Chapter One

Why Study Stellar Atmospheres?

A central objective of astrophysics is to use physical theory to simulate the conditions in astrophysical objects. Today's astrophysicist must, in principle, be familiar with just about all of physics: elementary particle theory and nuclear physics; quantum mechanics and atomic/molecular physics; classical electrodynamics, quantum electrodynamics, and plasma physics; hydrodynamics and magnetohydrodynamics; classical gravitational mechanics, special relativity, and general relativity. The goal of this book is decidedly more modest: (1) to present the physical and mathematical tools needed to make models of stellar atmospheres that are realistic enough to fit closely the observed spectrum of a star, and (2) to show how they can be used to make a reliable quantitative spectroscopic analysis of the physical structure and chemical composition of its outer layers. The problems to be faced in achieving it are developing correct formulations of the physics of spectral line and continuum formation and the transport of radiation.

The complex systems of nonlinear equations that describe cosmic objects can be solved using the extremely fast computers now available. In addition, we now have access to most of the electromagnetic spectrum, ranging from radio wavelengths to gamma rays. These observational data provide a solid basis upon which we can construct theoretical structures to interpret them. It is a reasonable metaphor to say that today our picture of the Universe is developing as rapidly, and as radically, as if Galileo and Newton had lived and worked at the same time.

But given the crowded curriculum faced by astrophysics students one may ask, "Why take time to study the outer layers of stars? Is this work merely a 'cottage industry' of no great relevance to the rest of astrophysics?" This question was put pointedly to one of us (D. M.) nearly 50 years ago by E. Salpeter: "Why in the world would *anyone* want to study stellar atmospheres? They contain only about 10^{-10} of the mass of a star. Surely such a negligible fraction of its mass cannot affect its overall structure and evolution!" His query is reasonable and deserves a good answer.

1.1 A HISTORICAL PRÉCIS

Salpeter's question must be put in the perspective of his own seminal work on stellar structure, stellar evolution, and nucleosynthesis. Relative to those studies, the theory of stellar atmospheres at that time had not produced spectacular results, and did not yet have a sound theoretical basis. To provide some context, we outline below a few milestones in the development of today's observational techniques; the theory of astrophysical radiative transfer and quantitative spectroscopy; and the theory of stellar evolution. Other authors would doubtless select different topics than we have chosen here.

Instrumentation

At the beginning of the 20th century, the most powerful telescopes were long-focus refractors, the largest being the 40″ diameter telescope at Yerkes Observatory, built under the direction of G. E. Hale. They were used for visual observations of the orbits of binary stars (which give data for the determination of stellar masses); photographic observations of *radial velocities* (the component of a star's velocity, relative to the Sun, along the line of sight); *proper motions* (which are proportional to a star's velocity, relative to the Sun, perpendicular to the line of sight); and stellar *parallaxes* (which give a star's distance from the Sun).

After Hale left Yerkes to found the Mt. Wilson Observatory, he directed the construction of the 60″ and 100″ reflectors on Mt. Wilson and the 200″ reflector and 48″ wide-angle Schmidt camera on Palomar Mountain. Each increase in aperture permitted fainter objects to be observed. The instrumentation for these large reflectors was at the cutting edge of technology then available. For example, their spectrographs contained large blazed gratings made with interferometrically controlled grating-ruling engines at the headquarters of the observatory in Pasadena. These had higher efficiency and resolution, and better freedom from ghosts, than any previously made. But until the 1950s astronomical telescopes had little automation other than drives to track the apparent motion of stars across the sky. After about 1950, the sophisticated electronic measurement and control techniques of physics laboratories began to invade the mountaintop redoubts of astronomers.

In the past two to three decades observational astrophysics has enjoyed unparalleled growth. We now have telescopes with apertures of several meters, which have thin mirrors whose optical figures can be adjusted to minimize the effects of turbulence in the Earth's atmosphere and corrected for flexure and other transient defects, in real time, using high-speed computers. Soon, it will be possible to restore images to nearly the diffraction limit of a telescope. With such instruments, we can observe objects at low light levels that were previously inaccessible. The far infrared, ultraviolet, X-ray, and gamma-ray regions of the spectrum can now be observed with space telescopes.

Today's echelle spectrographs can capture spectra at high resolution in many orders simultaneously. Photographic plates have been replaced with CCDs (*charge-coupled devices*), which have very high light detection efficiency and a linear response that permits precise calibration and measurement of images of stars, extended objects, and echelleograms. With them we can now obtain high signal-to-noise spectra of extremely faint objects.

At radio wavelengths, interferometric techniques developed for the Very Large Array (VLA) and the Very Long Baseline Interferometer (VLBI) have produced exquisite pictures showing that many galaxies have massive *black holes* at their centers, which spew immense jets of material racing outward at relativistic speeds. These data imply the existence and continuing modification of an intergalactic medium from which new galaxies form. In short, it is now possible to observe, and begin to model, phenomena totally unknown only a few years ago.

Observations

The two most important observational activities in astrophysics at the start of the 20th century were *photometry* and *spectral classification*. Strenuous efforts were made to set up accurate photometric brightness scales[1] for stars, but they were thwarted by the nonlinear response of photographic emulsions to exposure. Through the 1940s, the apparent brightnesses of stars could not be measured to much better than 10%, and often only to 20%–30%. Brightness measured in different wavelength bands could be used to make estimates of stellar *colors*, which give a low-resolution measure of the distribution of light in their spectra.

The arrival of *photomultipliers* in the early 1950s gave astronomers very sensitive *linear* receivers accurate to 1% and revolutionized stellar photometry. With them, standard photometric systems were established and used for incisive analyses of stellar properties (see § 2.4–§ 2.6). By the mid-1960s, photoelectric *spectrophotometers* calibrated against standard sources having known energy distributions put spectrophotometry on an absolute energy scale (see § 2.3). Today, spectrophotometric measurements of the continua of stars can be matched with high precision to results obtained from physically realistic theoretical model atmospheres that allow for the effects of many thousands of spectral lines. With arrays of CCDs containing thousands of individual detectors one can measure simultaneously the apparent brightness of huge numbers of stars in large fields of the sky.

The state of stellar spectroscopy was better. In the early 1900s, it was found that stellar spectra could be arranged in a sequence that correlates closely with a star's *effective temperature*.[2] This classification scheme was quickly supplemented with additional criteria based on subtle effects that correlate with the average density in a star's atmosphere. These phenomena could be interpreted using theoretical work in statistical mechanics [344]. By 1914, E. Hertzsprung and H. Russell [488, 915] showed that stars fall in definite sequences in the *Hertzsprung-Russell diagram* (or *HR diagram*), a plot of their *luminosity* versus effective temperature. This discovery had profound implications for the development of a theory of stellar structure and evolution.

