Preface

Not simple, but as simple as possible

> Physics should be made as simple as possible,
> but not any simpler.
>
> —A. Einstein

Einstein gravity should be made as simple as possible, but not any simpler.

My goal is to make Einstein gravity* as simple as possible. I believe that Einstein's theory should be readily accessible to those who have mastered Newtonian mechanics and a modest amount of classical mathematics. To underline my point, I start with a review of $F = ma$.

Seriously, what do you need to know to read this book? Only some knowledge of classical mechanics and electromagnetism! So I fondly imagine, perhaps unrealistically. More importantly, you need to be possessed of what we theoretical physicists call sense—physical, mathematical, and also common.

I wrote this book in the same spirit as my *Quantum Field Theory in a Nutshell*.[1] In his *Physics Today* review of that book, Zvi Bern wrote this lovely sentence aptly capturing my pedagogical philosophy: "The purpose of Zee's book is not to turn students into experts—it is to make them fall in love with the subject." I might extend that to "fall in love with the subject so that they might desire to become experts." Here I am echoing William Butler Yeats, who said, "Education is not the filling of a pail, but the lighting of a fire."

* Also known as general relativity.

A portion of this book can be used for an undergraduate course. I have done it, and I provide a detailed course outline later in this preface.

Accessible is not to be equated with dumbed-down or watered-down. Also, accessible is not necessarily the same as elementary: in the last parts of the book, I include some topics far beyond the usual introductory treatment.

My strategy to make Einstein gravity as simple as possible has two prongs. The first is the emphasis on symmetry. As some readers may know, I have written an entire book[2] on the role of symmetry in physics, and I absolutely love how symmetry guides us in constructing physical theories, a notion that started with Einstein gravity, in fact. The second is the extensive use of the action principle. The action is invariably simpler than the equations of motion and manifests the inherent symmetry much more forcefully. I can hardly believe that some well-known textbooks on Einstein's theory barely mention the Einstein-Hilbert action. Symmetry and the action principle constitute the two great themes of theoretical physics.

To get a flavor of what the book is about, you might want to glance at the recaps first; there is one at the end of each of the ten parts of the book.

How difficult is Einstein gravity?

Any intelligent student can grasp it without too much trouble.
—A. Einstein, referring to his theory of gravity

When Arthur Eddington returned from the famous 1919 solar eclipse expedition that observed light from a distant star bending in agreement with Einstein gravity, somebody asked him if it were true that only three people understood Einstein's theory. Eddington replied, "Who is the third?" The story, apocryphal[3] or not, is one of many[4] that gives Einstein's theory its undeserved reputation of being incomprehensible.

I believe that in some cases, people like to persist in believing that Einstein's theory is beyond them. A renowned philosopher who is clearly well above average in intelligence (and who understands things that I find impossible to understand) once told me that he was tired of popular accounts of general relativity and that he would like to finally learn the subject for real. He also emphasized to me that he had taken advanced calculus[5] in college, as if to say that he could handle the math. I replied that, for a small fee, my impecunious graduate student could readily teach him the essence of general relativity in six easy lessons. I never heard from the renowned philosopher again. I was happy and he was happy: he could go on enunciating philosophical profundities about relative truths[6] and physical reality.

The point of the story is that it is not that difficult.

For whom is this book intended

Experience with my field theory textbook suggests that readers of this book will include the following overlapping groups: students enrolled in a course on general relativity, students and others indulging in the admirable practice of self-study, professional physicists in other research specialties who want to brush up, and readers of popular books on Einstein gravity who want to fly beyond the superficial discussions these books (including my own[7]) offer. My comments below apply to some or all of these groups.[8]

Personally, I feel special sympathy for those studying the subject on their own, as I remember struggling[9] one summer during my undergraduate years with a particularly idiosyncratic text on general relativity, the only one I could find in São Paulo back in those antediluvian times. That experience probably contributed to my desire to write a textbook on the subject. From the mail I have received regarding *QFT Nut,* I have been pleasantly surprised, and impressed, by the number of people out there studying quantum field theory on their own. Surely there are even more who are capable of self-studying Einstein gravity. All power to you! I wrote this book partly with you in mind.

Serious students of physics know that one can't get far without doing exercises. Some of the exercises lead to results that I will need later.

Quite naturally, I have also written this book with an eye toward quantum field theory and quantum gravity. While I certainly do not cover quantum gravity, I hope that the reader who works through this book conscientiously will be ready for more specialized monographs[10] and the vast literature out there.

