection of light from a moving mirror). Starting with the equation $\sin(\alpha) - \sin(\beta) = -k\sin(\alpha + \beta)$, where k is a constant and both α and β are in the interval 0 to $\pi/2$, solve for β as a function of k and α. Despite its convoluted appearance, this is not a transcendental equation. I think Feynman would have had fun working through this calculation, and I hope you do, too, but fair warning: prepare yourself for at least some mental exertion (there is significantly more computation involved than in the first problem). As a partial check on your general result, notice that in the case of $k = 0$, your answer should obviously reduce to $\beta = \alpha$.

CP 1.3. When I was a freshman at Stanford I did well enough during the first two terms of calculus to be allowed to transfer into the honors section of the course. (That's when I found out *lots* of my fellow freshmen were at least as good at math as I was!) One of the homework problems in that course (Math 53, Spring Quarter 1959) was the following: prove that the shortest path length between any two given points in a plane is that of the straight line segment connecting the two points. This is, of course, "obvious" to anybody with a body temperature above that of an ice cube, but the point of the problem was to construct a proof. I remember sitting at my desk in the dorm one night wondering just how I might attack this problem—and then I hit upon what I thought an incredibly clever idea. Obviously the professor intended us to use what we had been discussing in lecture, and that just happened to be the general formula for the arc length of a smooth curve (*smooth* means the curve has a tangent at every point, which means the curve has a derivative at every point). In fact, if we have the curve $y = y(x)$, then the length L of the curve segment connecting points A and B, with coordinates (x_A, y_A) and (x_B, y_B), respectively, in the usual Cartesian coordinate system, with $x_A \leq x_B$, is given by $L = \int_{x_A}^{x_B} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$. Well, I reasoned, the length L is *independent* of the particular choice of coordinate axes, that is, we can choose the orthogonal x- and y-axes to be at any orientation we wish (and place the origin anywhere we want, too) and we'll always get the same L. (Physicists call L a *physical invariant*, while the choice of coordinate axes is a *mathematically arbitrary* decision.) In particular, then, let's pick the x-axis to be such that $y_A = y_B$, which we clearly can always do. Then the straight line segment joining A and B will have zero slope, and so $\frac{dy}{dx} = 0$ *always*. But clearly, "$\frac{dy}{dx} = 0$ *always*" is just what is needed to minimize the integrand *at every point in the interval of integration*, and so L itself will be minimized. I recall I wrote that all up and finished it off in a flourish with a big "QED" (just to impress the professor with my cultural sophistication and mastery of Latin!). Alas, my homework paper came back the next week with a big red cross through my brilliant proof (the word **NO** was also quite prominently displayed), along with a score of 1 (for trying) out of 10. I was too embarrassed by all that red ink to ask where I had gone wrong (which of course was a silly reaction). So, there's *your* challenge: explain where I went wrong fifty years ago. *Note:* In my book *When Least Is Best* (Princeton, N.J.: Princeton University Press, 2004, corrected edition 2007), written when I had become a somewhat more sophisticated fellow than I was in 1959, I solve this problem (pp. 238–239) using the Euler-Lagrange formulation of the calculus of variations. And in my book *Dr. Euler's Fabulous Formula* (Princeton, N.J.: Princeton University Press, 2006) I solve it again, that time using Fourier analysis (pp. 181–187). But surely my Math 53 professor didn't expect his *freshman* students to use either of those approaches. At least I don't think he did.

CP 1.4. (a) A black-and-white $m \times n$ digital TUPA image consists of mn pixels, each with k bits of gray-level coding ($2^k - 1 \Longrightarrow$ whitest white and $0 \Longrightarrow$ blackest black). How many such pictures are in TUPA? (b) If mn = ten million (that is, if a black-and-white TUPA picture is ten megapixels, comparable to what, as I write, a good digital camera produces), and if $k = 11$ bits (there are $2,048$ gray levels), a black-and-white TUPA image would be a very crisp, ultra-sharp, high-resolution photo. How many such images are in TUPA? (c) If we had a computer that could automatically generate one hundred trillion of these TUPA images per nanosecond, then what fraction of TUPA would have been generated by now since the start of time at the instant of the Big Bang fifteen billion years ago?

