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INTRODUCING BLACK HOLES: EVENT HORIZONS AND SINGULARITIES

Black holes are extraordinary objects. They exert an attractive force that nothing can withstand; they stop time, turn space inside out, and constitute a point of no return beyond which our universe comes to an end. They address issues that have always fascinated humans—literature and philosophy in all times and cultures explore irresistible lures, the limits of the universe, and the nature of time and space. In our own time and place, science has become a dominant force both intellectually and technologically, and the scientific manifestation of these ancient themes provides a powerful metaphor that has come to permeate our culture—black holes abound not only in speculative fiction but in discussions of politics, culture, and finance, and in descriptions of our internal and public lives.

But black holes are not just useful metaphors or remarkable constructs of theoretical physics; they actually exist. Over the past few decades, black holes have moved from theoretical exotica to a well-known and carefully studied class of astronomical objects. Extensive data archives reveal

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the properties of systems containing black holes, and many details of their behavior are known. In the current astronomical literature, the seemingly bizarre properties of black holes are now taken for granted and are used as a basis for understanding a wide variety of phenomena.

The title of this book is oxymoronic. The defining property of black holes is that they do not emit radiation (hence "black")—so they cannot "look like" anything at all. Nevertheless, black holes are the targets of a wide variety of observational studies. This paradox is of a piece with much of modern astrophysics, in which objects that cannot be observed directly are studied in detail. Cosmologists have found that more than 90% of the mass energy of the Universe is in the form of unobservable "dark matter" and "dark energy." Thousands of planets have been discovered orbiting stars other than our Sun, but only a tiny handful have been observed directly. So it is with black holes. They cannot be observed directly, and yet they can be studied empirically, in some detail.

My goal for this book is to describe how astronomers carry out empirical studies of a class of objects that is intrinsically unobservable, and what we have found out about them. I will focus on current observations and understanding of the astrophysical manifestations of black holes, rather than on the underlying physical theories. There are a number of excellent textbooks on the physical processes, and I will refer to them along the way. The first three chapters sketch some of the physics needed to understand and interpret the observational results. Subsequent chapters describe observed black holes, and thus provide an answer to the question, What do black holes look like?

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1.1 Escape Velocity and Event Horizons

One of the basic concepts to emerge from Newton's theory of gravity is the *escape velocity*, denoted V_{esc}. The escape velocity is the speed required to escape the gravitational attraction of a spherical object. It can be shown from basic principles that

$$V_{esc} = \sqrt{2GM/R}$$

where G is the gravitational constant (equal to $6.674 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$), and M and R are the mass and radius, respectively, of a spherical object.[1]

It is a simple matter to calculate the escape velocity for any combination of size and mass. For example, numbers approximating the size and mass of a human being (1 m and 50 kg) result in an escape velocity of just over $80\ \mu m\ s^{-1}$ (or about a foot per hour). While this result would apply precisely only to a spherical object with $R = 1$ m and $M = 50$ kg, an object of comparable mass and size would have a comparable escape velocity. Because the resulting escape velocity is much slower than the speeds associated with everyday life, gravitational effects between ordinary objects (people, cars, buildings) can generally be ignored. By contrast, the Earth, with a mean radius of a bit less than 6400 km and a mass of 5.9×10^{24} kg, has an escape velocity of $11\ km\ s^{-1}$—much faster than everyday speeds. So, without mechanical assistance we remain bound to the Earth.

[1]Technically, the escape velocity thus calculated applies to test particles attempting to escape from the surface of the sphere.

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The most conceptually straightforward description of a black hole is an object whose escape velocity is equal to or greater than the speed of light. Such objects had already been contemplated in the eighteenth century.[2] In such a case, we can rewrite the escape velocity equation as

$$R \leq R_s = 2GM/c^2,$$

where c is the speed of light, and R_s is the *Schwarzschild radius* (named after the early twentieth-century physicist Karl Schwarzschild). In the context of Newtonian physics such objects have no particularly striking physical qualities other than their small size (R_s of a mass equal to that of the Earth is only about a centimeter). Presumably, light would not be able to escape from them, so they would be hard to observe. But the fascinating physics associated with black holes emerged only when *general relativity* was developed.

Nevertheless, it is amusing to play with the Newtonian concept of black holes as objects with $V_{esc} \geq c$ and to notice how the size and density of black holes vary with their mass. The density ρ of an object is defined as mass/volume, so the density of a black hole must be

$$\rho_{bh} \geq \frac{3}{32\pi} \frac{c^6}{M^2 G^3}.$$

Thus the density required to form a black hole decreases as the mass of the black hole increases—masses 10^8 times that of the Sun (which, as we will see, are common in

[2] The eighteenth century British philosopher John Michell is generally credited with the first published consideration of objects with escape velocities greater than c. See Gary Gibbon 1979, "The Man Who Invented Black Holes," *New Scientist* 28 (June) 1101.

