Chapter One

Introduction

1.1 Large-Scale Interconnected Dynamical Systems

Modern complex dynamical systems[1] are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication network constraints. The sheer size (i.e., dimensionality) and complexity of these large-scale dynamical systems often necessitates a hierarchical decentralized architecture for analyzing and controlling these systems. Specifically, in the analysis and control-system design of complex large-scale dynamical systems it is often desirable to treat the overall system as a collection of interconnected subsystems. The behavior of the aggregate or composite (i.e., large-scale) system can then be predicted from the behaviors of the individual subsystems and their interconnections. The need for decentralized analysis and control design of large-scale systems is a direct consequence of the physical size and complexity of the dynamical model. In particular, computational complexity may be too large for model analysis while severe constraints on communication links between system sensors, actuators, and processors may render centralized control architectures impractical. Moreover, even when communication constraints do not exist, decentralized processing may be more economical.

In an attempt to approximate high-dimensional dynamics of large-scale structural (oscillatory) systems with a low-dimensional diffusive (non-oscillatory) dynamical model, structural dynamicists have developed thermodynamic energy flow models using stochastic energy flow techniques. In particular, statistical energy analysis (SEA) predicated on averaging system states over the statistics of the uncertain system parameters have been extensively developed for mechanical and acoustic vibration problems [109,119,129,163,173]. Thermodynamic models are derived from large-scale dynamical systems of discrete subsystems involving stored energy flow among subsystems based on the assumption of weak subsystem coupling or identical subsystems. However, the ability of SEA to predict the dynamic behavior of a complex large-scale dynamical system in terms of pairwise subsystem interactions is severely limited by the coupling strength of the remaining subsystems on the subsystem pair. Hence, it is not surprising

[1]Here we have in mind large flexible space structures, aerospace systems, electric power systems, network systems, communications systems, transportation systems, economic systems, and ecological systems, to cite but a few examples.

that SEA energy flow predictions for large-scale systems with strong coupling can be erroneous.

Alternatively, a deterministic thermodynamically motivated energy flow modeling for large-scale structural systems is addressed in [113–115]. This approach exploits energy flow models in terms of thermodynamic energy (i.e., ability to dissipate heat) as opposed to stored energy and is not limited to weak subsystem coupling. Finally, a stochastic energy flow *compartmental model* (i.e., a model characterized by conservation laws) predicated on averaging system states over the statistics of stochastic system exogenous disturbances is developed in [21]. The basic result demonstrates how compartmental models arise from second-moment analysis of state space systems under the assumption of weak coupling. Even though these results can be potentially applicable to large-scale dynamical systems with weak coupling, such connections are not explored in [21].

An alternative approach to analyzing large-scale dynamical systems was introduced by the pioneering work of Šiljak [159] and involves the notion of *connective stability*. In particular, the large-scale dynamical system is decomposed into a collection of subsystems with local dynamics and uncertain interactions. Then, each subsystem is considered independently so that the stability of each subsystem is combined with the interconnection constraints to obtain a *vector Lyapunov function* for the composite large-scale dynamical system, guaranteeing connective stability for the overall system.

Vector Lyapunov functions were first introduced by Bellman [14] and Matrosov[2] [133] and further developed by Lakshmikantham *et al.* [118], with [65, 127, 131, 132, 136, 159, 160] exploiting their utility for analyzing large-scale systems. Extensions of vector Lyapunov function theory that include matrix-valued Lyapunov functions for stability analysis of large-scale dynamical systems appear in the monographs by Martynyuk [131, 132]. The use of vector Lyapunov functions in large-scale system analysis offers a very flexible framework for stability analysis since each component of the vector Lyapunov function can satisfy less rigid requirements as compared to a single scalar Lyapunov function. Weakening the hypothesis on the Lyapunov function enlarges the class of Lyapunov functions that can be used for analyzing the stability of large-scale dynamical systems. In particular, each component of a vector Lyapunov function need not be positive definite with a negative or even negative-semidefinite derivative. The time derivative

