1
Arithmetic and Geometry

I do not mean by beauty of form such beauty as that of animals or pictures . . . but straight lines and circles, and the plane and solid figures that are formed out of them by turning-lathes and rules and measures of angles; for these I affirm to be not only relatively beautiful like other things, but they are eternally and absolutely beautiful.

—Plato, *Philebus*, 360–347 BC

WHAT ARE NUMBERS? How does mankind know them? What is art? Why do people sing, dance, and draw pictures? Biologists and anthropologists have sought satisfactory answers to these notoriously difficult questions by describing the evolution of *Homo sapiens* into a species that perceives patterns and seeks pleasure.

The ability to recognize quantity—the number of bananas in a bunch or predators in a pack—has an advantage in the struggle for survival. Biologists have demonstrated that many birds and mammals alive today, as well as human infants, have an innate ability to distinguish between small numbers of discrete objects (1, 2, 3, 4, and "many") and to add and subtract these simple sums.[1] *Homo sapiens*' numerical competence is based on neural circuitry for counting and reasoning that they share with animal relatives at least as distant as birds and rodents. This means that when early humans evolved from apes, they inherited the preverbal mechanisms for counting, addition, and subtraction—the basis of arithmetic.

Mankind's ape-like ancestors—and apes still today—could chip the edges of stone to make tools, but the marks they made did not produce symmetrical patterns (plate 1-2). But around 1.4 million years ago, modern man's hominid ancestor *Homo erectus* came down from the trees to live in Africa's tropical grasslands, walked upright, and made the first flat stone tools with balanced halves and symmetrical outlines (plate 1-3).[2] After another million years *Homo sapiens* began chipping three-dimensional symmetrical tools in flaked stone, and, by 300,000 years ago, he carved tools in the round with striking symmetry (plate 1-4). This suggests that the human perceptual/cognitive system evolved in that interval, between 1.4 million and 300,000 years ago. In humans living today, the recognition of flat, two-dimensional shapes (which *Homo erectus* needed to impose a symmetrical outline) does not require the high functioning of the cerebral cortex. However, the perception of left and right and the ability to perceive three-dimensional forms, rotate them in the mind, and judge whether they can be exactly superimposed (which *Homo sapiens*

1-1. Illustration of the Genesis story of creation in a *Bible moralisée*, French, ca. 1208–15. Egg tempera and gold leaf on vellum. Österreichische Nationalbibliothek, Vienna, Bildarchiv, Codex 2554, fol. 1v.

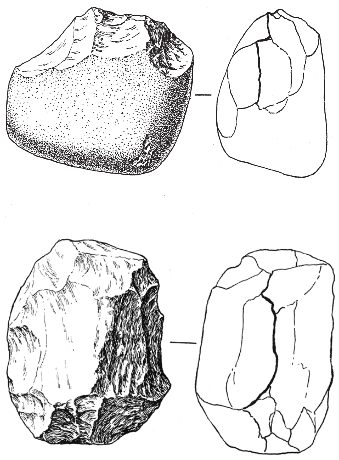
Using a compass, the Creator shaped the cosmos from chaos by drawing its spherical boundary, and then he formed the red sun and golden moon. Next he will use his compass to shape the lump of earth that lies at the center of his cosmos. This is the frontispiece of a *Bible moralisée*, a manuscript containing paraphrases of biblical verses together with moralizing interpretation, which was created for the French royal family by scholarly clergy in Paris in the early thirteenth century. Above the picture the iconographer instructs the royal family that only almighty God (as opposed to mortal astronomers) can encompass the entire cosmos, declaring (in Old French), ICI CRIE DEX CIEL ET TERRE SOLEIL ET LUNE ET TOZ ELEMENZ (Here God creates heaven and earth, sun and moon, and all the elements).

needed to carve stone tools in the round) requires a high level of processing in the cerebral cortex, the part of the brain that evolved most recently.[3] Anthropologists hypothesize that the evolution of this spatial perceptual/cognitive system was part of a more general increase in intelligence, and it may have been selected in reproduction, if potential mates saw the ability to make symmetrical tools as a sign of sharp wits and good health.[4] Today newborns have the neurological repertoire to recognize and mentally manipulate simple shapes and forms, and as they grow, children develop the ability to distinguish more complex patterns of shapes and forms—the basis of geometry.

When *Homo erectus* began walking upright, this change in body position prompted not only the development of the perceptual/cognitive system but also the evolution of aesthetic behavior, according to the following anthropological theory.[5] When the womb and birth canal of female bipedal hominids shifted to a vertical orientation (as opposed to horizontal in lower mammals), the result was that as the evolving *Homo sapiens* brain got heavier, the force of gravity pulled the fetus "prematurely" from the womb. Even after birth, the brain of infant *Homo sapiens* continues to grow, its skull bones knitting for another twelve months, during which the mother invests enormous time and energy into caring for the child. When other family members and friends interact with the baby, they elicit a response by speaking in high, lilting sounds or moving rhythmically (clapping, swaying). A responsive infant who joins in this "performance" by smiling and cooing at the pleasant sounds or playing in rhythm with the adult—all of which are *aesthetic* behaviors—receives more attention for longer periods of time from the aunts, uncles, cousins, and neighbors with whom the baby interacts. In other words, the "performing" baby receives more caretaking during its first year of life, and as a result the infant thrives. In this way aesthetic behavior evolved into an innate human trait.

