Preface

1 What Is *The Companion*?

The Princeton Companion to Applied Mathematics describes what applied mathematics is about, why it is important, its connections with other disciplines, and some of the main areas of current research. It also explains what applied mathematicians do, which includes not only studying the subject itself but also writing about mathematics, teaching it, and influencing policy makers.

The Companion differs from an encyclopedia in that it is not an exhaustive treatment of the subject, and it differs from a handbook in that it does not cover all relevant methods and techniques. Instead, the aim is to offer a broad but selective coverage that conveys the excitement of modern applied mathematics while also giving an appreciation of its history and the outstanding challenges. *The Companion* focuses on topics felt by the editors to be of enduring interest, and so it should remain relevant for many years to come.

With online sources of information about mathematics growing ever more extensive, one might ask what role a printed volume such as this has. Certainly, one can use Google to search for almost any topic in the book and find relevant material, perhaps on Wikipedia. What distinguishes *The Companion* is that it is a self-contained, structured reference work giving a consistent treatment of the subject. The content has been curated by an editorial board of applied mathematicians with a wide range of interests and experience, the articles have been written by leading experts and have been rigorously edited and copyedited, and the whole volume is thoroughly cross-referenced and indexed.

Within each article, the authors and editors have tried hard to convey the motivation for each topic or concept and the basic ideas behind it, while avoiding unnecessary detail. It is hoped that *The Companion* will be seen as a friendly and inspiring reference, containing both standard material and more unusual, novel, or unexpected topics.

2 Scope

It is difficult to give a precise definition of applied mathematics, as discussed in WHAT IS APPLIED MATHEMATICS? [I.1] and, from a historical perspective, in THE HISTORY OF APPLIED MATHEMATICS [I.6]. *The Companion* treats applied mathematics in a broad sense, and it cannot cover all aspects in equal depth. Some parts of mathematical physics are included, though a full treatment of modern fundamental theories is not given. Statistics and probability are not explicitly included, although a number of articles make use of ideas from these subjects, and in particular the burgeoning area of UNCERTAINTY QUANTIFICATION [II.34] brings together many ideas from applied mathematics and statistics. Applied mathematics increasingly makes use of algorithms and computation, and a number of aspects at the interface with computer science are included. Some parts of discrete and combinatorial mathematics are also covered.

3 Audience

The target audience for *The Companion* is mathematicians at undergraduate level or above; students, researchers, and professionals in other subjects who use mathematics; and mathematically interested lay readers. Some articles will also be accessible to students studying mathematics at pre-university level.

Prospective research students might use the book to obtain some idea of the different areas of applied mathematics that they could work in. Researchers who regularly attend seminars in areas outside their own specialities should find that the articles provide a gentle introduction to some of these areas, making good pre- or post-seminar reading.

In soliciting and editing the articles the editors aimed to maximize accessibility by keeping discussions at the lowest practical level. A good question is how much of the book a reader should expect to understand. Of course "understanding" is an imprecisely defined

concept. It is one thing to read along with an argument and find it plausible, or even convincing, but another to reproduce it on a blank piece of paper, as every undergraduate discovers at exam time. The very wide range of topics covered means that it would take a reader with an unusually broad knowledge to understand everything, but every reader from undergraduate level upward should find a substantial portion of the book accessible.

4 Organization

The Companion is organized in eight parts, which are designed to cut across applied mathematics in different ways.

Part I, "Introduction to Applied Mathematics," begins by discussing what applied mathematics is and giving examples of the use of applied mathematics in everyday life. THE LANGUAGE OF APPLIED MATHEMATICS [I.2] then presents basic definitions, notation, and concepts that are needed frequently in other parts of the book, essentially giving a brief overview of some key parts of undergraduate mathematics. This article is not meant to be a complete survey, and many later articles provide other introductory material themselves. METHODS OF SOLUTION [I.3] describes some general solution techniques used in applied mathematics. ALGORITHMS [I.4] explains the concept of an algorithm, giving some important examples and discussing complexity issues. The presence of this article in part I reflects the increasing importance of algorithms in all areas of applied mathematics. GOALS OF APPLIED MATHEMATICAL RESEARCH [I.5] describes the kinds of questions and issues that research in applied mathematics addresses and discusses some strategic aspects of carrying out research. Finally, THE HISTORY OF APPLIED MATHEMATICS [I.6] describes the history of the subject from ancient times up until the late twentieth century.