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the center of large galaxies) will become black holes even if their density is no greater than that of water. Black holes with masses comparable to those of typical stars must attain densities comparable to those of atomic nuclei, that is, much greater than the density of ordinary matter. Less massive black holes would require densities far beyond that of any known substance.

In the general theory of relativity, the Schwarzschild radius becomes fundamentally important. Gravity is not considered a force in general relativity but, rather, is a consequence of the curvature of space-time. Mass causes space-time to curve, and this curvature affects the trajectories of objects. In situations where the distances between objects are large compared with their Schwarzschild radii, the predictions of general relativity become indistinguishable from those of Newtonian gravity, and all the familiar Newtonian results can be recovered. However, as distances between objects approach R_s, objects begin to behave differently from Newtonian predictions. Indeed, the first observational evidence supporting general relativity came from slight anomalies in the orbit of Mercury, the planet closest to the Sun. Mercury's mean distance to the Sun is about 20 million times the Schwarzschild radius associated with the mass of the Sun, so the deviations are quite small, but the orbits of the planets are known very precisely, so the deviation was already known before Einstein developed his theory. Closer to the Schwarzschild radius, the differences between Newtonian and relativistic physics become greater, leading eventually to drastic qualitative differences in behavior. These dramatic effects cannot be observed in Earth-bound laboratories, or indeed anywhere in the solar

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system, because all nearby objects have radii that are many orders of magnitude bigger than R_s. But as we will see, black holes and the dramatic physical effects associated with them can be found in other astronomical contexts.

For objects that fit inside their Schwarzschild radius, the spherical surface where $r = R_s$ is often referred to as the *event horizon*. This name comes about because information from inside the event horizon cannot propagate to the outside world. Consequences of events that occur at $r < R_s$ cannot be seen by an observer outside R_s. The interior of the event horizon is thus causally disconnected from the rest of the Universe—in a sense, it is not part of our Universe. The behavior of matter and energy inside the event horizon can be explored mathematically by assuming that the equations of general relativity apply and then interpreting the results of mathematical manipulations of these equations. However, the laws that lead to the equations also categorically prohibit them from being tested by experiments or observations conducted by observers located at $r > R_s$. Thus from an epistemological point of view, physics inside an event horizon is a different kind of science from physics in parts of the Universe that are causally connected to us.

1.2 The Metric

We will not explore the details of the mathematics associated with general relativity here.[3] But simply looking at the

[3]For a good introduction at the undergraduate level, see Bernard Schutz, *A First Course in General Relativity* (Cambridge: Cambridge University Press, 2009).

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form of some of the relevant equations can reveal some of the remarkable qualities of black holes.

Mathematically, the curvature of space-time is defined by a *metric*. A metric defines a line element ds, and the separation between two space-time events is given by the integral of ds. In general, this integral depends on the trajectory taken by the object. In the absence of external forces, objects follow trajectories that minimize the separation.[4] In an uncurved space-time, this behavior is in accordance with Newton's first law of motion, which requires that (in the absence of forces) objects move in straight lines (the closest distance between two points). In relativity, objects in a gravitational field follow a curved trajectory in space not because of a "gravitational force" that redirects their motion but rather because of the curvature of space-time itself: the minimum separation between two space-time events follows a curve in spatial coordinates.

A single point mass generates a space-time curvature associated with the so-called Schwarzschild metric:

$$ds^2 = -(1 - R_s/r)c^2dt^2 + \frac{dr^2}{1 - R_s/r} + r^2d\Omega^2.$$

Here space is measured in polar coordinates (r, Ω, where $d\Omega = \sin\theta d\theta d\phi$), with the point mass at the origin. To be specific, dt is the time interval seen at infinity, and R_s is the circumference around the black hole divided by 2π. By looking at the limiting cases of this equation, we can gain some insight into how black holes behave.

[4]Formally, the "proper time" is maximized.

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$r \to \infty$: When r is large, the gravitational influence is small. In this case the spatial coordinates of the metric approximate polar coordinates of a Euclidean space, and time and space decouple. In this limit, the equations of general relativity reduce to the familiar results of Newtonian physics.

$r \to R_s$: As r approaches the Schwarzschild radius, the term $(1 - R_s/r)$ goes to zero. The time term of the metric thus becomes zero, and the radial term becomes infinite. Something very peculiar must happen at $r = R_s$! Indeed, as an object falls toward the black hole, it appears to an outside observer that time is slowing down. That is, a clock mounted on the infalling object runs slower than an identical clock that remains at a large distance from the black hole. This observed slowness applies also to the frequencies of emitted radiation. Radiation emitted near a black hole will be observed to have lower frequencies, and thus longer wavelengths. This effect is called *gravitational redshift* and can be observed in a variety of ways (see chapter 8). However, for an observer on the infalling object, local clocks appear to be accurate, the Universe far from the black hole appears to speed up, and the radiation from distant objects appears to be blueshifted.