Measurement of stellar spectra progressed from mere eye estimates of "line intensities" in the early 1900s to photographic measurements of *line profiles* and *equivalent widths* in the 1930s. In contrast to the absolute photometry needed to determine stellar magnitudes, these measurements require only *relative* photometry, i.e., comparing the light at several wavelengths in a line to the local continuum. Hence the results were more accurate. Today, with linear receivers such as CCDs, very precise measurements can be made simultaneously for a large range of wavelengths.

[1] Measured in *magnitudes*; see § 2.4 for their definition.

[2] A representative temperature of the material in its atmosphere; see equation (2.3) for its formal definition.

A breakthrough came in 1944 when W. Baade [73, 74] (see especially the excellent review [938]) made the seminal discovery of two *stellar populations* in the Galaxy, whose properties were determined by, and give information about, the formation and subsequent development of the Galaxy. His work integrated our observational picture of the stellar content of galaxies and also provided critical guidance to stellar evolution theory. He found that the distributions of the two populations of stars in the HR diagram are distinctively different. HR diagrams for *Population I* and *Population II* stars are shown in figures 2.6 and 2.7, respectively.

Population I objects in our Galaxy are typified by (1) *galactic clusters* (loose clusters containing $\sim 10^2 - 10^3$ stars), which include (a) a *main sequence* extending from massive, hot, very luminous *blue dwarfs* to cool, less massive, much less luminous *red dwarfs* (see table 2.4); (b) very luminous, cool, red *supergiants* in young clusters; and (c) *subgiants* and *red giants* in older clusters; (2) *Cepheid pulsating variables*; and (3) *interstellar material*. These objects are located near the central plane of the disk of the Galaxy. In other spiral galaxies, they are found in bright *spiral arms* bordered by dark *dust lanes*.

The great majority of stars in the solar neighborhood belong to Population I and are on the main sequence. In their cores, these stars are converting hydrogen to helium in thermonuclear reactions. This process releases the largest amount of energy per reaction, and hydrogen is the most abundant element in stellar material. Therefore, a star spends more time "burning" hydrogen to helium than in any other stage in its evolutionary history, so most stars will be found in this phase. Population I stars have relatively small velocities with respect to the Sun. They move on high angular momentum, nearly circular orbits around the Galactic center. They have near-solar abundances of "*metals*" (astrophysical jargon for elements with $Z \geq 6$; Z being the atomic number) [379, 472, 473, 474, 475, 476, 1129].

Typical Population II objects in our Galaxy are (1) *globular clusters* (gravitationally bound spherical systems containing $\sim 10^5 - 10^6$ stars); (2) individual *halo stars*, weak-lined *high-velocity stars*, and *subdwarfs* in the solar neighborhood; (3) *RR Lyrae pulsating variables*; and (4) *planetary nebulae* and their *nuclei*, which are old stars in late stages of their life. These objects are not strongly concentrated in the Galactic plane. Indeed, the distribution of globular clusters is roughly isotropic around the center of the Galaxy. Population II stars make up the central bulge in other spiral galaxies and most elliptical galaxies. They have low angular momentum and low velocities around the center of the Galaxy; hence they have high velocities relative to the Sun. The most extreme of these stars, and globular clusters, move on almost radial "plunging" orbits with respect to the Galactic center [120], [chapter 10]. This kinematic behavior gives clues about the formation of the Galaxy [305]. In 1951, the extreme subdwarfs HD 19445 and HD 140283 were found [211] to have "metal" abundances smaller by at least a factor of 25 to 40 than the Sun (a much too conservative estimate; see the discussion in [938, p. 433]). Since then, spectroscopic analyses of many Population II stars have been made, which give metal abundances factors of 10 to 10^5(!) smaller than solar; see, e.g., [85, 206, 258, 273, 347, 635, 802, 1022, 1024, 1036, 1117, 1130, 1131, 1132, 1133, 1136]. These stars presumably represent a primeval population.

At the opposite extreme, some (quite young?) *super-metal-rich* (SMR) Population I stars have a higher metal abundance than the Sun; see, e.g., [204, 293, 327, 1020, 1070, 1134]. It is clear that as a function of time there has been a progressive enrichment of the metal abundance of the material from which stars form.

The conventional notation used to indicate a star's metal abundance relative to the Sun is

$$[\mathrm{Fe/H}] \equiv \log\left[(\mathrm{Fe}_*/\mathrm{H}_*) \big/ (\mathrm{Fe}_\odot/\mathrm{H}_\odot) \right]. \tag{1.1}$$

Here iron is used as a proxy for all elements with $Z \geq 6$. This notation is oversimplified because there are variations from star to star in the ratio of the abundance of any chosen element to that of iron. Using criteria based on their distribution, kinematics, and abundances, the idea of stellar populations has been elaborated into a picture having several groups intermediate between the extremes represented by the most metal-poor globular clusters and youngest galactic clusters [800, 805].

The earliest generation of stars was composed of about 90% hydrogen by number; about 10% helium by number; and very small amounts of some isotopes of Li and Be, which were formed from primeval hydrogen in the *Big Bang*. Modern work shows that the He/H ratio is about the same in the interiors of both Population I and Population II stars. This fact implies that essentially all He was formed in the Big Bang. On the other hand, the existence of young and very old stars, having high and low metal abundances, respectively, shows that *there has been a progressive enrichment of elements with* $Z \geq 6$ *in the interstellar material from which stars form*. The heavy elements in the interstellar medium are created by *nucleosynthesis* [160, 1137], i.e., by thermonuclear processing of material in the cores of highly evolved, massive stars. This material is deposited into the interstellar medium by *supernovae*.

Stellar Structure and Evolution

The earliest models of the internal structure of stars were based on Ritter and Emden's theory of polytropic gas spheres with self gravity [318]. By assuming that the gas pressure $p_{\mathrm{gas}} = K\rho^{(n+1)/n}$, where ρ is the mass density [gm/cm^3], the equations for hydrostatic pressure balance can be combined with Poisson's equation for the gravitational field to get a single second-order differential equation for $\rho(r)$, the variation of density with radius in the star. It can be solved analytically for $n = 0$, 1, and 5 and by numerical integration for other values of n.