[2]Even though the theory of vector Lyapunov functions was discovered independently by Bellman and Matrosov, their formulation was quite different in the way that the components of the Lyapunov functions were defined. In particular, in Bellman's formulation the components of the vector Lyapunov functions correspond to disjoint subspaces of the state space, whereas Matrosov allows for the components to be defined in the entire state space. The latter formulation allows for the components of the vector Lyapunov functions to capture the whole state space and, hence, account for interconnected dynamical systems with overlapping subsystems.

of the vector Lyapunov function need only satisfy an element-by-element vector inequality involving a vector field of a certain comparison system. Moreover, in large-scale systems several Lyapunov functions arise naturally from the stability properties of each subsystem. An alternative approach to vector Lyapunov functions for analyzing large-scale dynamical systems is an input-output approach, wherein stability criteria are derived by assuming that each subsystem is either finite gain, passive, or conic [5, 122, 123, 168].

In more recent research, Šiljak [161] developed new and original concepts for modeling and control of large-scale complex systems by addressing system dimensionality, uncertainty, and information structure constraints. In particular, the formulation in [161] develops control law synthesis architectures using decentralized information structure constraints while addressing multiple controllers for reliable stabilization, decentralized optimization, and hierarchical and overlapping decompositions. In addition, decomposition schemes for large-scale systems involving system inputs and outputs as well as dynamic graphs defined on a linear space as one-parameter groups of invariant transformations of the graph space are developed in [178].

Graph theoretic concepts have also been used in stability analysis and decentralized stabilization of large-scale interconnected systems [34, 45]. In particular, graph theory [51, 63] is a powerful tool in investigating structural properties and capturing connectivity properties of large-scale systems. Specifically, a directed graph can be constructed to capture subsystem interconnections wherein the subsystems are represented as nodes and energy, matter, or information flow is represented by edges or arcs. A related approach to graph theory for modeling large-scale systems is bond-graph modeling [35, 107], wherein connections between a pair of subsystems are captured by a bond and energy, matter, or information is exchanged between subsystems along connections. More recently, a major contribution to the analysis and design of interconnected systems is given in [172]. This work builds on the work of bond graphs by developing a modeling behavioral methodology wherein a system is viewed as an interconnection of interacting subsystems modeled by tearing, zooming, and linking.

In light of the fact that energy flow modeling arises naturally in large-scale dynamical systems and vector Lyapunov functions provide a powerful stability analysis framework for these systems, it seems natural that dissipativity theory [170, 171] on the subsystem level, can play a key role in unifying these analysis methods. Specifically, dissipativity theory provides a fundamental framework for the analysis and design of control systems using an input, state, and output description based on system energy[3] related considerations [70, 170]. The dissipation hypothesis on dynamical systems results in a fundamental constraint on their dynamic behavior wherein a dissipative dynamical system can deliver to its surroundings only a fraction of its energy

[3]Here the notion of energy refers to abstract energy for which a physical system energy interpretation is not necessary.

and can store only a fraction of the work done to it. Such conservation laws are prevalent in large-scale dynamical systems such as aerospace systems, power systems, network systems, structural systems, and thermodynamic systems.

Since these systems have numerous input, state, and output properties related to conservation, dissipation, and transport of energy, extending dissipativity theory to capture conservation and dissipation notions on the subsystem level would provide a natural energy flow model for large-scale dynamical systems. Aggregating the dissipativity properties of each of the subsystems by appropriate storage functions and supply rates would allow us to study the dissipativity properties of the composite large-scale system using *vector storage functions* and *vector supply rates*. Furthermore, since vector Lyapunov functions can be viewed as generalizations of composite energy functions for all of the subsystems, a generalized notion of dissipativity, namely, *vector dissipativity*, with appropriate vector storage functions and vector supply rates, can be used to construct vector Lyapunov functions for nonlinear feedback large-scale systems by appropriately combining vector storage functions for the forward and feedback large-scale systems. Finally, as in classical dynamical system theory [70], vector dissipativity theory can play a fundamental role in addressing robustness, disturbance rejection, stability of feedback interconnections, and optimality for large-scale dynamical systems.