So, I prevaricated a little earlier. In the latter part of the book, occasionally you will need to know more than classical mechanics and electromagnetism. But, to be fair, how do you expect me to talk about Hawking radiation, a quintessentially quantum phenomenon, in chapter VII.3? Indeed, how could we discuss natural units in the introduction if you have never heard of quantum mechanics? For the readers with only a nodding acquaintance with quantum mechanics, the good news is that for the most part, I only ask that you know the uncertainty principle.

I do not doubt that some readers will encounter difficult passages. That's because I have not made the book "any simpler"!

In the preface to the second edition of my quantum field theory book, I mentioned that Steve Weinberg and I, each referring to his own textbook, each said, "I wrote the book that I would have liked to learn from." So this is the book I would have liked as an undergrad* eager to learn Einstein gravity. I would have liked having at least a flavor of what the latest

* In a letter to the editors of *Physics Today* in 2005, A. Harvey and E. Schucking wrote that, in view of the "monumental lip service" paid to Einstein in the physics community, "it is a scandal" that Einstein gravity is still not regularly taught to undergraduates. I find it even more of a scandal that many physics professors proudly profess ignorance of Einstein gravity, saying that it is irrelevant to their research. Yes, maybe, but this is akin to being proudly ignorant of Darwinian evolution because it is irrelevant to whatever you are doing.

excitement was all about. In this spirit, I offer chapter X.6 on twistors, for example, trusting the reader to be sophisticated enough to know that all one should expect to get from a single textbook chapter is an entry key to the research literature rather than a complete account of an emerging area.

The importance of feeling amazed, and amused

I am amazed that students are not amazed.

The action principle amazed Feynman when he first heard about it. In learning theoretical physics, I was, and am, constantly amazed. But in teaching, I am amazed that students are often not amazed. Even worse, they are not amused.

Perhaps it is difficult for some students to be amazed and amused when they have to drag themselves through miles of formalism. So this exhortation to be amazed is related to my attempt to keep the formalism to an absolute minimum in my textbooks and to get to the physics.

To paraphrase another of my action heroes, students should be required to gasp and laugh[11] periodically. Why study Einstein gravity unless you have fun doing it?

As much fun as possible

Bern started his review of my quantum field theory textbook thus:

> When writing a book on a subject in which a number of distinguished texts already exist, any would-be author should ask the following key question: What new perspectives can I offer that are not already covered elsewhere? . . . perhaps foremost in A. Zee's mind was how to make *Quantum Field Theory in a Nutshell* as much fun as possible.

Good question! My answer remains the same. I want to make Einstein gravity as much fun as possible.

Sidney Coleman, my professor in graduate school and thesis advisor, once advised me that theoretical physics is a "gentleman's diversion." I was made to understand that I should avoid doing long sweaty calculations. This book reflects some of that spirit. Thus, in chapter VI.1, instead of deriving Einstein's field equation as a true Confucian scholar would, I try to get to it as quickly as possible by a method I dub "winging it southern California style." Similarly, in chapter VI.2, I get to cosmology as quickly as possible.

This invariably brings me to the dreaded topic of drudgery in general relativity. Many theory students in my generation went into particle physics rather than general relativity to avoid the drudgery of spending an entire day calculating the Riemann curvature tensor. I did.[12] But that was the old days. Nowadays, students of general relativity can use ready-made symbolic manipulation programs[13] to do all the tedious work. I strongly urge you, however, to write your own programs, as I did, rather than open a can. It also goes without saying that you should calculate the Riemann curvature tensor from scratch at least a few times to know how all the cogs fit together.

You make the discoveries

My pedagogical philosophy is to let students discover certain things on their own. Some of these lessons evolved into what I call extragalactic fables. For example, in part IV, I let the extragalactic version of you discover electrodynamics and gravity. In chapter IV.3, you discover that gravity affects the flow of time.

I also whet your appetite by anticipating. For example, I mention the Einstein-Rosen bridge already in chapter I.6. In working out the shortest distance between two points in chapter II.2, I mention that you will encounter the same equations when you study motion around black holes. In part II, I note that the peculiar replacement of a simple equation by a more complicated looking equation foreshadows Einstein's deep insight about gravity to be discussed in part V.