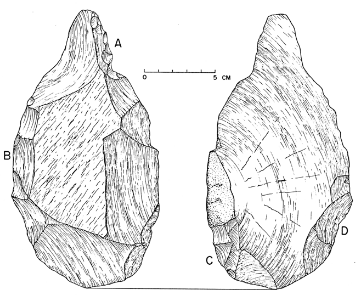
Although the ability to do simple arithmetic was hard-wired into mankind's brain millions of years ago, it was only around 300,000 years ago that the cognitive hardware evolved to perceive and manipulate shapes and forms, which is the basis of geometry, and to possess a preverbal desire for pleasure and human bonding, which is the basis of art. By about 200,000 years ago, the brain of the modern human being—*Homo sapiens*—was fully evolved in terms of its size and shape.

In Africa, the Near East, Europe, and Asia between about 40,000 and 10,000 years ago, human artifacts suddenly show extensive evidence of symbolic thought (plates 1-7 and 1-8). Signs of the use of algorithms (step-by-step procedures to accomplish a task) also appeared, such as carving a pattern of holes in a bone (plate 1-6).[6] These abstract systems must develop in a cultural context, passed on from generation to generation, because although an infant is born with the neurological circuitry to perceive three-dimensional form, to seek pleasure, and to compose sentences, a child does not spontaneously produce mathematics, art, and language. To develop these abstract cognitive skills, a child needs to imitate others in a community. Algorithms were used to apply decorative patterns throughout the Neolithic era between around 10,000 and 2500 BC (plate 1-5), when complex societies formed permanent communities, and the evolutionary drive shifted from biology to culture.

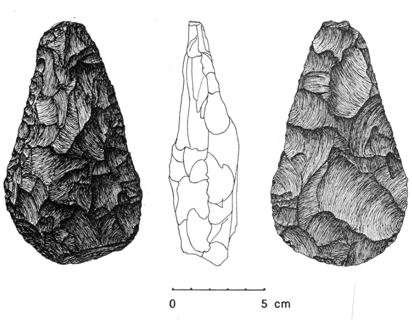


1-2. Asymmetrical tools, 1.8 million years old. Mary D. Leakey, *Olduvai Gorge* (Cambridge: Cambridge University Press, 1971), 3, fig. 9 on 27, and fig. 11 on 29. © 1971 Cambridge University Press. Used with permission.

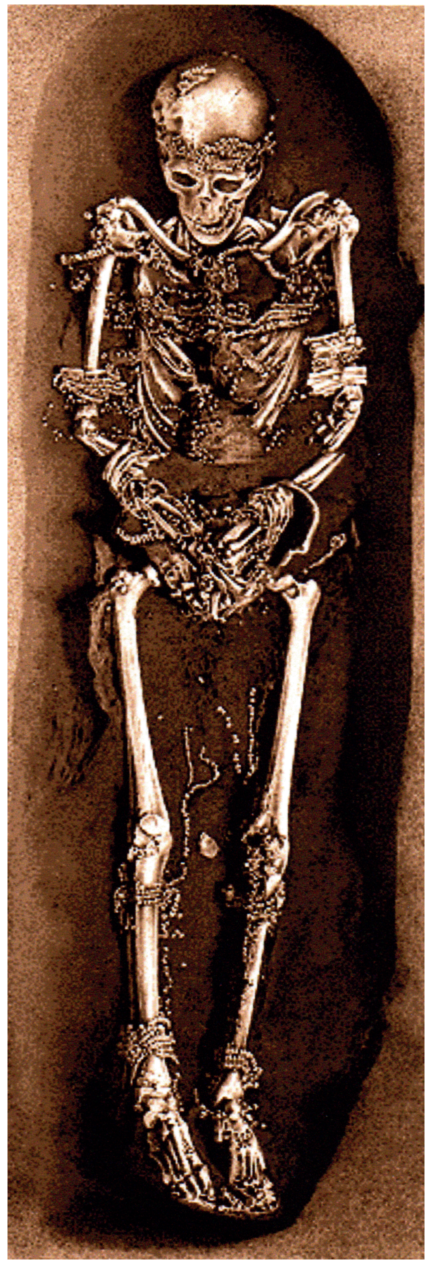
Front and side views (above) of a side chopper made from a water-worn rounded piece of lava, and a two-edged chopper made from lava (below).