The design and implementation of control law architectures for large-scale interconnected dynamical systems is a nontrivial control engineering task involving considerations of weight, size, power, cost, location, type, specifications, and reliability, among other design considerations. All these issues are directly related to the properties of the large-scale system to be controlled and the system performance specifications. For conceptual and practical reasons, the control processor architectures in systems composed of interconnected subsystems are typically distributed or decentralized in nature. Distributed control refers to a control architecture wherein the control is distributed via multiple computational units that are interconnected through information and communication networks, whereas decentralized control refers to a control architecture wherein local decisions are based only on local information. In a decentralized control scheme, the large-scale interconnected dynamical system is controlled by multiple processors operating independently, with each processor receiving a subset of the available subsystem measurements and updating a subset of the subsystem actuators. Although decentralized controllers are more complicated to design than distributed controllers, their implementation offers several advantages. For example, physical system limitations may render it uneconomical or impossible to feed back certain measurement signals to particular actuators.

Since implementation constraints, cost, and reliability considerations often require decentralized controller architectures for controlling large-scale systems, decentralized control has received considerable attention in the literature [17, 22, 48, 96–99, 104, 125, 126, 145, 150, 154, 158–160, 162]. A straightforward decentralized control design technique is that of *sequential optimization* [17, 48, 104], wherein a sequential centralized subcontroller design procedure is applied to an augmented closed-loop plant composed of the actual plant and the remaining subcontrollers. Clearly, a key difficulty with decentralized control predicated on sequential optimization is that of dimensionality. An alternative approach to sequential optimization for decentralized control is based on *subsystem decomposition* with centralized design procedures applied to the individual subsystems of the large-scale system [96–99, 125, 126, 145, 150, 154, 158–160]. Decomposition techniques exploit subsystem interconnection data and in many cases, such as in the presence of very high system dimensionality, are absolutely essential for designing decentralized controllers.

1.2 A Brief Outline of the Monograph

The main objective of this monograph is to develop a general stability analysis and control design framework for nonlinear large-scale interconnected dynamical systems, with an emphasis on vector Lyapunov function methods and vector dissipativity theory. The main contents of the monograph are as follows. In Chapter 2, we establish notation and definitions and develop stability theory for large-scale dynamical systems. Specifically, stability theorems via vector Lyapunov functions are developed for continuous-time and discrete-time nonlinear dynamical systems. In addition, we extend the theory of vector Lyapunov functions by constructing a generalized comparison system whose vector field can be a function of the comparison system states as well as the nonlinear dynamical system states. Furthermore, we present a generalized convergence result which, in the case of a scalar comparison system, specializes to the classical Krasovskii-LaSalle invariant set theorem.

In Chapter 3, we extend the notion of dissipative dynamical systems to develop an energy flow modeling framework for large-scale dynamical systems based on vector dissipativity notions. Specifically, using vector storage functions and vector supply rates, dissipativity properties of a composite large-scale system are shown to be determined from the dissipativity properties of the subsystems and their interconnections. Furthermore, extended Kalman-Yakubovich-Popov conditions, in terms of the subsystem dynamics and interconnection constraints, characterizing vector dissipativeness via vector system storage functions, are derived. In addition, these results are used to develop feedback interconnection stability results for large-scale nonlinear dynamical systems using vector Lyapunov functions. Specialization of these results to passive and nonexpansive large-scale dynamical systems is also provided.

In Chapter 4, we develop connections between thermodynamics and large-scale dynamical systems. Specifically, using compartmental dynamical system theory, we develop energy flow models possessing energy conservation and energy equipartition principles for large-scale dynamical systems. Next, we give a deterministic definition of *entropy* for a large-scale dynamical system that is consistent with the classical definition of entropy and show that it satisfies a Clausius-type inequality leading to the law of nonconservation of entropy. Furthermore, we introduce a new and dual notion to entropy, namely, *ectropy*, as a measure of the tendency of a dynamical system to do useful work and grow more organized, and show that conservation of energy in an isolated thermodynamic large-scale system necessarily leads to nonconservation of ectropy and entropy. In addition, using the system ectropy as a Lyapunov function candidate, we show that our large-scale thermodynamic energy flow model has convergent trajectories to Lyapunov stable equilibria determined by the system initial subsystem energies.