The return of Confusio

Readers of *QFT Nut* might be pleased to hear that Confusio makes a return appearance, together with other characters, such as the Smart Experimentalist. Some other friends of mine, for example the Jargon Guy, also show up. Here I am alluding to what Einstein referred[14] to as "more or less dispensable erudition."

An outline of this book

This book appears to start at a rather low level, with a review of Newtonian mechanics in part I. The reason is that I want to treat two topics more thoroughly than usual: rotations and coordinate transformations. A good understanding of these two elementary subjects allows us to jump to the Lorentz group and curved spacetime later. My pedagogical approach is to beat 2-dimensional rotations to death. Depending on how mechanics is taught, students typically miss, or fail to grasp, some of the material in the chapter on tensors. I repeat the discussion of tensors under various guises and in different contexts. One of my students who read the book points to various places where I appear to repeat myself, but I told her that it is better to hear some key point for the third time[15] than not to have understood it at all. A respected senior colleague and pioneer in Einstein gravity said to me that a good teacher is someone who never says anything worth saying only once.

I devote part II to a discussion of the all-important action principle, because I believe that it provides the quickest, and the most fundamental, way to Einstein gravity (and to quantum field theory). Part III is devoted to special relativity but, in contrast to some elementary treatments, the emphasis is on geometry and completion, not on a collection of paradoxes. In part IV, as was mentioned earlier, I let you discover electromagnetism and gravity, and so the treatment is somewhat nonstandard. Thus, even if you feel that you already know special relativity, you might want to take a quick look at part III and part IV.

Many readers probably pick up this book because of a burning desire to learn Einstein gravity. These readers would have already mastered Newtonian mechanics and special relativity, and they could probably cut to the chase and skip directly to part V. To them, the first four parts may appear to be a rather leisurely preparation for Einstein gravity. Still, I would counsel skimming, rather than skipping, the first four parts. At the very least, parts I–IV set down the conventions and notation. More importantly, they offer up the ideology of this text, an ideology that can be simply stated: action!

While I appear to start slow in parts I–III, I am actually setting things up so that we can go fast in parts V and VI. For example, all the discussion about coordinate transformation and curved spaces is to prepare the reader for a quick plunge into curved spacetime in chapter V.1. Similarly, the action principle enables the geodesic equation to be introduced early on, in part II, so that it is "ready to trot" when needed in part V. In considering whether to sign up for my course that grew into this book, some students ask how fast I will be zooming through special relativity to get to the "good stuff." But special relativity is good stuff! In particular, it is essential to understand special relativity as the geometry of spacetime* before moving on to general relativity.

The essence of Einstein gravity is explained in parts V and VI. The rest of the book contains what may be regarded as applications of the theory as developed in part VI. Part X contains extras that some might consider beyond the scope of an introductory text. The title is thus something of a misnomer, but to please my publisher, I am obliged to keep up a running joke I started with my field theory book. A better title might be *Gravity from Newton to the Brane World*.

The role of appendices

As a textbook writer, I am torn between being concise and being complete. One way out is to place numerous topics in appendices to various chapters. Some are fun, such as Einstein's derivation of $E = mc^2$ in his 1946 Haifa lectures (see chapter III.6), which, unfortunately, is in danger of being forgotten and which I much prefer to his 1905 derivation. Another example is Weyl's shortcut to the Schwarzschild solution (see chapter VI.3). Some are results I will need later, but often much later. For example, I talk about the speed of sound in an appendix to chapter III.6, but I won't need it until I get to the cosmic microwave background. Some appendices are peripheral or technical. When possible, I try to give an intuitive and heuristic understanding before launching into a long development, such as

* A multitude of books treat special relativity, but while they all get the job done, they differ widely in conceptual clarity. Besides the geometrical view of special relativity, I also want to emphasize the Lorentz action as leading to a unified approach to both massive and massless particles.

the treatment of Fermi normal coordinates. Some are for enrichment. In sum, the use of appendices represents my effort to appeal to a broad range of readers with enormously different levels of knowledge and sophistication. The reader should not feel obliged, upon first reading this book, to study all the appendices. Each should exercise his or her own judgment.

Still, a book this size is inevitably incomplete, and so it comes down to the author's choice (of course). So many beautiful results, so little space and time! I regard certain topics, though important, as better covered in more specialized tomes, such as gravitational lensing, and prefer to include some topics not discussed in several standard textbooks, such as anti de Sitter spacetime, brane worlds, and twistors.

