Extended Introduction with Online Resources

Mircea Pitici

This is the sixth anthology in our series of recent writings on mathematics selected from professional journals, general interest publications, and Internet sources. All pieces were first published in 2014, roughly in the form we reproduce (with one exception). Most of the volume is accessible to readers who do not have advanced training in mathematics but are curious to read well-informed commentaries about it.

What do I want by sending this book into the world? What kind of experience I want the readers to have? On previous occasions I answered these questions in detail. To summarize anew my intended goal and my vision underlying this series, I use an extension of Lev Vygotsky's concept of "zone of proximate development." Vygotsky thought that a child learns optimally in the twilight zone where knowing and not knowing meet—where she builds on already acquired knowledge and skills, through social interaction with adults who impart new knowledge and assist in honing new skills. Adapting this idea, I can say that I aspire to make the volumes in this series ripe for an optimal impact in the imaginary zone of proximal reception of their prospective audience. This means that the topics of some contributions included in these books might be familiar to some readers but novel and instructive for others. Every reader will find intriguing pieces here.

Besides offering a curated collection of articles, each book in this series doubles into a reference work of sorts, for the recent nontechnical writings on mathematics—with the caveat that I decline any claim to being comprehensive in this attempt. The list of book titles I give at the end of the introduction and the list of notable writings at the end of the volume contain a few entries published prior to the 2014 calendar year, in an acknowledgment that in previous volumes I overlooked materials worth mentioning. The same is surely the case for this year. The fast pace of the series, the immense quantity of literature I survey, and the convention subtly ensconced in calling a "year" the interval from January 1 to December 31 not only make such lapses inevitable but to a high degree determine the content of the book(s). Were we to look at the same literature from July 1 of one year until June 30 of the next year, the books in this series would look very different from what you can read between these covers. That is why each volume should be seen in conjunction with the others, part of a serialized enterprise meant to facilitate the access to and exchange of ideas concerning diverse aspects of the mathematical experience.

In this volume a greater number of contributions than in the previous volumes concern mathematical games and puzzles. For many centuries and in many cultures, recreational mathematics used to be seen as a benign amusement of no immediate utility. That enduring but now old-fashioned perception has gradually changed over the past century because of at least two broad phenomena. First, the history of the most salient branches of contemporary mathematics (algebra, modern algebra, geometry, probability, number theory, graph theory, knot theory, topology, combinatorics, and even calculus) has been either rooted into or decisively influenced by "recreational" problems. Second, talented writers and popularizers of recreational mathematics (the most famous of whom was Martin Gardner) found a large audience in the public, enjoyed appreciation from select but remarkable mathematicians, and built a devoted following of like-minded authors who carry on working in the same vein, encouraged by the lasting impact of their predecessors. Recreational mathematics has a rich and sophisticated history studied in the past by a few authors who contributed brief works (notably David Singmaster); recently the scholarship is growing rapidly, as illustrated by a special issue of the journal *Historia Mathematica* dedicated entirely to recreational mathematics. Nowadays good recreational mathematics is placed midway between the intelligent but mathematically untrained public and the mathematics professionals, by virtue of linking easy-to-understand problems to serious mathematics. In other words the problems posed in high-quality recreational mathematics are comprehensible to the layperson, while pursuing and understanding the ideas developed in the solutions occasioned by the problems might require an independent learning effort the reader is free to undertake or not. Thus the context of good recreational mathematics straddles the popular and the pedagogical, having the dual value of intellectual entertainment and optional instructional use. At its best, recreational mathematics illustrates the synergetic encounter between the ludic and the serious aspects of mathematics, as well as the one between the amateur and the professional strands of doing mathematics—just two of the many polarities of contrary/complementary features that meet and mesh in mathematical thinking and practice. I am glad that this year "recreational" mathematics is well represented in the anthology.

There is much more in this book. Some problems of recreational mathematics are at least two millennia old, representing well the nature of mathematics as *we have known it*—a type of thinking that endures forever, the timeless and unchanging mathematics of a realm at rest, in equilibrium. In that endurance we feel a flavor of the times when only a few million people lived on this planet, scattered over the Earth, and few of them thought mathematically—and those who did lived far apart, isolated from each other. But now we are in the billions, we instantly communicate with each other, and we put numbers on more things than can be measured. The mathematics adequate to this dynamic world in ceaseless disequilibrium has to have a basis different from the old mathematics, at least in some respects. It has to be a mathematics that treats the dynamic phenomena from the inside, not from the outside (just as Albert Einstein thought up relativity of motion by asking himself what would happen if one traveled as fast as light does). Glimpses of a more statistical bend in mathematical thinking are obvious in some contributions to this volume and attest not only to the ever-changing nature of mathematics but also to the merits of interpreting mathematics in broad personal and societal contexts.

Interpreting mathematics is a further stage of thinking mathematically—not a "higher" stage or a "lower" stage, just an essentially *different* one. Interpreting mathematics is not about mathematical truth (or any other truth); it is a *personal* take on mathematical facts, and in that it can be true or untrue, or it can even be fiction; it is vision, or it is rigorous reasoning, or it is pure speculation, all occasioned by mathematics; it is imagination on a mathematical theme; it goes back several millennia and it is flourishing today, as I hope this series of books lays clear. Fragments in Plato's philosophical dialogues qualify as interpreting mathematics, and so does Edwin Abbott's *Flatland*, and (for example) the work of our contemporaries Ian Stewart, Steven Strogatz, Edward Frenkel, Jordan Ellenberg, and many others, including the contributors to this volume. Interpreting mathematics is a creative domain, inclusive but also competitive—and certainly potent, despite its neglect in academic circles. Interpreting mathematics points toward protean qualities of mathematics not immediately obvious in doing mathematics per se. An accepted mathematical result is merely the egalitarian premise from which each of us can part with the commonly shared view by interpreting it idiosyncratically, as we please or even as it suits us. While mathematics is a "great equalizer" in a sociological sense (as Jaime Escalante famously proclaims in the 1988 movie *Stand and Deliver*), interpreting mathematics is a subtle ability, as much competitive as it is differential. Interpretation sets each of us apart, even when we speak about the same mathematical fact. Interpreting mathematics applies to a part of human affairs where opportunities rule, not constraining mathematical rules. To speak about interpreting mathematics sounds odd, but it seems so only because the customary indoctrination served by our school system pervades the common views of mathematics, both among mathematicians and the lay public. For decades the penury of talented authors able to interpret mathematics in original ways has affected the interaction between mathematics and domains in which mathematical methods were coopted. Lack of reflection on the proper context of applied mathematical thinking perverted the humanities, the social sciences, and even the study and the practice of law—to name just a few areas.