Beginning in the late 19th century, and through the 1930s, intellectual giants such as S. Chandrasekhar, A. Eddington, R. Emden, R. Fowler, E. Hopf, H. Lane, E. Milne, A. Ritter, A. Schuster, and W. Thomson (Lord Kelvin) developed and refined the theory of polytropes. With the assumption that the material is a perfect gas in which radiation pressure is unimportant, their work yielded realistic lower bounds on a star's central and average pressure: $p_c > 1.3 \times 10^{15}(\mathsf{M}^2/\mathsf{R}^4)$ and $\langle p \rangle \geq 5.4 \times 10^{14}(\mathsf{M}^2/\mathsf{R}^4)$ dynes/cm^2, and its average temperature: $\langle T \rangle \geq 4.6 \times 10^6 \mu(\mathsf{M}/\mathsf{R})$ K. In these inequalities M and R are a star's total mass and its radius in units of the solar mass and radius and μ is the "effective mean molecular weight" of the gas. They also showed that the ratio of radiation pressure to total pressure increases monotonically toward a star's center, and with increasing mass. For the Sun this ratio is < 0.03. A rigorous discussion of the reasoning leading to these bounds is given in Chandrasekhar's classic book *An Introduction to the Study of Stellar Structure* [223].

The publication of Eddington's book *The Internal Constitution of the Stars* in 1926 [302] was a milestone in studies of stellar structure. Eddington showed that (1) stars have central energy sources; (2) for massive stars, energy is transported to the surface by radiation; (3) and for these stars, radiation pressure produces the dominant force supporting them against gravity. His book contains good discussions of the thermodynamics of radiation, then-current quantum theory, ionization, diffusion, his pioneering work on the opacity of stellar material, and speculations about stars' energy source, which he could not identify but suspected must come from subatomic processes. He developed a "standard model" that assumes that the average rate of energy release per unit mass, $[L(r)/\mathsf{L}]/[M(r)/\mathsf{M}]$, within a volume of radius r, times the opacity coefficient k, is a constant. This assumption is ad hoc, but it allowed him to derive a mass-luminosity relation for main-sequence stars in fair agreement with observation and to derive the basic relation $P\sqrt{\langle\rho\rangle} =$ constant for Cepheid variables; here P is the star's pulsation period and $\langle\rho\rangle$ is its average density.

Also in 1926, Fowler [341] realized that *white dwarfs*, being $\sim 10^{-4}$ times as bright as normal stars of the same effective temperature, must have radii about 10^{-2} times smaller and are such compact, dense objects that electrons in the material are *degenerate*, i.e., obey Fermi-Dirac statistics. Further analysis [33, 212, 213, 1055, 1056] showed that degenerate material behaves like a polytrope with $p \propto \rho^{5/3}$ if the electrons' speeds are non-relativistic and $p \propto \rho^{4/3}$ if they are relativistic.[3] In the early 1930s Chandrasekhar [212, 213, 215] wrote a number of fundamental papers in which he represented white dwarfs in terms of polytropes, using the exact formula for the transition from non-relativistic to relativistic degeneracy with increasing mass. He found the remarkable result[4] that if a star's mass exceeds $\mathsf{M}_{\text{lim}} = (5.84/\mu_e^2)\,\mathsf{M}_\odot$, the *Chandrasekhar limit*, relativistic degeneracy pressure is insufficient to support the star, and it will collapse to "zero" radius, becoming either a *neutron star* or a black hole. Here μ_e is the number of atomic mass units per free electron in the gas, ≈ 2 for material with $Z \geq 2$, which is appropriate for material that has been processed in thermonuclear reactions. Thus the mass of a stable white dwarf at the end of nuclear burning must be $\lesssim 1.46\,\mathsf{M}_\odot$.

A persistent question from the beginning of studies of stellar structure was "What is the energy source that supports a star's luminosity?" The classical answer was that it is the release of *gravitational potential energy*. The gravitational potential energy between two particles with masses m_1 and m_2 separated by a distance r is

$$\mathsf{V} = -\frac{G m_1 m_2}{r}. \tag{1.2}$$

[3] That is, with a polytropic index $n = 3/2$ if non-relativistic and $n = 3$ if relativistic.
[4] For which he received the Nobel Prize in 1983.

Here, G is the gravitational constant. Thus the total potential energy of a star of radius R with a radial mass distribution $M(r)$ is

$$\mathsf{V} = -\int_0^{\mathsf{R}} \frac{GM(r)}{r}\, dM(r). \tag{1.3}$$

v and V are negative because particles are more tightly bound gravitationally as they approach one another.

At each position, particles in non-degenerate material have a Maxwellian velocity distribution at the local temperature. Let T be the total *thermal energy* in the star, i.e., the sum of the kinetic energies of all the particles. Consider the star to be evolving so slowly that it is in quasi-static equilibrium at any instant. Then we can apply the *virial theorem*, which states that

$$2\mathsf{T} + \mathsf{V} = 0 \quad \text{or} \quad \Delta\mathsf{T} = -\tfrac{1}{2}\Delta\mathsf{V}. \tag{1.4}$$

From (1.4) we see that if a star contracts as a whole, so that V becomes more negative, gravitational binding energy is released; half of it is converted into thermal energy (heats the material), and the other half is radiated away.[5] To get an order of magnitude estimate of the timescale of this process for the Sun, suppose we compute V by taking $M(r) \sim \frac{4}{3}\pi\,\overline{\rho}\,r^3$, and $dM(r) \sim 4\pi\,\overline{\rho}\,r^2\,dr$, where $\overline{\rho} \approx 1.4\ \mathrm{gm/cm^3}$ is the Sun's average density. With these approximations

$$\mathsf{V} = -\frac{16\pi^2 G\overline{\rho}^2}{3}\int_0^{\mathsf{R}} r^4\, dr = -\frac{3}{5}\frac{G\mathsf{M}^2}{\mathsf{R}} \sim -2.3\times 10^{48}\ \mathrm{erg}. \tag{1.5}$$

The Sun's luminosity is 4×10^{33} erg/sec $\approx 1.2\times 10^{41}$ erg/year. So the maximum length of time that luminosity can be supported by release of the Sun's entire potential energy, the *Kelvin-Helmholtz timescale*, is $\tau_{KH} \equiv -\frac{1}{2}\mathsf{V}/\mathsf{L}_\odot \approx 10^7$ years. In the absence of any evidence to the contrary, this result was regarded as reasonable. But it soon became clear that τ_{KH} is about 100 times smaller than the geological timescale derived from careful analyses of rock strata at many sites around the world.