1-3. Hand ax with symmetrical outline, 1.4 million years old. Courtesy of Thomas Wynn.



1-4. Symmetrical tool in the round, 150,000–300,000 years old. Courtesy of Thomas Wynn.



ABOVE LEFT

1-5. Storage jar, Gansu Yangshao culture, Banshan type, Gansu or Qinghai Province, China, ca. 3000–1000 BC. Earthenware painted with red and black slip, 15⅝ in. (40 cm) high, diameter 13¾ in. (35 cm), without handles. Asia Society, New York, Mr. and Mrs. John D. Rockefeller III Collection of Asian Art, 1979.125.

ABOVE CENTER

1-6. Flute found in the Hohle Fels cave in the Danube River valley near Ulm, Germany, 43,000–42,000 years old. Bone, ca. 8½ in. (21.6 cm) long.

One of the first anatomically modern humans in central Europe made this flute by carving a row of finger holes in a bone from the wing of a griffon vulture, a large bird of prey. Thin and hollow but very strong, bird bones are especially suited to making flutes. At one end of the bone the musician cut a double-notched, V-shaped hole, which is assumed to be the mouthpiece, suggesting that this flute was played in a vertical position, like a modern recorder. This and other flutes (bones with a similar pattern of holes) found in nearby caves along the Danube River are the oldest known musical instruments in the world.

ABOVE RIGHT

1-7. Bison painted on the wall of the Upper Paleolithic cave at Lascaux, France, ca. 15,000 BC.

RIGHT

1-8. Burial of a Cro-Magnon man in Sunghir, Russia, 30,000–28,000 years old. Institute of Archaeology, Moscow.

This early modern man was laid to rest in a garment sewn with 3,000 beads, each individually carved from ivory. He was a member of a Cro-Magnon tribe of humans—*Homo sapiens*—who lived in Europe and Russia.

