PREFACE

OVERVIEW

We have attempted to write a concise modern treatment of differential equations emphasizing applications and containing all the core parts of a course in differential equations. A semester or quarter course in differential equations is taught to most engineering students (and many science students) at all universities, usually in the second year. Some universities have an earlier brief introduction to differential equations and others do not. Some students will have already seen some differential equations in their science classes. We do not assume any prior exposure to differential equations.

The core of the syllabus consists of Chapters 1 and 2 on linear differential equations. The use of Laplace transforms to solve differential equations is described in Chapter 3 since many engineering faculty use them. Series solutions of differential equations used to be part of the course, but the trend is to not do this, and we have decided to omit series. By doing so, we are communicating that linear and nonlinear systems are more important in a differential equations course for the future. Our book is also less expensive without series.

Most universities do some systems in their differential equations course. Some do a little, some do a lot. We have tried to present systems in an elementary introductory way, so that the beginning student understands the material, and also in a flexible way, so that universities that only spend two weeks on systems will be satisfied, and those that spend more will also have options. We also present the phase plane for linear systems in an elementary way. Most universities will want to do systems because present technology makes them very graphically exciting to students and faculty. Each university will use technology in its own unique way. We do not provide expensive software because the software would be useful for only a short portion of the standard differential equations course. Similarily, while numerical simulation is important, it is easily done with numerous software packages, so that only a brief introduction is needed in a first course.

Our book discusses standard topics for differential equations in the standard way, so that it can easily be adopted by most universities. In addition, we have focused on the essential areas, so that the text is concise. Our book is unique in the way that some of the essential topics are discussed, as we now describe. The most important concept for students in our text is to understand linear differential equations and how to solve or analyze them in an elementary way. We have attempted to make the entire book easy for students to read.

CHAPTER 1

Some of the presentations in Chapter 1 are unique and intended to simplify the overall linearity concepts. The key sections are very early in the text. Section 1.6 on first-order linear differential equations discusses all the concepts of linearity for first-order equations, and confirms the form of the general solution using an integrating factor. It shows that the general solution is a particular solution plus a constant times the homogeneous solution. The section is not unique, but it is very well written and easy for students to understand. Ideas of homogeneous and nonhomogeneous equations are introduced early. The unique section is the short Section 1.7 on elementary methods to solve first-order differential equations with constant coefficients and constant input. By our approach, students are taught that differential equations are sometimes easy, not hard and mysterious. All differential equations will be easier after beginning with our approach.

First-order differential equations have many important applications. Sections 1.8–1.9 discusses population growth, radioactive decay, Newton's law of cooling, and mixture problems. All the differential equations that appear in these applications have constant coefficients. In other books these equations are solved by an integrating factor, and students sometimes are confused and get the wrong answer. In our presentation, the elementary method of Section 1.7 is used to solve all the application problems, so that the student believes that the solutions to differential equation are applicable and sometimes easy. All students can benefit by being introduced to differential equations in this way.

CHAPTER 2

Chapter 2 covers linear second- and higher-order differential equations. This is the most important chapter in our book and in most courses in differential equations. The necessary theory in this chapter is difficult in all books. Our students will not be confused here because they have been introduced to the linearity idea in Chapter 1, where enough examples have been done for them already to understand some of the ideas. In Section 2.1, we immediately introduce the idea that the general solution is a particular solution plus a linear combination of homogeneous solutions. We discuss the Wronskian and its relationship to the initial value problem in Section 2.2. Section 2.3 presents reduction of order, so that it we can use it in Section 2.4 to obtain homogeneous solutions of constant coefficient differential equations. In Section 2.5, the vibrations of spring-mass systems are discussed with great care. The differential equations are formulated using Newton's law. Detailed presentations are given of mechanical vibration with no damping, along with presentations of the three cases of damped mechanical vibrations, especially appropriate for science and engineering students. Section 2.6 presents the method of undetermined coefficients with greater clarity than most books. Section 2.7 is a carefully written presentation of forced vibrations including a very detail oriented discussion of mechanical resonance. Section 2.8 is a nice separate section devoted to linear electric circuits, including a derivation from first principles of RLC (resistors, inductor, capacitors) circuits and the phenomena of electrical resonance. Section 2.9 is a short presentation of Euler equations. Chapter 2 finishes with the method of variation of parameters (including a more advanced presentation for higher-order systems).

CHAPTER 3

Chapter 3 discusses how to solve differential equations using Laplace Transforms. Students learn how to use a table. We present both a short and long table, so that instructors can provide the type they wish. In Section 3.1, we emphasize the use of two theorems for Laplace transforms, multiplying by an exponential and multiplying by t. In Section 3.2 we develop the systematic inverse Laplace transform based on roots, quadratics, and partial fractions. We solve many differential equations using Laplace transforms in Section 3.3. To illustrate the science and engineering orientation of our presentation, we do a resonance example. More in-depth discussion of Laplace transforms is possible. For many, the inclusion of discontinuous forcing with Heaviside step functions (Section 3.4), periodic forcing (Section 3.5), the convolution theorem (Section 3.6), and impulsive forcing using the delta function (Section 3.7) are highly recommended.