The source of energy production in stars could not be understood until G. Gamow showed [360, 361, 413] that quantum mechanical tunneling allows a positively charged particle to penetrate a nucleus even though its thermal energy is smaller than the Coulomb barrier between the two particles. This theory was applied by R. Atkinson [43, 44] to a three-step *proton-proton,* or *p-p, reaction* in which four protons are synthesized into a He^4 nucleus. There are three branches for this reaction. The dominant one starts with a proton penetrating the Coulomb barrier between it and another proton, fusing them into a deuteron (H^2) plus a positron and neutrino; followed by the deuteron fusing with another proton to make a He^3 nucleus and a

[5] This result shows that gravitationally bound systems effectively have a *negative specific heat* in the sense that when energy is removed from the system by being radiated away, the material left behind becomes *hotter*!

gamma ray; finally, the He^3 nucleus fuses with another He^3 to make He^4 plus two protons, with a release of 26.2 MeV of energy:

$$\begin{aligned} &\mathrm{p} + \mathrm{p} \rightarrow \mathrm{H}^2 + \beta^+ + \nu, \\ &\mathrm{H}^2 + \mathrm{p} \rightarrow \mathrm{He}^3 + \gamma, \\ &\mathrm{He}^3 + \mathrm{He}^3 \rightarrow \mathrm{He}^4 + 2\mathrm{p}. \end{aligned} \tag{1.6}$$

He showed that at the conditions that apply in its core ($T \sim 10^7$ K and $\rho \sim 100$ gm/cm^3) this process would produce the Sun's luminosity.

In 1939, H. Bethe [115] found another path to generate thermonuclear energy: the fusion of four protons into a He^4 nucleus in the *CNO cycle*, a chain of six reactions with isotopes of carbon, nitrogen, and oxygen, which releases 25.0 MeV of energy.[6] The dominant form of this reaction is

$$\begin{aligned} &\mathrm{C}^{12} + \mathrm{p} \rightarrow \mathrm{N}^{13} + \gamma, &&\mathrm{N}^{13} \rightarrow \mathrm{C}^{13} + \beta^+ + \nu, \\ &\mathrm{C}^{13} + \mathrm{p} \rightarrow \mathrm{N}^{14} + \gamma, &&\mathrm{N}^{14} + \mathrm{p} \rightarrow \mathrm{O}^{15} + \gamma, \\ &\mathrm{O}^{15} \rightarrow \mathrm{N}^{15} + \beta^+ + \nu, &&\mathrm{N}^{15} + \mathrm{p} \rightarrow \mathrm{C}^{12} + \mathrm{He}^4. \end{aligned} \tag{1.7}$$

In this process the CNO isotopes act as *catalysts* and recover their initial abundances by the end of the cycle.

To estimate the Sun's *nuclear timescale*, assume it is composed of 100% H, which is all converted to He. $m_H = 1.67 \times 10^{-24}$ g and $M_\odot = 2 \times 10^{33}$ g, so it contains $\sim 1.2 \times 10^{57}$ protons. Each cycle of both reaction chains "consumes" 4 protons, so there can be $\sim 3 \times 10^{56}$ reactions, each with a yield $\sim 4 \times 10^{-5}$ erg. Hence the Sun's nuclear energy reservoir is $\sim 1.2 \times 10^{52}$ erg. With these assumptions, the solar luminosity of 1.2×10^{41} erg/year could be sustained for $\sim 10^{11}$ years, about 10^4 times longer than τ_{KH}, and 10 times longer than the currently estimated lifetime of the Universe. Modern stellar evolution calculations show that only about 10% of the Sun's hydrogen is converted to helium, so the lifetime of a solar-type star is actually $\sim 10^{10}$ years, in agreement with the ages estimated for the oldest globular clusters.

Near $T \approx 1.5 \times 10^7$ K, the rate of energy release in the p-p chains can be fit with a power law: $\epsilon_{\mathrm{pp}} \approx \epsilon_0 \rho X_{\mathrm{H}}^2 T_6^4$. Here X_{H} is the fraction, by mass, of the material that is hydrogen; ρ is its density, and $T_6 \equiv T/10^6$. Near the same temperature the rate of energy release in the CNO cycle can be fit with the power law $\epsilon_{\mathrm{CNO}} \approx \epsilon_0' \rho X_{\mathrm{H}} X_{\mathrm{CNO}} T_6^{20}$, where X_{CNO} is the total mass fraction of the material that is carbon, nitrogen, and oxygen. Low-mass stars have lower central temperatures than more massive stars, so the p-p reaction is active in the cores of stars with masses $\lesssim 1.2\, \mathrm{M}_\odot$; at higher masses the CNO cycle is dominant in the core.

Because of the relatively weak temperature dependence of the p-p reaction, the temperature gradient in the cores of low-mass stars is small enough for them to be in *radiative equilibrium* and the released energy is initially transported outward by *absorption and emission of photons*. In this process, high-energy photons are systematically degraded to lower energies, but at each position the *total amount*

[6] This reaction was also noticed by C. von Weizsäcker [1146]; see [678] for historical commentary.

of radiant energy emitted is exactly balanced by the total amount absorbed. The envelopes of these stars generally have zones in which hydrogen ionizes, or helium becomes once or twice ionized, so the opacity of the material is very large. Then radiative energy transfer is inefficient, and energy is transported outward by a *turbulent convective flow* in which hot, less dense "parcels" of hotter material rise, expand, release energy to the surroundings, cool, compress back to a higher density, and sink. The material is then in *convective equilibrium*.[7] In some cases (e.g., a red giant) practically the entire outer envelope of a star is convective. In contrast, because the carbon cycle has such a steep temperature dependence, stars more massive than about $1.2\mathsf{M}_\odot$ have convective cores, whereas their envelopes are in radiative equilibrium.

The physical structure of a star is determined by its mass M, age, and chemical composition. Stars on the *zero-age main sequence* (ZAMS), where fusion of hydrogen into helium in their cores has just begun, have the same chemical composition throughout, i.e., are *chemically homogeneous*. As thermonuclear fusion converts one element ("fuel") to another ("ash"), a star's chemical composition begins to vary with depth. As hydrogen is converted to helium, the molecular weight of the gas rises, and to sustain the pressure needed to support the overlying hydrogen envelope, the material in the core is compressed to a higher density.

Eventually a star completely exhausts all the hydrogen at its center. It develops an inert, isothermal, helium core surrounded by a hydrogen-burning shell. In 1938, E. Öpik [818] showed that the core undergoes contraction, and in accordance with the virial theorem, half of the released gravitational potential energy goes into thermal energy that heats the core, and half raises the star's luminosity. The heating of the core causes the shell to generate additional ("excess") energy, which is absorbed in the star's outer envelope, causing it to expand, and lowers the surface temperature. Thus in the HR diagram, a star moves upward, toward higher luminosity, and to the right, toward lower $T_{\rm eff}$. A star having about a solar mass evolves up the subgiant branch toward becoming a red giant. The higher the luminosity of a star, the more rapidly it consumes its hydrogen. This means that the more massive (hence more luminous) a star is, the faster it departs from the main sequence. Consequently, there is a characteristic *turnoff point* at the upper end of the main sequence in the HR diagram of a cluster, where stars move toward the domain of red giants (see figure 2.7).