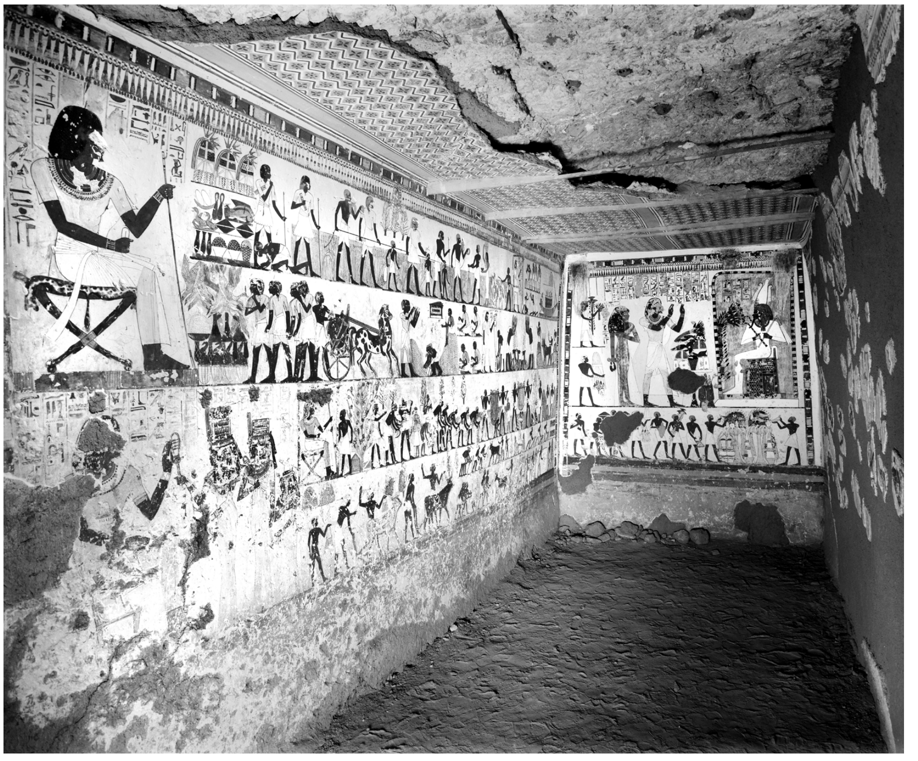
ANCIENT FOUNDATIONS OF MATHEMATICS: ABSTRACTION AND GENERALIZATION

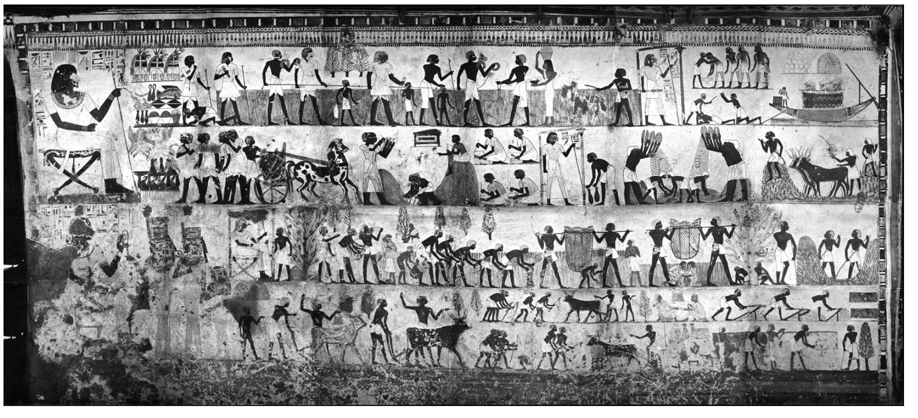
By 3000 BC, in settlements in the Nile, Tigris-Euphrates, and Yellow River valleys, scribes kept numerical records by making marks, such as | for one, || for two, ||| for three, and so on. To avoid a profusion of marks, others were invented for larger numbers, such as the Egyptian ∩ for ten, so that twelve could be written ||∩. Other marks—perhaps a simple picture—signified objects, such as ⌓ for a loaf of bread (in the Egyptian hieroglyph ||⌓

The Tomb of Menna

Menna, whose title was Scribe of the Fields of the Lord of the Two Lands, was overseer of all agricultural activities on the royal estates, which he observes seated at the upper left of the top and middle images. Menna watches farmers plant, harvest, and transport crops. In the center of the top register of the middle image, his surveyors measure the fields preliminary to assessing taxes that the farmers must pay under threat of being beaten for underpayment (top right). Surveyors are at the front and back of the rope (upper register, and in the color detail), each carrying a coiled rope around his shoulder, and they pull taut a rope that is knotted at regular intervals for measuring units of distance. The front of the rope is partly coiled, ending in a knot in the hand of the lead surveyor, and the rope ends in a knot that dangles from the hand of the rear surveyor.

Egyptians measured with a unit of length—a cubit—that was originally based on the length of a forearm, although it was longer than most people's forearms, measuring about 20 1/2 in. (52 cm); the cubit was subdivided into seven palms and four fingers. The knots in this example appear to be 3 cubits apart (a little over 5 ft.). The surveyors are accompanied by a boy with a bag (probably for the rope) and two scribes, who carry pallets on which to record the size of the field of grain shown in the background. An old man with a staff and two boys walk alongside the survey team, which is met by a couple carrying food and drink.





OPPOSITE

1-9. Tomb of Menna, Sheikh Abd el-Qurna, Thebes, Eighteenth Dynasty, late fifteenth–early fourteenth century BC.

Above: Black and white photograph of the Tomb of Menna, taken around 1920 during the Metropolitan Museum of Art Expedition (1907–35). Metropolitan Museum of Art, New York, inv. no. T807.

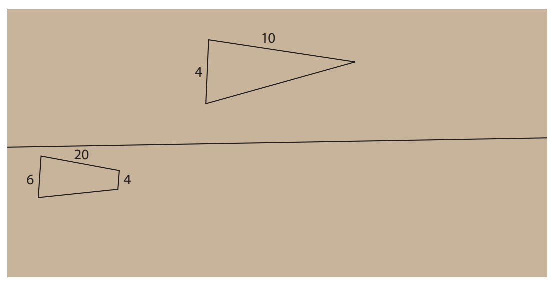
Middle: Black and white photograph of the left wall, taken around 1920 during the British Expedition (1902–25), led by Robert L. Mond. Griffith Institute, University of Oxford, inv. no. Mond 11.22.

Below: As a member of the Metropolitan Museum of Art Expedition, the job of the American artist Charles K. Wilkinson was to document the ancient paintings in color. Around 1920, Wilkinson did this painting of harvest scenes in the upper register of the left wall of the Tomb of Menna. Tempera on paper, $29\frac{15}{16}$ × 73¼ in. (76 × 186 cm). Metropolitan Museum of Art, New York, Rogers Fund, 1930.

ABOVE

1-10. Rhind papyrus scroll, ca. 1650 BC. Ink on papyrus, height 12½ in. (31.7 cm). This section of the scroll includes problems 49–55 (right) and problems 56–60 (left). © The Trustees of the British Museum, acc. no. 10057r.

The Rhind papyrus scroll, which was found at Thebes, is written in hieratic (an abridged form of hieroglyphics) and includes geometric figures that are among the oldest known. Egyptologists estimate that the geometric knowledge embodied in these figures is at least one thousand years old, dating to the time of the pyramid-builders of the Old Kingdom.



1-11. Diagram of problems 51 and 52 from the Rhind papyrus.