CHAPTERS 4–6

Chapters 4–6 cover linear and nonlinear systems, an introduction to so-called dynamical systems. The material on linear and nonlinear systems has great flexibility. Chapter 4 includes a very careful introduction to linear systems of differential equations (two first-order linear equations with two unknowns) and the phase plane for these linear systems. It is our anticipation that most students in an elementary differential equations course will at least cover Chapter 4. Chapter 5 is a very brief chapter discussing mostly nonlinear first-order differential equations. It introduces equilibrium, stability, and one-dimensional phase lines. Chapter 6 discusses nonlinear systems of differential equations in the plane. It has been written to require Chapter 4 but not Chapter 5. Some instructors may wish to discuss Chapter 5 before Chapter 6, some may wish to go directly from Chapter 4 to Chapter 6. Equilibrium, linear stability, and phase plane analysis are carefully presented. These chapters have been written so that instructors can discuss as much of these three chapters as they wish. To keep the presentation relatively simple and the cost relatively low, only systems in the plane are discussed.

CHAPTER 4

Chapter 4 introduces linear systems of two differential equations and their phase planes. Section 4.2 shows in an elementary way how solving a linear system of differential equations reduces to solving a linear system of algebraic equations involving a 2×2 matrix. It is not assumed that students have seen matrices before. Section 4.2 introduces eigenvalues and their corresponding eigenvectors for 2×2 systems in an elementary way with many examples. The emphasis is on finding the general solution to the linear system of differential equations. Careful elementary presentations based on examples are discussed for the cases where the eigenvalues are real and distinct and where they are complex. Only an elementary motivational example is given if the eigenvalues are real and repeated. In Section 4.3, the phase plane is introduced but only for linear systems. The presentation is elementary. The first examples are of elementary systems of differential equations that are not coupled; we then discuss examples that require eigenvalues and eigenvectors. Many examples of 2×2 systems of differential equations with real distinct eigenvalues are given, including a systematic discussion of the phase plane associated with stable and unstable nodes and saddle points. Detailed examples of the phase plane for the case of stable and unstable spirals and centers are also given. Some courses will not go any further than Chapter 4.

CHAPTER 5

This innovative chapter on mostly nonlinear first-order differential equations is optional and not required for Chapter 6. In Section 5.2, the concept of equilibrium and linearized stability analysis is introduced in a very elementary context of first-order nonlinear differential equations. In Section 5.3, the very elementary (but very powerful) method of one-dimensional phase lines for first-order nonlinear problems is introduced to determine the stability of the equilibrium and other qualitative behavior of the solution of the differential equation. An application to an elementary nonlinear biological growth model is presented in Section 5.4.

CHAPTER 6

In Chapter 6, we give a comprehensive presentation of nonlinear systems of differential equations in the plane. It is anticipated that different universities will cover varying amounts of this chapter. In Section 6.2, we discuss equilibriums, and we show that the stability of an equilibrium is determined from the eigenvalues of the linearization (Jacobian) matrix. Furthermore, we show that the phase plane for the nonlinear system of differential equations is usually approximated near each equilibrium by the phase plane of a linear system. Thus, the linear systems studied in Chapter 4 apply to nonlinear systems near equilibria. Section 6.3 discusses in depth competing species and predator-prey population models. Section 6.4 discusses the mechanical system corresponding to a nonlinear pendulum. For such conservative systems, we derive conservation of energy, and we show how to obtain periodic solutions and the phase plane using the potential energy.

APPLICATIONS

Many fundamental problems in biological and physical sciences and engineering are described by differential equations. We believe that many problems of future technologies will be described in the same way. Physical problems have motivated the development of much of mathematics, and this is especially true of differential equations. In this book, we study the interactions between mathematics and physical problems. Thus, in our presentation, we devote some effort to mathematical modeling, deriving the governing differential equations from physical principles. We take four major applications and in most cases carry them throughout. They are population growth, mixing problems, mechanical vibrations, and electrical circuits. In this way, the student has the chance to understand the physical problem.

EXERCISES

Differential equations cannot be learned by reading the text alone. We have included a large number of problems of various types and degrees of difficulty. Most are straightforward illustrations of the ideas in each section. The answers are provided in the back for all the odd problems. A student solutions manual exists in which the solutions to the odd exercises are carefully worked out. Frequently, even problems are similar to neighboring odd problems so that the answer to an odd problem can often be used for guidance. An instructor's manual exists with the answers (and some discussion) to all the exercises. Exercises have been class tested.

TECHNOLOGY

The increasing availability of technology (including graphing and programmable calculators, computer algebra systems, and powerful personal computers) has caused some to question the existing syllabi in university courses in differential equations. However, we believe that the importance of applications will continue to motivate the study of differential equations. This book has been written with that in mind. Courses with a strong emphasis on applications can use our book. In addition, those that wish a greater presence of technology can use our book supplemented by increasingly available web based resources or computational supplements.

ACKNOWLEDGMENTS

Many people have had input into this book, including an excellent group of reviewers, users of an earlier edition by both of us, users of an even earlier edition by one of us, and an experienced editorial staff. However, two groups have especially influenced us: the many students to whom we have taught differential equations and the colleagues with whom we have worked in differential equations, electrical and mechanical engineering.

Finally, the book could not have written without the support and understanding of our wives, Gail and Liz, and our children (two each) Matthew and Eric and Ken and Vicki.

Stephen L. Campbell
Richard Haberman