In 1942, M. Schönberg and Chandrasekhar showed that one cannot construct simple core-envelope models if the core mass exceeds about 10% of the star's total mass [968] because then the star begins to contract on a Kelvin-Helmholtz timescale. The contraction could be halted if the core material becomes degenerate. But in this phase the hydrogen shell source generally cannot support the star's luminosity, so the core continues to contract and heats further. In 1957, Salpeter [937] showed that when a star's central temperature rises to $\sim 10^8$ K, the *triple-alpha reaction*, in which three He^4 nuclei are fused to make C^{12}, comes into operation. The energy released in this reaction halts the contraction, leaving the star with a hydrogen-burning shell and a helium-burning core. The triple-alpha reaction is less robust than the p-p reaction

[7] The criteria separating radiative and convective equilibria are given in chapter 16.

or CNO cycle because the beryllium isotope created in the intermediate reaction $\alpha + \alpha \rightarrow \text{Be}^{8*}$ is in an excited state (hence that part of the reaction is endothermic) and is unstable. Thus an equilibrium with sufficient numbers of this isotope must be reached for the final reaction, $\text{Be}^{8*} + \alpha \rightarrow \text{C}^{12}$, to proceed, with a net release of about 7.4 MeV of energy.[8]

In the 1930s, B. Strömgren [1060] computed improved opacities applicable at the temperatures and densities in stellar interiors. After World War II better values became available from work at U.S. government laboratories. Currently, the best results come from the Opacity Project (OP) [1192–1232]; its successor, the Iron Project (IP) [1233–1253]; and the Livermore Opacity Library (OPAL), [1254–1271].

Mathematically, computations of stellar structures pose two-point boundary value problems for a system of four nonlinear first-order differential equations. In the early 1950s, models could be made for chemically homogeneous stars and stars with a single change in composition between core and envelope. The solution was obtained by integrating a family of (dimensionless) trial solutions from the center toward the surface, and another from the surface toward the center. The value and slope of a solution in one set were fitted to a solution in the other set at some intermediate point. A discussion of this technique is given in [982, § 13 and § 14]. In 1952, calculations by A. Sandage and M. Schwarzschild [940] showed that after core hydrogen exhaustion, a star evolves quite rapidly from a main-sequence dwarf into a red giant. Their results were consistent with observed *color-magnitude diagrams* for globular clusters and indicated that those clusters must be of the order of 3 billion years old (current work gives ages of about 14 billion years).

As a star ages, it can develop a very complex internal structure. Starting with hydrogen, it burns a nuclear fuel in its core, leaving behind the product of the reaction (in this case helium). Eventually all of that fuel in the core is exhausted, and energy generation shifts to a *shell source* surrounding an inert core (e.g., a hydrogen-burning shell surrounding a helium core). As the star ages further, the shell source becomes thinner (less massive) and eventually is unable to support the luminosity of the star. At that point the star contracts, which releases gravitational potential energy, and its central density and temperature rise until the next possible exoergic thermonuclear reaction can proceed, in this case the triple-alpha reaction, converting helium to carbon. The same series of events takes place until the new core fuel is exhausted and starts burning in another shell source around an inert core. For example, the star may have an unburned hydrogen outer envelope and then a hydrogen shell source, surrounding an inert helium shell, below which there is a helium-burning shell, and an inert carbon core, etc. If a star is massive enough, this sequence may be repeated several times, and the star develops an "*onion skin*" structure with a number of shell sources burning simultaneously. In addition, different zones can be in either radiative or convective equilibrium.

The computational procedure outlined above works when a star's interior structure is relatively simple. But it becomes difficult to use, or fails altogether, for stars that

[8] Note that helium burning produces less energy per reaction than hydrogen burning.

have several shells with different compositions, types of energy release, or modes of energy transport. When fast digital computers arrived in the 1960s, L. Henyey and his coworkers [480, 481] made a gigantic step forward by developing an efficient two-point boundary-value linearization method. This strongly convergent method can handle models of great structural complexity given a reasonable starting model, and it was immediately adopted by all workers in the field. Over the past 50 years, comprehensive calculations have been carried out tracing a star's evolution from its formation in the interstellar medium to its ultimate fate as either a supernova or a white dwarf.

For most stars, the rate of mass loss is negligible during the time they are near the main sequence, so its mass M can be used as a basic input parameter for a model of their interior structure. But for the most massive upper main-sequence stars, the mass-loss rate is large enough that it must be taken into account from the time a star is formed. As a massive star evolves away from the main sequence and becomes a red supergiant, its outermost layers becomes less strongly bound gravitationally, and the star may develop a very strong *stellar wind*, driven by momentum deposited in the material by photons emerging from the star. The mass-loss rate in the wind may be large enough to strip away much, or even most, of the star's envelope, thus exposing material that has undergone nuclear burning.

Stars of near-solar mass develop similar complex structures and winds in the very late phases (red giant and beyond) of their evolution. We can track this process with quantitative spectroscopic analysis of their atmospheres because we begin to see successive layers in which the relative abundances of elements have been altered in the "burned" material. Indeed, in the most extreme cases known, we can observe a bare stellar core; see [1155, 1157].

Calculation of the most advanced stages of stellar evolution is at best very difficult. A number of good books and review articles about the subject are available, e.g., [233, 234, 558, 559, 619, 725, 773, 949, 952].

Precise models of stellar structure can also yield very important, unexpected results. For example, for some time the rate of neutrino emission from the Sun calculated with even the most refined models differed by an order of magnitude from observations made at neutrino observatories. This discrepancy ultimately led to the conclusion that neutrinos are *not* massless, as previously supposed in "standard" elementary particle theory, but having a finite mass, they can oscillate from one type to another (electron, τ, and μ neutrinos) in the time it takes them to travel from the Sun to the Earth.

Stellar Atmospheres

The theory of *radiative transfer* started with the work of A. Schuster [979], who formulated the *transfer equation*[9] for radiation passing through a *scattering* medium. His work was extended by K. Schwarzschild [980, 981], who made the distinction

[9] The terms "radiative transfer" and "transfer equation" are used when the material is static. The terms "radiation transport" and "transport equation" are used in problems in which the material is in motion or is time-dependent. See [1002] for an interesting review of the development of radiative transfer theory.

between scattering and *thermal absorption and emission* processes,[10] developed an elegant integral equation formalism to describe the transfer of radiation; showed that the variation of the brightness of the solar continuum from the center to the edge of the Sun's disk is consistent with radiative, not convective equilibrium; and demonstrated that the frequency variation of the depths of the H and K Fraunhofer lines of ionized calcium are consistent with them being formed by scattering.