These problems give a lesson in how to calculate the area of a plot of land. The upper figure is a triangle with a base of 4 units and a height of 10. Assuming the height and base are perpendicular (despite the slight deviation in the Rhind papyrus figure), the accompanying text calculates the area of the right triangle as 20 units, which corresponds to the value given today using the modern formula: $A = \frac{1}{2} bh$, which reads "A equals one-half of b times h, where A is the area, b is the base, and h is the height." In the figure below, students of surveying were taught how to calculate the area of a truncated triangle (a trapezoid), with a base of 6, height of 20, and a second base (a cut-off side) of 4. The calculated result of 100 corresponds to the value obtained by the modern formula for finding the area (A) of a trapezoid:

$$A = \left(\frac{b_1 + b_2}{2}\right) h$$

where b_1 and b_2 are the lengths of the two bases and h is the height of the trapezoid.

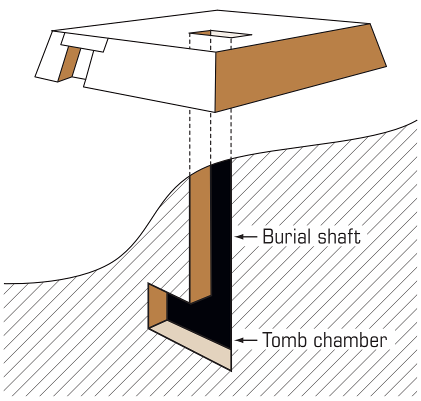
RIGHT

1-12. Imhotep (Egyptian, ca. 2650–2600 BC), *Step Pyramid of Zoser*, Saqqara, Egypt, 2630–2611 BC.

This first Egyptian pyramid, the Step Pyramid at Saqqara, is the oldest surviving monumental stone structure in the world. The architect Imhotep designed the tomb by placing truncated pyramids—mastabas—of decreasing size on top of each other. The original base mastaba was first extended laterally, and then the "steps" were added vertically. Students of the Rhind papyrus were taught the surveyor's rule for calculating the area of a trapezoid, and scribes have recorded that Egyptian builders knew the related rule for finding the volume of a truncated pyramid. Thus when Old Kingdom workers began constructing tombs in the form of a mastaba, they were able to estimate the amount of material needed.

BELOW

1-13. A typical Old Kingdom mastaba tomb has the form of a truncated pyramid.



A new light flashed upon the mind of the first man (be it Thales or some other) who demonstrated the properties of the isosceles triangle.

—Immanuel Kant,
***The Critique of Pure Reason*, 1781**

for "two loaves"). The predominance of multiples of ten in ancient numerical systems is undoubtedly because human hands, on which counting began, have ten fingers.

The Egyptians and other ancient cultures employed whole numbers (1, 2, 3, . . .), ratios of numbers, written today as either 1 : 2, 1 : 3, 1 : 4, . . . (the ratios of one to two, one to three, one to four, . . .) or as 1/2, 1/3, 1/4, . . . (the fractions one-half, one-third, one-fourth, . . .), and the operations of addition, subtraction, and multiplication, the basis of arithmetic. Ancient peoples also began using triangles and rectangles to measure land and to set out right angles for the corners of buildings. For example, Egyptian surveyors kept careful records of the parcels of farmland along the Nile, so that each year after the floodwaters receded they could redivide the river valley using geometry (from the Greek for "measure the earth"). Surveyors could calculate the area of plots of land and measure the volume of architectural forms (plates 1-9, 1-10, 1-11, 1-12, and 1-13).

The abstraction of counting and measuring from rules of thumb to a systematic arrangement of general solutions or proofs—the step into pure mathematics—took place in Greek culture between the sixth and fourth centuries BC. Egyptian surveyors could find the area of a *particular* plot of land, but they did not move beyond mere calculation into the realm of abstract thought and develop a proof of how to find the area of *all* triangles or *all* rectangles.[7] In the sixth century BC, the Ionian Greek philosopher Thales of Miletus (in present-day Turkey) made that critical move beyond considering only particulars and generalizing to *all*. Thales brought Egyptian surveying methods to Ionia, where, according to ancient sources, he "attacked some problems in a general way"[8] and demonstrated properties of *any* right triangle and *any* triangle with two equal sides, so-called isosceles (from the Greek for "equal-legged") (plate 1-14).[9] In the sixth century BC Anaximander, a follower of Thales who was also from Miletus, declared that the structure of the cosmos is based on proportion and symmetry and that key facts about the natural world, such as the position of the sun, moon, and stars, can be expressed in numbers.[10] This Ionian Greek outlook marks the beginning of the scientific worldview in which nature is understood as embodying a mathematical structure that is discernable by human reason.