In Chapter 5, we introduce the notion of a *control vector Lyapunov function* as a generalization of *control Lyapunov functions* [6], and show that asymptotic stabilizability of a nonlinear dynamical system is equivalent to the existence of a control vector Lyapunov function. Moreover, using control vector Lyapunov functions, we construct a universal decentralized feedback control law for a decentralized nonlinear dynamical system that possesses guaranteed gain and sector margins in each decentralized input channel. Furthermore, we establish connections between the notion of vector dissipativity developed in Chapter 3 and optimality of the proposed decentralized feedback control law. The proposed control framework is then used to construct decentralized controllers for large-scale nonlinear systems with robustness guarantees against full modeling uncertainty. In Chapter 6, we extend the results of Chapter 5 to develop a general framework for finite-time stability analysis based on vector Lyapunov functions. Specifically, we construct a vector comparison system whose solution is finite-time stable and relate this finite-time stability property to the stability properties of a nonlinear dynamical system using a vector comparison principle. Furthermore, we design a universal decentralized finite-time stabilizer for large-scale dynamical systems that is robust against full modeling uncertainty.

Next, using the results of Chapter 5, in Chapter 7 we develop a stability and control design framework for time-varying and time-invariant sets of nonlinear dynamical systems. We then apply this framework to the problem of coordination control for multiagent interconnected systems. Specifically, by characterizing a moving formation of vehicles as a time-varying set in the state space, a distributed control design framework for multivehicle coordinated motion is developed by designing stabilizing controllers for time-varying sets of nonlinear dynamical systems. In Chapters 8 and 9, we present discrete-time extensions of vector dissipativity theory and system thermodynamic connections of large-scale systems developed in Chapters 3 and 4, respectively.

In Chapter 10, we provide generalizations of the stability results developed in Chapter 2 to address stability of impulsive dynamical systems via vector Lyapunov functions. Specifically, we provide a generalized comparison principle involving hybrid comparison dynamics that are dependent on the comparison system states as well as the nonlinear impulsive dynamical system states. Furthermore, we develop stability results for impulsive dynamical systems that involve vector Lyapunov functions and hybrid comparison inequalities. In addition, we develop vector dissipativity notions for large-scale nonlinear impulsive dynamical systems. In particular, we introduce a generalized definition of dissipativity for large-scale nonlinear impulsive dynamical systems in terms of a hybrid vector inequality, a vector hybrid supply rate, and a vector storage function. Dissipativity properties of the large-scale impulsive system are shown to be determined from the dissipativity properties of the individual impulsive subsystems making up the large-scale system and the nature of the system interconnections. Using the concepts of dissipativity and vector dissipativity, we also develop feedback interconnection stability results for impulsive nonlinear dynamical systems. General stability criteria are given for Lyapunov, asymptotic, and exponential stability of feedback impulsive dynamical systems. In the case of quadratic hybrid supply rates corresponding to net system power and weighted input-output energy, these results generalize the positivity and small gain theorems to the case of nonlinear large-scale impulsive dynamical systems.

Using the concepts developed in Chapter 10, in Chapter 11 we extend the notion of control vector Lyapunov functions to impulsive dynamical systems. Specifically, using control vector Lyapunov functions, we construct a universal hybrid decentralized feedback stabilizer for a decentralized affine in the control nonlinear impulsive dynamical system that possesses guaranteed gain and sector margins in each decentralized input channel. These results are then used to develop hybrid decentralized controllers for large-scale impulsive dynamical systems with robustness guarantees against full modeling and input uncertainty. Finite-time stability analysis and control design extensions for large-scale impulsive dynamical systems are addressed in Chapter 12.

In Chapter 13, a novel class of fixed-order, energy-based hybrid decentralized controllers is proposed as a means for achieving enhanced energy dissipation in large-scale vector lossless and vector dissipative dynamical systems. These dynamic decentralized controllers combine a logical switching architecture with continuous dynamics to guarantee that the system plant energy is strictly decreasing across switchings. The general framework leads to hybrid closed-loop systems described by impulsive differential equations [82]. In addition, we construct hybrid dynamic controllers that guarantee that each subsystem-subcontroller pair of the hybrid closed-loop system is consistent with basic thermodynamic principles. Special cases of energy-based hybrid controllers involving state-dependent switching are described, and several illustrative examples are given as well as an experimental test bed is designed to demonstrate the efficacy of the proposed approach. Finally, we draw conclusions in Chapter 14.