In the 1920s, work by Fowler, E. Milne, A. Pannekoek, and M. Saha; see, e.g., [343, 344, 757, 760, 836, 932], showed that excitation and ionization of material in *thermodynamic equilibrium* can be described with statistical mechanics. Using this theoretical framework, they made the first correct interpretation of the observed spectral sequence. Cecilia Payne showed in her Ph.D. thesis, later published as the monograph *Stellar Atmospheres* [849], that hydrogen is the most abundant element in stellar atmospheres, a result confirmed in an analysis of solar hydrogen line profiles by A. Unsöld [1099].

Milne [753, 754] derived approximate results for the temperature distribution in, and emergent radiation from, a *gray* (i.e., the opacity of the stellar material is independent of frequency) plane parallel atmosphere in radiative equilibrium and *local thermodynamic equilibrium* (LTE).[11] E. Hopf, M. Bronstein, and C. Mark found the exact solution to this problem; see [150, 151, 511, 711]. It provides a valuable benchmark for evaluating approximate methods of solving transfer equations. S. Rosseland showed that radiative transfer in a stellar interior can be described as a diffusion process with an average opacity known as the *Rosseland mean* [907]. And in a prescient paper B. Gerasimovic opened the question of departures from LTE in stellar atmospheres [372].

In the 1930s, W. McCrea [719] constructed the first model stellar atmosphere by numerical integration of the structural equations, allowing for the depth dependence of pressure, temperature, ionization, and opacity. In this era radiative transfer in spectral lines was treated with schematic models based on the paradigm of pure absorption and *coherent* scattering processes; see, e.g., [301, 756, 837, 1071, 1175]. This work showed that these two mechanisms result in very different emergent spectra. In reality, the scattering of photons in lines is *not* coherent. Some simplified studies of line formation allowing for noncoherent scattering were made by several authors. But at a more basic level, the picture of "absorption" and "scattering" processes is oversimplified. In reality more complicated chains of transitions, which cause *interlocking* of spectral lines [1174], are implicit in the equations of *kinetic equilibrium*; they did not receive satisfactory treatment until the 1970s. Observational and theoretical studies of stars showing strong emission lines had also started [97, 98, 373].

Despite the disruption of research by World War II, important advances were made in the 1940s. Chandrasekhar developed the *discrete-ordinate method* to solve the

[10] This distinction is extremely important. Its physical and mathematical implications are described in chapters 5, 6, 9–11, 13–15.

[11] LTE hypothesizes that gradients of physical conditions in stellar material are so small that its properties can be computed using thermodynamic equilibrium formulae at the local value of the temperature and density. This approximation is generally invalid in the observable outer layers of stars.

transfer equation subject to the constraint of radiative equilibrium in gray material; see, e.g., [218]. An outcome of this work was the conclusive identification of H^- as the dominant opacity source in the solar atmosphere. This method is a powerful and flexible numerical tool that has been applied in most astrophysical radiation transport problems.

An outstanding breakthrough in this era was made by V. Sobolev, working in near isolation in the Soviet Union. Through extraordinary physical insight, he recognized that the velocity gradient in the rapidly expanding envelopes of *Wolf-Rayet stars* and *novae* actually *simplifies* the problem of spectral line formation, even under extreme non-equilibrium conditions. The reason is that a steep velocity gradient limits the geometric size of the *interaction sphere* within which a line photon emitted at one point in the atmosphere can interact with material at another point before it is Doppler-shifted out of the line's bandwidth. This insight allowed him to derive an analytical expression for a photon's escape probability in terms of the *local* properties of the gas and the value of the velocity gradient. With it, he was able to compute realistic emission line spectra for expanding stellar envelopes and nebulae. His work was first published only in Soviet journals that were unavailable in the West during wartime, and it remained almost unnoticed until the 1960s when it was translated into English [1028, 1029]. The "Sobolev approximation" is still a very useful tool in astrophysics.

After World War II ended, Unsöld and his coworkers pushed forward detailed studies of stellar spectra and atmospheric structure, based on the hypothesis of LTE; see, e.g., [1100–1103]. But in the 1950s, astrophysicists began to take a more iconoclastic view of the assumption of LTE and the classical picture of line formation and to examine critically the effect of abandoning these a priori assumptions. In 1957, R. Thomas made his classic study of line formation for a simplified model atom having two bound levels plus continuum, a good first caricature for strong resonance lines [1075]. His work showed that photons emitted in the core of a line are redistributed so efficiently over the line's absorption profile that transfer in the line cannot be treated as coherent. A much better approximation is the limit of *complete redistribution*, i.e., where the frequency of the photon emerging from the scattering process has no correlation with that of the incoming photon.

The important physical consequence of this difference is that line-core photons absorbed and emitted at physical depths where they would be trapped by the high opacity at line center can be redistributed into a line's wings, where the opacity is smaller, and escape to the surface. This leakage allows the source term in the transport process to differ (by orders of magnitude) from its LTE value at the surface. A dark absorption line is produced even in an isothermal atmosphere, whereas no absorption feature would exist in LTE. Subsequently, in a series of seminal papers, J. Jefferies and Thomas studied non-LTE (NLTE) effects in the formation of resonance lines in a medium having a "chromosphere" (i.e., an outward rise in temperature) and delineated the behaviors of *collision dominated* and *photoionization dominated* lines. These papers [579–581] demolished earlier arguments that LTE is valid for line formation calculations and set the stage for the developments of the next 40 years.

As was the case for stellar evolution, work on stellar atmospheres changed radically in the 1960s with the arrival of fast computers that had compilers able to convert a mathematics-like language into machine code. It became possible to use more realistic descriptions of radiative transfer (which is strongly nonlinear), and theoretical work morphed into a combination of analysis and computation. A flood of new results followed. Algorithms to solve the central problem of finding the temperature distribution for a nongray atmosphere in radiative equilibrium were developed by E. Avrett, M. Krook, and L. Lucy [67, 703]. A major step forward was the development of an efficient algorithm for solving very general transfer equations by P. Feautrier [323–325], who generalized Schuster's second-order differential equation form of the transfer equation to allow for an angle- and frequency-dependent radiation field, a depth-dependent opacity, and arbitrary scattering functions. It did for stellar atmospheres modeling what Henyey's method did for stellar evolution. Large grids of LTE models were calculated for various classes of stars and used to estimate element abundances. Computations were made of the effects of "blanketing" by H lines in the visible [728] and by strong lines in the UV of hot stars; see, e.g., [141, 238, 747, 865]. S. Strom and R. Kurucz showed how to treat blanketing by millions of spectral lines by constructing *opacity distribution functions*, ODFs [1059]. In parallel, other investigators, e.g., [595, 866], developed direct *opacity sampling* techniques. Both methods are powerful tools.