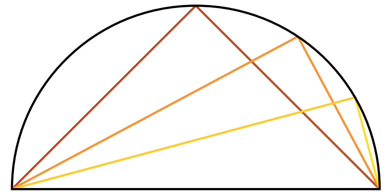
What led mankind to take this leap of the imagination from a particular object to its underlying form? The Greek philosophers' sense that the natural world embodies a mathematical structure is grounded in ancient myths that celebrate the victory of a celestial, divine mind over chaos; the world came into being when reason (spirit) imposed structure and laws on the formless void of primordial matter. The Babylonians believed that the divine Marduk ripped the goddess of chaos, Tiamat, in half to form earth and sky, positioned the sun, moon, and stars, and then "ordained the year into sections and he divided it; for the twelve months he fixed three stars" (*Enûma Eliš*, 1500–1100 BC). The god of the Hebrew prophets "laid the foundations of the earth," and in order to contain the rivers and the sea he "set a bound that they may not pass over," putting in place a body of laws to govern the natural world (Psalms 104:5 and 9, ca. 1000 BC; see plate 1-1, chapter frontispiece).[11] In the late eighth century BC, the Greek poet Hesiod told how heaven and earth separated themselves out from a primordial abyss—the chasm of Chaos—that was closed by Eros (sexual desire), creating harmony in the cosmos (*Theogony*, ca. 700 BC).

Babylonian, Hebrew, and Greek people set out to find the structure that had been imposed on formless chaos, and gradually, piece by piece, they found order in the movement of the stars and patterns in numbers. Since these ancient peoples all looked at the same natural world and contemplated the same mathematical-world-out-there, albeit from different cultural perspectives, their observations of nature and their insights about mathematics translated from culture to culture, and over the centuries a cumulative body of knowledge formed that is the basis of Western science and mathematics.

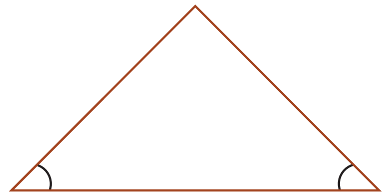
In the late sixth century BC the citizens of Athens on the Greek mainland threw off rule by aristocrats who had inherited their authority and tyrants who had seized power, and, led by Cleisthenes, they invented a new form of government—democracy (from the Greek for "government by the people")—that is historically linked to a scientific approach because of its anti-authoritarian assumption that all humans are equal and its appeal to human reason. In the political reforms that began under Cleisthenes in 508–507 BC, Greek male citizens were selected to hold public office by lottery (on the assumption that all were equally capable). Greek citizens practiced *isonomia* ("equal before law"); there were no judges in the courts but rather a jury of citizens, who were selected by drawing lots from a large pool. Public policy was set at meetings of the Assembly, a public forum open to all citizens, each of whom could speak and cast a vote to decide the issue. As in science and mathematics, truth in Greek democracy was determined by reasoned argument, not on the basis of governmental authority or because it was the custom of the land.

In this political climate, the next generation of Greek citizens began the artistic and literary achievements of the Golden Age of Athens, including the building of the Parthenon, the sculpture of Phidias, and the plays of Aeschylus, Sophocles, and Euripides. Then, in the late fifth and fourth centuries, the scientific approach that had been initiated by Thales and Anaximander in Ionia was developed in Athens to an extraordinary level of abstraction and generalization by Socrates, Plato, and Aristotle, culminating in Euclid.

1-14. Thales's theorems.



Any angle inscribed in a semicircle makes a right triangle.



The base angles of an isosceles triangle are equal.

Majority rule is called by the fairest of terms: Equality before the Law. Next, it requires something the tyrant never allows: people hold office by lot, they are accountable for the actions of their administration, and their deliberations are held in public. I propose, therefore, that we abolish the monarchy and increase the power of the people, for in the many is all our strength.

—**Herodotus,**
***The Histories*, 450–420 BC**

Greek religion of the sixth and fifth centuries BC centered on the worship of mythological heroes such as Heracles, nature deities such as Zeus, god of sky and thunder, and civic deities, such as Zeus's daughter Athena Parthenos (Athena the virgin), the goddess of Athens. But during the Golden Age, the Athenian understanding of the cosmos in terms of the unpredictable whims of these personal gods gave way to the concept of the natural world as the product of impersonal forces that were discernable by the human mind. After the outbreak of the Peloponnesian War (431–404 BC), confidence in Athena Parthenos was seriously undermined by decades of civil war that devastated Athens and caused poverty throughout the Greek islands.

Does any say there are gods in heaven? No! there are none if man will not be fool enough to credit the old tale. Let not my words guide your judgment; see for yourselves. I say that tyranny slays its thousands and despoils their goods, and men who break their oath cause cities to be sacked; and, doing so, they are happier than men who walk quietly in the ways of piety from day to day.