(The answer to the buckled railroad track problem is several yards. Are you surprised? For an analysis, see the Solutions section at the end of the book.)

Notes and References

1. I never personally talked with Feynman, but when I was in high school my father took me to a public lecture Feynman gave one weekend at Caltech. It must have been sometime in 1957 or so. Brea, my hometown in 1950s Southern California, is only an hour's drive from Pasadena, and my father (holder of a PhD in chemistry, but with scientific interests that ranged far beyond chemistry) had heard that Feynman was "the next Einstein." So, off we went. That experience gave me a firsthand encounter with Feynman's personality. I had never before heard anyone like Feynman. He certainly wasn't boring, dull, or pompous! He talked like a New York City wiseguy, cracked jokes, and clearly had a good time. I recall being initially shocked (and then enormously entertained) by how irreverent he was during the talk, which long after I recognized as presenting material that later appeared in his famous, equally irreverent *Lectures on Physics* (Addison-Wesley, 1963). It is in volume 2 of *Lectures* that you'll find (p. 7–2) this comment by Feynman, which I think correctly illustrates his real feelings about math: "Now we come to a

miraculous mathematical theorem which is so delightful we shall leave a proof of it for one of your courses in mathematics." The reference is to the Cauchy-Riemann equations from complex variable theory (although Feynman didn't tell his young students that), about which you can find more in my book *An Imaginary Tale: The Story of* $\sqrt{-1}$ (N.J.: Princeton University Press, Princeton, 1998 [corrected printing 2007], pp. 191–194).

2. For a Feynman comment on EEs, see pp. 23-6 and 23-7 in volume 1 of *Lectures*. He does make an apology of sorts to electrical, mechanical, and civil engineers on pp. 16-8 to 16-10 of volume 2 of *Lectures* when he describes his impressions of Boulder Dam (which Feynman rightfully thought to be an engineering marvel). On at least one occasion Feynman got back at least as good as he gave. The great probabilist Mark Kac (1914–1984) once gave a lecture at Caltech, with Feynman in the audience. When Kac finished, Feynman stood up and loudly proclaimed, "If all mathematics disappeared, it would set physics back precisely one week." To that outrageous comment, Kac shot back with that yes, he knew of that week; it was "Precisely the week in which God created the world."

3. I have taken the two opening quotes from Lax's talk; see Peter D. Lax, "Mathematics and Physics" (*Bulletin of the American Mathematical Society*, January 2008, pp. 135–152). The quotation from Lax himself repeats what the great English mathematician J. J. Sylvester (1814–1897) wrote decades earlier in an 1879 paper, "The object of pure Physic is the unfolding of the laws of the intelligible world; the object of pure Mathematic that of unfolding the laws of human intelligence."

4. Edward Witten, "Magic, Mystery, and Matrix" (*Notices of the American Mathematical Society*, October 1998, pp. 1124–1129).

5. See, for example, my book *Time Machines: Time Travel in Physics, Metaphysics, and Science Fiction*, 2nd ed. (New York: Springer, 1999, pp. 19, 80–84, and 489–495), and Wolfgang Rindler, "Gödel, Einstein, Mach, Gamow, and Lanczos: Gödel's Remarkable Excursion Into Cosmology" (*American Journal of Physics*, June 2009, pp. 498–510).

6. Nicholas Metropolis, "The Age of Computing: A Personal Memoir" (*Daedalus*, Winter 1992, pp. 119–130). Metropolis will appear again, in a central role, in Chapter 3.

7. In volume 2 of Feynman's *Lectures*, p. 41–12.

8. See my book *Oliver Heaviside: The Life, Work, and Times of an Electrical Genius of the Victorian Age* (New York: IEEE Press, 1988; repr., Baltimore: Johns Hopkins University Press, 2001, pp. 25–26). Since Heaviside's time, better