Enormous progress was also made in the physical theory of spectral line formation. Accurate computations of Stark broadening of the H and He lines (used to determine the effective temperature, surface gravity, and helium abundance of a stellar atmosphere) became available; see, e.g., [80, 90, 91, 376, 399, 404, 1120, 1121, 1122]. And accurate descriptions of noncoherent scattering in spectral lines were worked out; see, e.g., [250, 251, 519, 530, 544, 546, 547, 699, 817, 1186]. This work put the physics of absorption and emission of line photons on a sound physical footing for the first time.

In the past 35 years great progress in modeling stellar atmospheres has resulted from (1) incisive physical formulations of radiative transfer; (2) calculation of high-quality cross sections for radiative and collisional processes in plasmas; (3) efficient algorithms to calculate the radiation field and material properties in a stellar atmosphere using simultaneous self-consistent solutions of the radiative transfer and kinetic equilibrium equations that allow for thousands of spectral lines. Stellar atmosphere modeling has also benefited greatly from the immense increases in computer speed and capacity.

In the area of algorithm development, the extreme difficulty of handling all of the *physical* coupling between radiative transfer and multi-level model atoms was solved by L. Auer and D. Mihalas in 1969 by the complete linearization (CL) method [53]. By the mid-1970s, calculations had been made for a number of NLTE models of O-stars. It was found that in NLTE, the strengths of the hydrogen and helium lines were considerably larger than those predicted by LTE, in agreement with observations for the first time [55, 56, 734]. But as originally formulated, the method was computationally too expensive to treat the complicated transition arrays of multielectron atoms and ions.

In the mid-1980s, several authors, e.g., [176, 918, 955, 957, 958, 959, 960], developed fast, approximate methods (which can be iterated to self-consistency) for solving transfer problems with large scattering terms. Generically, they are referred to as *Accelerated Lambda Iteration*(ALI) methods. In 1986, L. Auer, R. Buchler, and G. Olson [815] made a breakthrough by showing that inversion of only the *diagonal* of the matrix representing the depth-coupling in a transfer problem yields a convergent iterative method, which is *immensely* more efficient than direct solutions. It opened the door to constructing model atmospheres having huge numbers of spectral lines in their spectra. In the late 1980s, L. Anderson realized that we can group physically similar atomic levels into *superlevels* and represent entire transition arrays with a small number of *superlines* [29–32] between the superlevels. The combination of these three ideas (CL/ALI/superlevels) led to very powerful, fast model atmosphere codes; see, e.g., [432, 458, 523, 532, 533, 1148, 1149, 1150], that can handle atoms/ions having thousands of energy levels and millions of spectral lines, in the full non-equilibrium regime. It is now possible to compute theoretical spectra that fit observational data precisely; see, e.g., [290, 388, 427, 452, 453, 682, 683]. Indeed, they are reliable enough to use as absolute calibration standards for spectrophotometric observations; see, e.g., [124, 126, 679].

The effectiveness of photon momentum deposition in ultraviolet resonance lines to produce stellar winds from O-type and Wolf-Rayet stars was demonstrated conclusively by L. Lucy and P. Solomon in 1970 [705]. Soon after, J. Castor, D. Abbott, and R. Klein [199], using the Sobolev approximation to solve the transfer problem, and accounting for the huge numbers of lines present in the spectrum, made models that could give the massive flows actually observed. By the late 1980s, a number of groups in Germany produced models, e.g., [354, 355, 358, 430, 431, 433, 435, 436, 450], in which the photon momentum deposition in spectral lines in the expanding envelope of a star is treated self-consistently with the hydrodynamic flow it produces. ALI methods again had great impact on such computations. At present, we can even begin to analyze the exploding outer layers of novae and supernovae; see, e.g., [455, 459, 1004, 1065].

- *The essential problem faced in modeling stellar interiors is determining the depth variation of all physical variables in a star as a function of mass, age, and composition, in the face of having only two observational checks: the star's luminosity and radius.*
- *The essential problems faced in modeling stellar atmospheres are that the radiation field is formed in a non-equilibrium boundary layer and that the entire emergent spectrum must be computed as a function of depth, angle, and frequency, with a heavy burden of non-equilibrium physics.*

The tasks in both disciplines are very challenging!

1.2 THE BOTTOM LINE

Although stellar atmospheres theory originally lagged behind the theory of stellar structure and evolution, the progress made in this discipline in the past 40 years has put it on a firm physical foundation, and it now makes predictions in close agreement with observable data. The fact is, *the study of stellar atmospheres provides a perfect* *arena for the development of diagnostic tools that can be used in the analysis of radiation from all kinds of astrophysical and laboratory sources.*

The answers we can now make to Salpeter's question are more compelling today than they could have been 45 years ago:

1. The atmosphere of a star is what we can *see*. Once the gas in a stellar envelope becomes opaque, we cannot obtain *direct* information about conditions inside the star. In the case of the Sun, nonradial oscillation modes are observed on its disk, and some information about its internal structure can be inferred. But other stars are seen as unresolved points, so we get information only from their outer layers, and it must be used to the fullest.
2. Virtually everything known about *all* astrophysical objects is derived by analysis of their emitted radiation. The methods developed for stellar spectra can be applied to other objects, e.g., H II regions, planetary nebulae, neutron stars, and active galactic nuclei. Radiation is also an active ingredient in determining the structure and dynamics of some of these objects.
3. With a dependable theory of radiative transfer in a star's atmosphere that explicitly accounts for the coupling of the radiation field with the atomic/ionic occupation numbers of the material, one can now make accurate calculations for the emergent continuum and line spectra including the effects of millions of spectral lines. These allow reliable computation of the transformations between an observer's *absolute magnitudes* M_V and *colors*, e.g., *UBVRIJHK*, and a theoretician's *bolometric magnitude*, effective-temperature, and surface-gravity scales. As a result, the outputs of stellar structure calculations can be connected reliably with observed color-magnitude diagrams and thus provide critical tests of stellar evolution theory. Without a trustworthy theory of stellar atmospheres, such a connection would be extremely difficult. Further, these models provide good estimates of the ionizing fluxes that determine the physical state of the interstellar medium and gaseous nebulae.
4. Using accurate computations of spectral line strengths, we can make sound quantitative chemical analyses of stellar compositions for stars of different masses and ages. We are now able to perform such analyses even in the rapidly expanding, non-equilibrium atmospheres of very highly evolved stars. This work provides information about the internal structure and evolution of these stars and insight into the past history of the material in its interior that has undergone nucleosynthesis in thermonuclear reactions.
5. With line-blanketed NLTE model atmospheres and accurate stellar evolution tracks we can study stellar populations in galaxies by computing synthetic spectra for an assumed mixture of stars and comparing them with observed galaxy spectra [713]. When a good fit is achieved, one can infer information about their ages and chemical evolution. Here, very hot and massive stars have special importance because they are so luminous we can observe their spectra even in remote galaxies. With these results it is possible to map the change of

relative abundances of chemical elements in the Universe as a function of time with confidence. This work has provided strong observational support to, and tests of, the Big Bang picture.