—**Euripides, *Bellerophon*, ca. 430 BC**

With faith in Greek mythology shaken, an undercurrent of skepticism surfaced among Greek philosophers. Already in the late sixth century BC, Heraclitus had declared that there is no fixed, unchanging truth—"Everything flows"—from which his follower, Protagoras, inferred that each person's view is an equally valid description of the state of affairs: "Man is the measure of all things." Protagoras openly expressed doubt about the existence of Zeus and Athena: "As to the gods, I have no means of knowing either that they exist or that they do not exist. For many are the obstacles that impede knowledge, both the obscurity of the question and the shortness of human life."[12] The statesman and poet Critias (a relative of Plato) went so far as to suggest that a politician had invented the gods of Greek mythology to increase his power over citizens: "Although the laws kept them from open deeds of violence, men went on doing them in secret; and then it was, I believe, that some clever and sagacious man first invented for mortals the fear of the gods, so there might be something to frighten the wicked, even though their acts or words or thoughts were secret. . . . And for the dwelling of the gods he chose the place that would have the most startling effect on men, . . . the round sky above us, where he saw the lightning and the dreadful crash of thunder."[13]

In this atmosphere of doubt and suspicion about Heracles, Zeus, and Athena, some citizens of Athens perceived the scientific worldview as a threat to the Olympian pantheon. An example is the treatment of the Ionian philosopher Anaxagoras, who travelled from his homeland in Clazomenae (present-day Turkey) to Athens in the fifth century BC, and lectured in public about finding the structure of reality using reason. He described the cosmos as inert matter that had been put in motion by a purposive Mind (*Nous*), after which the natural world operated in predictable ways that could be discerned by man and described in physical (not divine) terms: the sun is an incandescent stone, and the moon is a mass of earth.[14] Fearing that such doctrine posed a serious threat to the authority of the sacred gods of Greek mythology, civic leaders arrested Anaxagoras and exiled him from Athens.[15] Fifty years later the Athenian state put the philosopher Socrates on trial, charging him with not believing "the sun and moon are gods, like other men. . . . since he says the sun is a stone and the moon made of earth."[16] Refusing to flee Athens, Socrates drank the fatal hemlock in 399 BC.

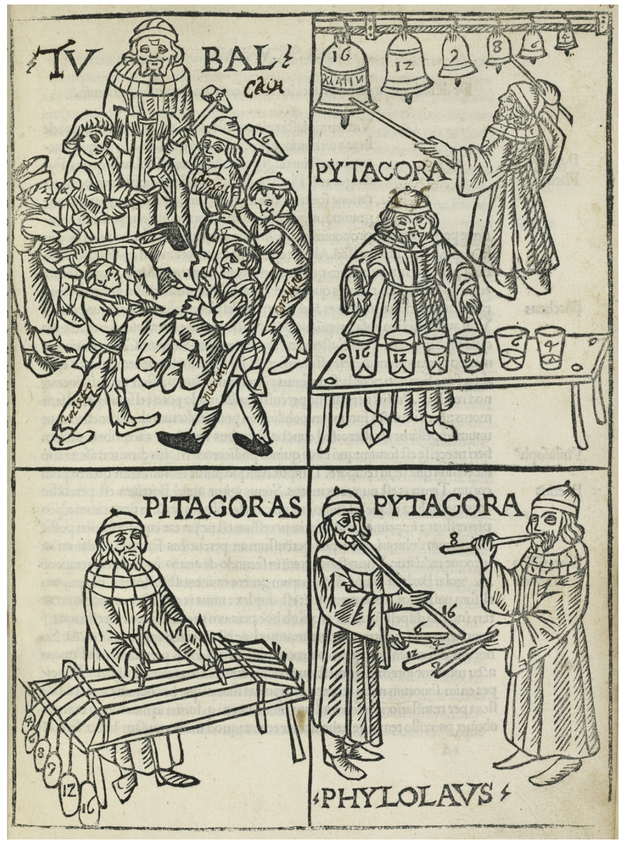
For with what aim did ye insult the gods, and pry around the dwellings of the moon?

—**Aristophanes, *Clouds*, later fifth century BC**

In addition to the scientific approach to understanding the natural world, a second tradition developed within Greek culture in so-called mystery cults, in which knowledge of the world was gained less by reason than in a moment of mystical insight, or intuition. Reason and intuition were used in both the scientific and mystical approaches, the difference being one of emphasis. From their observations of the cycles of nature—day and night followed by dawn, summer and winter followed by spring—members of the mystery cults of Dionysus, Orpheus, and Pythagoras believed that human life was also cyclical—life and death followed by rebirth. The Dionysus cult observed the life cycle of the grapevine, which produces vibrant, luscious grapes, whose dismembered bodies were fermented, producing intoxicating spirits that gave a taste of afterlife in the underworld. Returning to sobriety, they poured wine on the soil, giving libation to begin the cycle anew. The cycle of life-death-rebirth was also part of the cult of Orpheus, the legendary musician who descended into Hades in search of his beloved Eurydice and then returned to the land of the living.