methods for locating breaks have been developed. One of the grandmasters of science fiction, Robert Heinlein (1907–1988), used one of the new ideas in his first published story, "Life-Line" (*Astounding Science Fiction*, August 1939). Heinlein was no mere "penny-a-word" pulp magazine hack; he was in the top 10% of the U.S. Navy Academy's Class of 1929, and took graduate courses in physics and mathematics at UCLA after his early forced retirement (because of tuberculosis) from the Navy in 1934. Heinlein used his technical background to draw an analogy in his story between a space-time worldline and an electrical cable. The beginning and ending points in space-time of the worldline of a person (birth and death) are associated with breaks (faults) in the cable. By sending a brief pulse through the cable and measuring the time delay until the arrival of the echoes produced by any fault discontinuities in the cable, a technician can both detect and accurately locate faults. In a similar manner, Heinlein's story gadget locates the birth and death "faults" along a person's worldline. Knowledge of the death fault, in particular, causes financial chaos in the life insurance business, and an examination of that tension (not weird physics) is the point of the story.

9. Paul Nahin, "Last Word" (*Omni*, April 1982). (Please note the publication month.)

10. The monkey-and-typewriters parable can be traced back to a paper on statistical mechanics that the French scientist Émile Borel published in a 1913 physics journal (the concept of the Universal Library is even older, however—see the next note). The idea has reappeared many times since; for example, the English mathematical physicist A. S. Eddington used it in his 1927 book *The Nature of the Physical World*, as did Sir James Jeans in his 1930 book *The Mysterious Universe*. The American writer Russell Maloney updated the concept in his elegant 1940 short story "Inflexible Logic," published in The *New Yorker*. (Maloney's tale is reprinted in both the fourth volume of James R. Newman's *The World of Mathematics* and Clifton Fadiman's *Fantasia Mathematica*.) Three years later, in October 1943, the fictionalized idea moved from the glossy, high-class *New Yorker* to the cheap wood-pulp pages of *Astounding Science Fiction*, in Raymond F. Jones's "Fifty Million Monkeys." In this poorly written super-science tale the entire universe is doomed. To find the solution to the problem—which has eluded all human scientists—the hero builds a random machine to "try everything" in the spirit of Lully's machine (see the next note).

11. Lasswitz's short story "The Universal Library" was originally published in a 1901 German book, and its translation by Willy Ley is in Clifton Fadiman, *Fantasia Mathematica* (New York: Simon and Schuster, 1958). A few years later the Argentine writer Jorge Luis Borges gave a very detailed description of the Universal Library in his short story "The Library of Babel," collected in

Ficciones (New York: Grove Press, 1962). As Ley explains in a postscript to Lasswitz's story, Lully's thirteenth-century version of the Universal Library was actually in the form of a *constructable machine* that could automatically generate all possible hypotheses. This idea was elaborated in a famous 1947 book by physicist George Gamow, *One Two Three ... Infinity* (Ley's postscript gives a nice summary of Gamow's machine that, if you read it carefully, makes it plain that it isn't *quite* the Universal Library itself that is generated). More recently, Lully's idea was given a humorous treatment by Philip Cole in his paper "The Hypothesis Generating Machine" (*Epidemiology*, May 1993, pp. 271–273).

12. "Small flecks of brightness" is, of course, a terribly awkward phrase. My original draft of this essay simply defined a digital black-and-white TUPA image as an $m \times n$ array of k-bit numbers. That is, as *mn pixels* (picture elements) with k-bits of gray level (2^k levels, from 0 to $2^k - 1$). With some additional bits per pixel we could easily add color to TUPA, too, and so TUPA would contain not only all possible books but also the images of all possible paintings. The editors at *Omni*, however, decided that bits and pixels were "too technical" and substituted "small flecks of brightness." Still, since 1982 was back in the personal computer dark ages, and not today's world, in which six-year old, computer-savy kids routinely blast through Xbox360 or Playstation 3 video games on HD displays, maybe those long-ago editors were right.

13. There is just one image in TUPA in which every last pixel is the blackest black. But there are *many more* images in which almost all the pixels are the blackest black or nearly so, and those images too could be legitimately called "lens cap on" pictures.

14. Since *Omni* was published under the corporate umbrella of *Penthouse*, I had hopes for a while that *Penthouse* would hire me as a consultant, specifically to construct this exciting subset of TUPA photos for the magazine's centerfold feature. And, since TUPA is gender-neutral, TUPA also contains super-sizzling images that would make the famous centerfold shot of Burt Reynolds in the April 1972 *Cosmopolitan* look tame. Alas, neither magazine ever called.