The most incomprehensible thing about some physics textbooks

> The most incomprehensible thing about the physical world is that it is comprehensible.
>
> —A. Einstein

The most incomprehensible thing about some physics textbooks is that they are incomprehensible.

They manage to render the easily comprehensible into the nearly incomprehensible. Some textbook writers are simplifiers, others are what I call complicators. In defiance of Einstein's exhortation, many authors strive to make physics as complicated as possible, or so it seems to me. In the research literature, the cause of obscurity may be unintentional or intentional: either the author has not understood the issues involved completely (often laudably so, when the author is at the cutting edge), or the author wants to impress upon the reader the profundity of his or her paper by resorting to obfuscations. But in a textbook?

My task, and hope, in my textbooks is to make physics as simple as possible, as the "old man" with his toy[16] said. Having written both a textbook and a couple of popular books, I am perhaps qualified to express my opinions here. Popular books attempt to make physics simpler than it really is, thus in some sense deceiving the reader. Textbooks are different: they must make the reader work to master the subject. But making the reader work is not the same as making the reader suffer by rendering simple things obscure.

No bijective maps in this book

I am puzzled by students who profess no trouble with the physics but moan* about the math. All the "grown-ups" would say the opposite. The pros regard Riemannian geometry,

* Indeed, many of the postings on the sites of online booksellers regarding general relativity texts lament the difficulty of the math. At the other extreme, a few, by misguided individuals in my opinion, complain about the lack of rigor.

which is after all totally logical and algorithmic, as easy, but continue to lose sleep over Einstein's theory. Regarding the math, I can say, with only slight exaggeration, that mastery of the index notation and the chain rule almost suffices. Indeed, any serious student with a future in theoretical physics should be continually puzzled by the physics but not at all by the math.

Einstein did not say that physics should be made simple. Of course, physics is not simple, and understanding Einstein's theory does require effort. Surely you have heard that Einstein gravity involves curved spacetime, so there is no getting around learning the language needed to describe curvature. My strategy is to introduce math only when necessary, and then to illustrate the key concepts with plenty of examples. I dislike the Red Army[17] approach, and so I do not start by defining bundles on the tangent plane. I bring in the math gently and sneak in curvature early on via the familiar change of coordinates.

As for rigor, I will let yet another of my action heroes speak. "I'll differentiate any function, even the freaking delta function, as many times as I darn well please." So if you have to differentiate, just differentiate until the expression you are differentiating starts bleating for mercy. The trick is to know when it is absolutely necessary to be rigorous (which is seldom—I would never say never).

I respectfully submit that this book is not for those who want rigor.

While I realize the need for and the benefit of precise definition, for the most part I simply plead membership in the Feynman[18] "Shut up and calculate" school of physics.[19] Thus, I won't trouble your sleep with assertions such as "A bijective differentiable map of a manifold, whose inverse is also differentiable, is called a diffeomorphism." Regarding statements like this, I think that another Einstein quote may be apropos: "We should take care not to make the intellect our god; it has, of course, powerful muscles, but no personality."[20] Yet another relevant quote: "The people in Göttingen sometimes strike me, not as if they wanted to help one formulate something clearly, but instead as if they wanted only to show us physicists how much brighter they are than we."[21] Alas, "the people in Göttingen" have now gone off and multiplied,* and some even live in our midst. Precise definitions are indeed necessary occasionally, but by and large, they don't do much good in theoretical physics. Some things are better left undefined. In this connection, also keep in mind the distinction between true clarity and false clarity.[22] For example, I consider the insistence on saying "pseudo-Riemannian manifolds" in a book of this level false clarity at best.

As I was putting the finishing touches on this book, I read about some notes[23] Feynman scribbled to himself before teaching some course: "First figure out why you want the student to learn the subject and what you want them to know, and the method will result more or less by common sense." Well said! As it turned out, that was the method I followed when writing this book.

If you feel that bijection is indispensable for your existential essence, then I also respectfully submit that this book is not for you.

* One tribe is known to look at "old fashioned" indices with contempt. Only coordinate-free notations[24] are good enough for them.

But of course I am not against mathematics. For instance, I am all for differential forms (see chapters IX.7 and IX.8). However, when faced with a new formalism, I tend to be practical and ask, "For the time invested in learning it, what is the payoff?" How significant is it for the physics?