Pythagoras of Samos lived in the late sixth century BC on the Italian peninsula, where he was the legendary leader of a mystery cult whose members withdrew from society and adopted a strict ascetic lifestyle. Unlike other leaders of mystery cults, Pythagoras made pronouncements about the hidden meaning of numbers, and his cult had followers in the fifth and fourth centuries BC who made important contributions to mathematics.[17] Pythagoras himself wrote nothing, but his ideas survive in the writings of his followers. For example, Philolaus of Croton in southern Italy argued that the cosmos is made of unlimited continua such as fire, water, space, and time, which are limited by numbers, shapes, and forms that join together to form "harmonia"—a harmonious whole (*On Nature*, fifth century BC). To illustrate the process, Philolaus gave a musical example that the Pythagoreans had discovered: if a musician slides a stick up and down a vibrating string, it produces a continuum of sound (plate 1-15 lower left). But this unlimited continuum is limited according to ratios of whole numbers because the main musical consonances—an octave, a fifth, and a fourth—are expressible in ratios of the smallest whole numbers—1 : 2, 2 : 3, and 3 : 4, respectively. If one plucks a string and then divides it in half by pressing it in the middle, plucking the string again produces an octave and the listener experiences an agreeable feeling of finality—a consonance. Thus, according to Philolaus, numerical relations underlie harmony in both music and the soul (plates 1-15 and 1-16). A combination of tones not in a ratio of whole numbers—a dissonance—is experienced by the listener as unresolved. Since the natural world and man's inner being (the soul) are structured by numbers, we can, according to Philolaus, only gain true knowledge of the world and of ourselves by studying numbers.

Whereas philosophers who pursued a scientific approach put their trust in human reason, Greeks who joined mystery cults believed their lives were controlled by the irrational forces of fate—blind and without purpose. Since it was hopeless to even try to exercise rational control of their lives, cult members followed figures such as Dionysus, god of wine, and abandoned themselves to ecstasy and ritual madness. The suffering that they endured in their lives was alleviated by the promise that their lives had a hidden meaning—a mystery—which would be revealed to them in the form of a secret (a riddle, paradox, or absurd



1-15. Harmonic ratios in music, from Franchinus Gaffurius, *Theorica Musicae* (Milan: Phillipas Montegatius,1492), 18. Woodcut. Music Division, The New York Public Library for the Performing Arts, Astor, Lenox, and Tilden Foundations.

In this Renaissance book on music theory, Tubal-cain, the biblical forger (upper left), looks on as smiths make music by striking an anvil with a variety of hammers, while Pythagoras (upper right) strikes graduated bells and glasses filled with diminishing amounts of water. Pythagoras is shown on the lower left playing an instrument that has been tuned by hanging graduated weights from its six strings. As the weights increase from four to sixteen units, the strings are stretched ever tighter, producing increasingly higher pitches when plucked. On the lower right, Pythagoras plays a duet with his student Philolaus, whose flute is twice as long. Together they sound an octave, producing within them a satisfying feeling of resolution.

1-16. Fyodor Bronnikov (Russian, 1827–1902), *Pythagorean Hymn to the Rising Sun*, 1869. Oil on canvas, 39¼ × 63½ in. (99.7 × 161 cm). State Tretyakov Gallery Moscow.

The Russian Romantic artist Fyodor Bronnikov painted members of the Pythagorean mystery cult playing stringed instruments in harmony with cyclical rhythms that they observe in nature: the sun rises in the east, follows an arched path overhead, and sets in the west.

All things that are known have number. For it is not possible that anything whatsoever be understood or known without this.

—**Philolaus, *On Nature*, fifth century BC**

phrase) when they were initiated into the cult. They would understand the mystery not by reason but in an epiphany—a flash of insight. Initiates were also promised that their virtues would be rewarded in a life after death.

Mystery cults probably began as spring planting and fall harvest festivals, such as the Eleusinian cult of Demeter, goddess of agriculture, and her daughter Persephone, the goddess of fertility, who was abducted into the underworld by Hades, ruler of the gloomy land of the dead. In response to the pleadings of Demeter to her husband Zeus, the god of thunder allowed the goddess of fertility to return to her mother each spring, but in the fall Persephone had to go back for a cold season in the dark underworld. Because the Athenian members of the Eleusinian cult who paid tribute to Demeter and Persephone in the sixth century BC also followed Zeus and Athena, the cult did not provoke the wrath of public officials, whose only concern was that the power of the Olympian pantheon be preserved. When many Athenians became destitute in the later fifth century BC during the Peloponnesian War, the Eleusinian cult shifted its focus from partaking in a bountiful fall harvest to living in paradise after death, a shift that was repeated during troubled times in later mystery cults.[18]

Many mystery cults promised eternal life and featured figures such as Persephone and Orpheus, who returned alive after visiting the underworld. But whereas in the Eleusinian, Dionysian, and Orphic cults irrational absurdities were transformative, the secret phrases that moved the Pythagoreans were about numbers. Members of the Pythagorean cult had made the stunning and surprising discovery that music has a mathematical basis,

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