$r < R_s$: At radii smaller than R_s, the term $(1 - R_s/r)$ becomes negative. This means the signs of the time and radial terms of the metric are reversed. As a consequence, radial motion can be only unidirectional (as time is in ordinary

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> situations), while it is possible, in principle, to move forward and backward in time. Inside the Schwarzschild radius "time machines" are thus in principle possible. This property of black holes has led to a wide variety of speculative fiction, and some interesting physics as well. But inside the event horizon, it is not possible to move outward, any more than it is possible to move backward in time in less exotic regions of space-time. Any object that finds itself inside the Schwarzschild radius of a black hole will inexorably travel toward the center of the coordinate system at $r = 0$. Thus material will pile up in a point of zero volume and infinite density at the center of a black hole. This point is sometimes referred to as a *singularity*.

There are thus a number of situations related to black holes in which physical quantities should become infinite. At $r = R_s$, terms of the metric become infinite, and time stops.[5] At the center of the black hole, where $r = 0$, the density of matter becomes infinite. The existence of this central singularity suggests that the physical theory is likely to be incomplete.[6] In particular, there are likely to be quantum effects (which become important at small sizes) that need to be accounted for. But relativity is a continuous theory and does not fit easily with quantum mechanics.

[5]The mathematical divergence of the terms of the metric at $R_s = r$ can be avoided by an appropriate change in coordinates, but that does not change the predicted behavior of an infalling object as observed from a long distance away.

[6]An example is the "ultraviolet catastrophe" in radiation theory, in which classical physics requires the radiation emitted from a blackbody to become infinite at high frequencies. One of the first triumphs of quantum physics was to eliminate this infinity from the theory.

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The search for a "unified theory" which combines general relativity and quantum mechanics is ongoing. Until this search is complete, physical predictions of the behavior of matter and energy at the Schwarzschild radius or near the central singularity might plausibly be regarded as the results of an incomplete theory rather than as any sort of accurate representation of reality. One of the long-term goals of observational relativistic astrophysics is to probe this regime where current theory might encounter difficulties. However, as we will see, there are no current observations that contradict general relativity.

The Schwarzschild metric is actually a special case that applies only to black holes with no angular momentum. If the material forming the black hole has angular momentum (as one would expect to find in any physical object, particularly if it forms from the collapse of a much larger structure), then a more complicated metric known as the *Kerr metric* is the correct description of space-time. The Kerr metric has a key parameter in addition to the mass of the central object, namely, its angular momentum, usually given in dimensionless units as $a = J/(GM^2/c)$, where J is the angular momentum of the object. If $a \leq 1$, the Kerr metric generates an event horizon at

$$R_K = (GM/c^2)(1 + \sqrt{1 - a^2})$$

(note that this reduces to the Schwarzschild radius when $a = 0$). Situations in which event horizons exist and $a > 1$ are generally thought to be nonphysical. In Kerr black holes, the central singularity takes the form of a ring, rather than a point, and there is an additional critical surface

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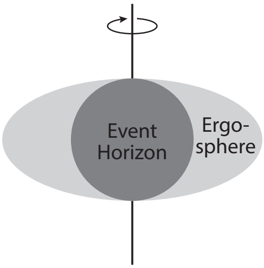

Figure 1.1. The event horizon and the ergosphere of a spinning black hole.

outside the event horizon called the *ergosphere*, inside of which objects cannot remain stationary but can escape from the black hole (see figure 1.1). Exchange of energy between particles within the ergosphere, some of which escape to infinity and some of which fall into the event horizon, allows much of the rotational energy of a Kerr black hole to be extracted and transferred to the outside universe.[7] The observable consequences of black hole spin will be explored further in chapter 8.

1.3 What Is a Black Hole?

So what exactly is a "black hole"? The term "black hole" is not defined in a technical way and is used in different contexts to mean different things. The phrase itself was popularized by the physicist John Archibald Wheeler to replace the cumbersome description "gravitationally

[7] R. Penrose, 1969, *Rivista del Nuovo Cimento 1* (Ser. 1): 252.

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completely collapsed object."[8] The term can be used to describe an object whose escape velocity is greater than the speed of light, which leads to a quasi-Newtonian description of such objects. Sometimes "black hole" is used to denote the volume inside the event horizon, that is, the region "outside our Universe." In this context it makes sense to discuss the "size" or "density" of a black hole, since there is a nonzero radial distance (R_s in the case of a nonspinning black hole) associated with the object. Sometimes "black hole" is used to refer specifically to the singularity, in which case such physical quantities are not well defined. Finally, "black hole" has become a commonly used metaphor for anything with an inexorable pull leading to destruction. As we will see, the assumptions about black hole behavior associated with these metaphors are often quite misleading when applied to the physical objects themselves.

[8] By his own account, Wheeler himself did not invent the term. Rather, the phrase was called out by an anonymous voice at a conference, and Wheeler adopted it then and afterwards. J. A. Wheeler, *Geons, Black Holes, and Quantum Foam* (New York: Norton, 2000), 296–97.

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