Part II, "Concepts," comprises short articles that explain specific concepts and their significance. These are mainly concepts that cut across different models and areas and provide connections to other parts of the book. This part is not meant to be comprehensive, and many other concepts are well described in later articles (and discoverable via the index).

Part III, "Equations, Laws, and Functions of Applied Mathematics," treats important examples of what its title describes. The choice of what to include was based on a mix of importance, accessibility, and interest. Many equations, laws, and functions not contained in this part are included in other articles.

Part IV, "Areas of Applied Mathematics," contains longer articles giving an overview of the whole subject and how it is organized, arranged by research area. The aim of this part is to convey the breadth, depth, and diversity of applied mathematics research. The coverage is not comprehensive, but areas that do not appear as or in article titles may nevertheless be present in other articles. For example, there is no article on geoscience, yet EARTH SYSTEM DYNAMICS [IV.30], INVERSE PROBLEMS [IV.15], and IMAGING THE EARTH USING GREEN'S THEOREM [VII.16] all cover specific aspects of this area. Nor is there a part IV article on numerical analysis, but this area is represented by APPROXIMATION THEORY [IV.9], NUMERICAL LINEAR ALGEBRA AND MATRIX ANALYSIS [IV.10], CONTINUOUS OPTIMIZATION (NONLINEAR AND LINEAR PROGRAMMING) [IV.11], NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS [IV.12], and NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS [IV.13].

Part V, "Modeling," gives a selection of mathematical models, explaining how the models are derived and how they are solved.

Part VI, "Example Problems," contains short articles covering a variety of interesting applied mathematics problems.

Part VII, "Application Areas," comprises articles on connections between applied mathematics and other disciplines, including such diverse topics as integrated circuit (chip) design, medical imaging, and the screening of luggage in airports.

Part VIII, "Final Perspectives," contains essays on broader aspects, including reading, writing, and typesetting mathematics; teaching applied mathematics; and how to influence government as a mathematician.

The articles within a given part vary significantly in length. This should not be taken as an indication of the importance of the corresponding topic, as it is partly due to the number of pages that could be allocated to each article, as well as to how authors responded to their given page limit.

The ordering of articles within a part is alphabetical for parts II and III. For part IV some attempt was made to place related articles together and to place one article before another if there is a natural order in which to read the two articles. The ordering is nevertheless somewhat arbitrary, and the reader should feel free to read the articles in any order. The articles within parts V–VIII are arranged only loosely by theme.

The editors made an effort to encourage illustrations and diagrams. Due to the cost of reproduction, color has been used only where necessary and is restricted to the color plates following page 364.

Despite the careful organization, the editors expect that many readers will flick through the book to find something interesting, start reading, and by following cross-references navigate the book in an unpredictable fashion. This approach is perfectly reasonable, as is starting from a page found via the table of contents or the index. Whatever the reading strategy, we hope the book will be hard to put down!

5 Relation to *The Princeton Companion to Mathematics*

This *Companion* follows the highly successful *Princeton Companion to Mathematics* (PCM), edited by Gowers, Barrow-Green, and Leader (2008), which focuses on modern pure mathematics. We have tried to build on the PCM and avoid overlap with it. Thus we do not cover many of the basic mathematical concepts treated in parts I and II of the PCM, but rather assume the reader is familiar with them.

Some crucial concepts that are already in the PCM are included here as well, but they are approached from a rather different viewpoint, typically with more discussion of applications and computational aspects.

Some articles in the PCM, listed in table 1, could equally well have appeared here, and the editors therefore made a decision not to solicit articles on these same general topics. However, particular aspects of several of these topics are included.

Table 1 Relevant articles from *The Princeton Companion to Mathematics* whose topic is not duplicated here.