Teaching from this book and self-studying

It would be ideal to teach a leisurely year-long course based on this book. But I have also taught Einstein gravity at the University of California, Santa Barbara, as a scandalously short one-quarter undergraduate course consisting of only 29 lectures. The students allegedly knew the action principle and special relativity, but I was appropriately skeptical. Here is the actual course plan.

Lecture 1 gives an overview. Lectures 2–6 cover chapters I.5 and I.6, starting with the notion of a metric and illustrated with numerous examples, including the Poincaré half plane, and ending with locally flat coordinates and a count of the components contained in the curvature tensor. Lectures 7 and 8 cover part II, and lectures 9 and 10 part III. In lectures 11 and 12, I let the students discover electromagnetism and gravity and derive how gravity affects the flow of time. Lectures 13–15 introduce the equivalence principle and cover part V up to chapter V.3, ending with closed, flat, and open universes.

The second half of the course proceeds as follows:

Lecture 16: the geodesic equation reduced to Newton's equation, gravitational redshift, spherically symmetric spacetime with time dependence

Lecture 17: the motion of particles and light in static spherically symmetric spacetime

Lecture 18: covariant differentiation, the geometrical picture

Lecture 19: to Einstein's field equation as quickly as possible

Lecture 20: the Riemann curvature tensor and its symmetry properties

Lecture 21: the Einstein-Hilbert action

Lecture 22: the cosmological constant and the expanding universe

Lecture 23: Schwarzschild metric, with precession of planets and radar echo delay described in words and pictures

Lecture 24: the energy momentum tensor

Lecture 25: general proof of energy momentum conservation

Lecture 26: the Einstein tensor and the Bianchi identity

Lecture 27: black holes in various coordinates

Lecture 28: the causal structure of spacetime

Lecture 29: Hawking radiation and a grand review

So it is entirely possible to cover the bulk of this book in a one-quarter course! I did it. Students were expected to do some reading and to fill in some gaps on their own. Of course, instructors could deviate considerably from this course plan, emphasizing one topic at the expense of another. They might also wish to challenge the better students by assigning the appendices and some later chapters.

Here I come back to those I applauded earlier for self-studying Einstein gravity. Some of you might want to know which chapters to read. The answer is of course that you should read them all, in an ideal world. But if you want to get "there" quickly, I suggest the following. You are on your own regarding the first three parts: it all depends on what you already know. So try starting with part IV and see how often you need to refer back to an earlier chapter. Part V is indispensable, particularly the equivalence principle and the tour of curved spacetimes. You need to understand the covariant derivative, but you could skip the somewhat heavier appendices in chapter V.6. After the covariant derivative, you are ready for the heart of the matter, Einstein's field equation, in chapter VI.1. The rest of part VI forms the core of a traditional course on general relativity, but my emphasis is somewhat less on working out orbits in detail. That's it! You would have then reached a certain level of mastery of Einstein gravity. You could then regard the rest of the book, parts VII–X, as a buffet of topics that you could browse at your leisure. Part X contains more speculative topics, including some that may not be of lasting value. Be warned!

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I wrote the bulk of this book in Santa Barbara, California, but some parts were written while visiting the Academia Sinica in Taipei, the Republic of China. I deeply appreciate the generous hospitality of Maw-kuen Wu, the director of the Institute of Physics. Bits and pieces were also written in Munich, Germany, and in Beijing, the People's Republic of China.

My editor at Princeton University Press, Ingrid Gnerlich, has always been a pleasure to talk to and work with. She has listened patiently to my ranting and raving for years. For my copyeditor, I am delighted to have, once again, Cyd Westmoreland, who worked on both editions of my *Quantum Field Theory in a Nutshell*. I am much impressed by the meticulous

work of Peter Strupp and Princeton Editorial Associates. I also thank Craig Kunimoto for his indispensable computer help.

I am deeply grateful to my wife Janice for her loving support and encouragement throughout the writing of this book. As this book was nearing completion, she gave birth to our son Max.