6. Comparison of physically realistic computations of stellar spectra with high-quality data can yield two constraints between the three fundamental parameters L, M, and R. If one of them can be found by an independent method, we can solve for all three. The results provide guidance to theories of stellar structure and evolution at its most fundamental level.
7. A star's mass M and luminosity L are well defined in the sense that they can, in principle, be measured by a remote observer. However, as will be shown later, if a star has an *extended envelope*, its radius R is not, even in principle, uniquely defined. Yet stars in the most sensitive stages of their evolution (giants, supergiants) have such envelopes! In stellar structure calculations, the outermost layers of a star have long been treated using a severely oversimplified model of radiation transport, which is invalid near the surface (i.e., several photon mean free paths into the material). As a result, the calculated values of R or T_{eff} may have significant uncertainties. Furthermore, the state of the material in such extended envelopes is far from LTE [87, 453, 1005]. *For giants and supergiants, it is an absolute necessity to take into account NLTE effects and stellar winds to model their outer envelopes, to interpret their observed radiation field (colors, spectral energy distribution, and total luminosity), and to provide realistic outer boundary conditions for the calculation of their internal structure*.
8. Stars in all parts of the HR diagram have been analyzed. The main sequence has been examined from massive, hot, very luminous stars, down to faint substellar objects that have very low surface temperatures and such small masses that their central temperatures are too low for thermonuclear fusion to generate enough energy to support their luminosity. The study of the post main-sequence evolution of stars has greatly profited from a very close interplay of stellar evolution calculations that tell us about the internal structure of a star and model atmosphere calculations that are now sufficiently accurate to make reliable direct connections to observed data.
9. Despite its small fractional mass, a star's atmosphere *can* affect its evolution profoundly through mass loss in stellar winds. We now know the following:
   a) Highly evolved massive stars have Wolf-Rayet spectra showing intense emission lines that indicate very rapid mass loss from their envelopes. Evolutionary calculations of the massive upper main-sequence stars and Wolf-Rayet stars *must* take mass loss in winds into account in order to get realistic evolution tracks in the HR diagram. Impressive self-consistent stellar structure and model atmosphere calculations can now be made; see, e.g., [233, 387, 725, 773, 947, 948, 951, 1039]. The fate of some of these stars will be to undergo collapse of a degenerate iron core, from which thermonuclear energy can no longer be extracted, and become supernovae. The remnant from this gigantic explosion may be a neutron star or a black hole.

b) Very old stars of about solar mass found on the red giant branch (RGB) of the HR diagram have a core in which hydrogen has been converted to helium, surrounded by a hydrogen-burning shell source, inside a hydrogen envelope [438, 439, 983, 987]. While the outer envelope expands, the core contracts; eventually it reaches temperatures at which the helium ignites and begins to produce a helium/carbon core. The star moves abruptly to the foot of the *asymptotic giant branch* (AGB). When the helium in the core is depleted, the star has both hydrogen-burning and helium-burning shells surrounding an inert carbon core. Instabilities occur in this *double shell-source* phase [440, 984], and hydrogen may be mixed into the helium-burning shell by helium-shell flashes [985]. In addition, AGB stars may develop extensive convective atmospheres that can *dredge up* material from deeper layers that has been processed through multiple stages of thermonuclear burning. Such episodes may explain the origin of spectral types C and S.

Copious mass loss strongly influences the evolution of these stars as they ascend the AGB branch into post-AGB phases that include the extremely hot PG 1159 stars; then to very hot *planetary nebula nuclei* [441]; and ultimately to white dwarfs, which have consumed all their thermonuclear fuel, are radiating away the heat in their interiors, and ultimately slide down the white-dwarf sequence in the HR diagram into a dark oblivion. Stellar atmospheres theory is now able to make fairly reliable calculations of these critical phenomena; see, e.g., [113, 483, 895, 896, 1152, 1153].

10. For theoretical reasons alone, a correct description of photon transfer in the outer layers of a star is a fascinating challenge, because here radiation emerges from the near-perfect thermodynamic-equilibrium stellar interior into the blackness of space. The observable atmospheric layers are a severely non-equilibrium environment and require detailed analysis of the role of radiation in determining the excitation/ionization state of the material, which, in turn, determines the opacity and emissivity of the material, and hence the radiation field. Attaining a consistent treatment of these tightly interlocked processes has been a significant scientific achievement.
11. Most of the physical insights and mathematical methods developed in the study of stellar atmospheres are applicable to the diagnosis of spectra from laboratory plasmas.
12. Finally, the ability to make quantitative spectroscopic analyses of cosmic objects can have profound implications. For example, current models of the structure of the Universe attribute only about 30% of its mass-energy to baryonic matter. The other 70% is given the name *Dark Energy*; its origin and nature are not understood within the current framework of physics. At present, astrophysicists are trying to interpret small departures of the observed redshift-distance relation (*Hubble diagram*) for Type Ia supernovae from a standard model for the expansion of the Universe. These may imply a time variation

of the dynamical effects of Dark Energy. The redshifts of the supernovae can be measured directly, but their distance must be deduced from their maximum luminosity. It is very important to continue to refine the models of the dynamic atmospheres of these objects in order to establish a precise relationship between the maximum brightness observed in their light curves and their absolute luminosities. It is possible that future modeling of these objects may modify some of the profound conclusions mentioned above.

On more general grounds, one must recognize that astrophysics is an intricate construct assembled using many different lines of reasoning. It is all connected: the activities in its different subfields must be viewed as *cooperative*, not competitive. For example, who would have guessed in the 1950s that the then "old-fashioned" discipline of celestial mechanics would re-emerge into the spotlight in the 1960s, when it was needed to plan the orbits of space vehicles? And who would have guessed that this magnificent 10- to 12-digit formalism developed by great mathematicians such as Brown, Euler, Gauss, Hill, Lagrange, Laplace, Newcomb, and Poincaré, coupled with precision measurement of distance by radar and radio astronomy, and precise timing using atomic clocks, would provide definitive proof of the correctness of general relativity?