Title	PCM article number
Mathematics and Chemistry	VII.1
Mathematical Biology	VII.2
Wavelets and Applications	VII.3
The Mathematics of Traffic in Networks	VII.4
Mathematics and Cryptography	VII.7
Mathematical Statistics	VII.10
Mathematics and Medical Statistics	VII.11
Mathematics and Music	VII.13

6 How to Use This Book

Authors were asked to make their articles as self-contained as possible and to define specific notation and technical terms. You should familiarize yourself with the material in THE LANGUAGE OF APPLIED MATHEMATICS [I.2], as this background is assumed for many of the later articles. If you are unsure about notation consult table 3 in I.2 or, for a definition, see if it is in the index. The editors, with the help of a professional indexer, have tried very hard to produce a thorough and usable index, so there is a good chance that the index will lead you to a place in the book where a particular piece of notation or a definition is clarified.

The extensive cross-references provide links between articles. A phrase such as "this vector can be computed by the FAST FOURIER TRANSFORM [II.10]" indicates that article 10 in part II contains more information on the fast Fourier transform.

In the research literature it is normal to support statements by citations to items in a bibliography. It is a style decision of *The Companion* not to include citations in the articles. The articles present core knowledge that is generally accepted in the field, and omitting citations makes for a smoother reading experience as the reader does not constantly have to refer to a list of references a few pages away. Many authors found it quite difficult to adopt this style, being so used to liberally sprinkling `\cite` commands through their LaTeX documents! Most articles have a further reading section, which provides a small number of sources that provide an entrée into the literature on that topic.

7 *The Companion* Project

I was invited to lead the project in late 2009. After the editorial board was assembled and the format of the book and the outline of its contents were agreed at a meeting of the board in Manchester, invitations to authors began to go out in 2011. We aimed to invite authors who are both leaders in their field and excellent writers. We were delighted with the high acceptance rate.

Ludwig Boltzmann was a contributor to the six-volume *Encyclopedia of Mathematical Sciences* (1898–1933) edited by Felix Klein, Wilhelm Meyer, and others. In 1905, he wrote, apropos of the selected author of an article:

> He must first be persuaded to promise a contribution; then, he must be instructed and pressed with all means of persuasion to write a contribution which fits into the general framework; and last, but not least, he must be urged to fulfill his promise in a timely matter.

Over a hundred years later these comments remain true, and one of the reasons for the long gestation period of a book such as this is that it does take a long time to sign up authors and collect articles. Occasionally, we were unable to find an author willing and able to deliver an article on a particular topic, so a small number of topics that we would have liked to include had to be omitted.

Of the 165 authors, at least two had babies during the course of preparing their articles. Sadly, one author, David Broomhead, did not live to see the project completed.

If the project has gone well, one of the reasons is the thoroughly professional and ever-cheerful support provided by Sam Clark of T&T Productions Ltd in London. Sam acted as project manager, copy editor, and typesetter, and made the process as smooth and painless for the editors and contributors as it could be. Sam played the same role for the *The Princeton Companion to Mathematics*, and his experience of that project was invaluable.

8 Acknowledgments

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It has been a pleasure to work with Sam Clark, with our level of cooperation being demonstrated by the 3000-plus emails that have traveled between us. I was constantly impressed not only by Sam's ability to find improvements in authors' submitted text during the copyediting phase but also by his ability to turn author-supplied figures of a widely varying standard into high-quality figures with a consistent style.

The original Princeton University Press editor for the project was Anne Savarese, who had also worked on *The Princeton Companion to Mathematics*. Anne's experience and advice was invaluable in formulating the plans for *The Companion*, and she shepherded the project through to the late stages with skill and patience. Vickie Kearn took over from Anne for the last eighteen months of the project, after Anne moved to another editorial role at the Press. A smooth transition was aided by Vickie's extensive knowledge of the world of mathematics publishing and by my having known her since the early 1990s when, as an editor at the Society for Industrial and Applied Mathematics (SIAM), she published my first book.

I would also like to thank Terri O'Prey, who oversaw the production at the Press, and indexer Julie Shawvan.

Finally, I thank my wife Françoise Tisseur and my sons Freddie and Thomas for uncomplainingly allowing me to spend many evenings and weekends working on *The Companion*.

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