Notes

1. Hereafter referred to as *QFT Nut*.
2. A. Zee, *Fearful Symmetry*. Hereafter, *Fearful*.
3. See chapter VI.3.
4. Chaim Weizmann, the first president of Israel and a chemist, once crossed the ocean with Albert Einstein on the same liner, and Einstein tried to explain the theory of relativity to him. When asked about this later, Weizmann said something like "I did not understand his theory, but he certainly convinced me that he did."
5. For the record, I took a philosophy course in college. To further emphasize that I am not totally lacking in "philosophical credentials," I was once invited by a philosophy professor to lecture, thanks to one of my popular books, to an auditorium full of philosophers. I like philosophers.
6. Einstein once said that he should have called his work "invariance theory" and lamented his use of the word "relative."
7. A. Zee, *An Old Man's Toy*. Hereafter, *Toy/Universe*.
8. In my introduction to Feynman's book on quantum electrodynamics, I wrote about three different kinds of readers of that book. Only part 0 of this book will be comprehensible to the first kind. See R. P. Feynman, *QED: The Strange Theory of Light and Matter,* with a new introduction by A. Zee, Princeton Science Library, 2006.
9. An undergrad friend had also deluded me into thinking that it was salutary to read Einstein in the original German!
10. Read J. Polchinski, *String Theory,* for example.
11. *QFT Nut,* p. 473.
12. For the record, I started my research career with John Wheeler, studying gravitational wave emission from neutron stars. For Wheeler's influence on his students, see Charles W. Misner, "John Wheeler and the Recertification of General Relativity as True Physics," in *General Relativity and John Archibald Wheeler,* ed. I. Ciufolini and R. Matzner, Springer, 2010.
13. See my remarks in chapter IX.9, for example.
14. A. Einstein, *Autobiographical Notes*, Open Court, 1999.
15. In any case, if you think that I talk too much about tensors, you could simply feel smugly superior to those poor souls who never get it.
16. See *Toy/Universe*. Also see figure 2b in the prologue to book two.
17. I learned this terminology (which, I should clarify, referred to the Russian, not the Chinese, version) in a conversation with Steve Weinberg about textbooks. It has something to do with lining up all the tanks first.
18. A colleague who got his doctorate at Caltech told me the following story. He was examined by a committee consisting of Feynman and a bunch of lesser lights. One of the lesser lights posed a question to my friend, who proceeded to answer it perfectly, outlining the calculation necessary and explaining the physical significance of the result. The lesser light then opined ominously, "You should have also said . . . " and hereforth issued from his mouth a long string of highfalutin hundred-dollar words. Feynman turned to the lesser light and announced to the rest of the room, "But that's exactly what he said!"

    Here is a totally gratuitous Feynman story that has nothing to do with the discussion at hand. During the exam, Feynman asked a question about quantum mechanics that the student was unable to answer. Feynman exploded, saying something like "Quantum mechanics was invented in the 1920s and it's now 1972; you really should have mastered quantum mechanics by now!" A committee member turned to Feynman and said softly, "Dick, Dick, it's now 1973."
19. A colleague told me his retort to Feynman: "Shut up and contemplate." Of course, Feynman is capable of doing both. Contrary to myth, Feynman won the national Putnam mathematics competition. Here we are talking about people who can only talk and not calculate.

20. The quote is possibly apocryphal.
21. Quoted in C. Reid, *Hilbert,* Springer, 1996, p. 142.
22. As one of my professors, an exceedingly distinguished theoretical physicist, used to say, the main purpose of all the talk about tangent bundles and pullback is to frighten young children. This is not entirely true, but, oh well.
23. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics,* volume III, Addison Wesley (Commemorative issue 2004), p. xi.
24. I am certainly not against coordinate-free notations. In physics, the only issue is which notation is best suited for the job at hand. Coordinate-free notations are great for proving general theorems but are not so good for calculating. In this connection, I might regale the reader with a story. At a recent Santa Barbara conference on black holes, dS, AdS, gravity dual, and so on—in short, the latest hot stuff—I was chatting at lunch with two leading young researchers, up and coming stars, not some aging curmudgeons with congealed opinions. When I mentioned how some people clamored for index-free notations, one of these two leading lights basically said to please get those people out of her sight. The other told me a more illuminating story. During grad school, to deepen his understanding of Einstein gravity, he enrolled in a course taught by a famous mathematician. As it happened, he was the only student able to do the problems in the final exam involving actual calculations: he did them by first using old fashioned indices and then translating back into the abstract notation used in the course.

    The index-free notation in Einstein gravity is somewhat analogous to using vectors without committing to any specific coordinate choice. For example, one can prove easily that $\vec{L} = \vec{r} \times \vec{p}$ is conserved, but try to do the spinning top on an oscillating inclined plane without setting up coordinates! The difference between the uninitiated and the misinformed is that the uninitiated is not acquainted with a particular formalism, while the misinformed insists that only the particular formalism he or she likes is any good.