This state of affairs is changing and *The Best Writing on Mathematics* series takes notice of the change. Criticisms of (ab)using mathematics and statistics crop up all the time, with a few included in this book. For me, this is payback of sorts. I once made the naive misstep to suggest, at a well-regarded business school, that mathematics relates to reality in subtler ways than is immediately apparent and that I would rather pursue my hunches than submit to the expensive dogma taught in the "mathematical finance" courses. For that I was not only disparaged but shown the door. Out I went, never to regret it—although, for a long while after that, I "pursued" only survival, as a lesson in what can happen if I take initiative in a place that nominally encourages it. I had been fooled by my preconceptions concerning the abstract notion of free inquiry; I had set up myself for the misadventures that come with resisting the enforcement of dogmatism. Those blunders did not shake my passion for inquiry, but it cured me of the tendency to speak my mind. (Other cures followed, also in the name of noble-sounding ideals; my favorite is the one administered by courts under the slogan "in the best interest of the child.") Since then I became a lot more cautions with my suggestions concerning mathematics; I learn from my errors, the hard way.

Yet I still venture a bit in talking about mathematics, here and there—now prudent, aware that attempting to crack the thought establishments is fraught with dangers. As an example, I can say that the process of editing each volume in this series is a lesson in working with uncertainty while at the same time interpreting mathematics. The contrast between my limited knowledge and the limitless possibilities available to all the people who gloss on mathematics offers me palpable practice for a general mnemonic that serves well in other endeavors. I generalized it into a theoretical and practical principle, which I call "the paradox of reward." The paradox of reward says that in a competitive, fair, unpredictable, and infinitely complex environment, the most valuable knowledge is to know how to be rewarded for ignorance; in other words, more reward is available for taking advantage of ignorance (if one finds such a path to reward) than it is for taking advantage of knowledge. Of course I am *not* saying that ignorance is preferable to knowledge; it is *not*. I am saying that in certain environments harvesting rewards off ignorance is (by far) more valuable than seeking rewards for knowledge. This might seem to have little to do with mathematics; yet to my mind it is nothing else *but* interpreting mathematics, in a world so complex that ignorance is unavoidable but ignoring its benefits is avoidable. This subject is a lot vaster than I sketched in these few sentences, and it has consequences not only for learning and teaching mathematics but also for incorporating the private interpretation of mathematics in strategic thinking. Yet I am mindful of the dangers of venturing too far in speaking unconventionally about mathematics and interpreting mathematics, so I leave it for another occasion.

Overview of the Volume

I feel rewarded to collect in this book thoughts and perspectives on mathematics I could never think up myself.

Michael Barany and Donald MacKenzie locate the center of the mathematical activity done in institutional settings (and occasionally in private homes) at the blackboard; they note that blackboards are key objects that influence the organizing of the research and teaching spaces, while chalk-writing on blackboards influences the logistics and the overall manner of mathematical communication.

Pradeep Mutalik finds that the repeated experience of feeling right when we suddenly comprehend the solution to a problem or a puzzle has had a positive evolutionary role in defining us as humans, both cognitively and emotionally.

Colm Mulcahy and Dana Richards write an informed centennial appreciation of the life and work of Martin Gardner, that remarkable polymath who inspired many mathematicians and laypersons to take up mathematical games and similar challenges.

Arthur Benjamin and Ethan Brown teach us how to construct an unlimited number of customized magic squares by improvising on a few ingenious templates.

Toby Walsh starts with the popular Candy Crush game as a guidepost for his discussion of the factors that determine the difficulty of solving computational problems in mathematics.

Marianne Freiberger takes us to the billiard room; she explains how the trajectory of a ball rolling on the pool table leads to mathematically complex problems related to chaos theory, the conductivity of metals, and other . . . infinite surprises.

Erik R. Tou models juggling numerically, to show that it is mathematically similar to the morphing game of transforming one word into a very different one by incremental steps that admit changes of only one letter at a time.

Scott Aaronson dissects the intricacies of the notion of randomness and connects it to the study of paradoxes, complexity, and quantum mechanics.

Dana Mackenzie describes how biologists, physicists, and mathematicians interact(ed) to overcome theoretical obstacles encountered in the birth and growth of synthetic biology.

In a similar vein, Natalie Wolchover describes the interdisciplinary efforts undertaken by researchers interested in the Tracy-Widom distribution associated with phase transitions in interactive systems of various types.

Eli Maor and Eugen Jost present (and illustrate beautifully) the basic geometric properties of the logarithmic spiral, cycloid, epicycloids, and hypocycloids—some of the best-known curves studied, over the centuries, in connection to natural phenomena and physical motions.

Burkard Polster analyzes the mathematical properties of several noncircular shapes of constant width and shows us how they have been applied to various gadgets and playful devices.

The quickest way to summarize the brief article by Annalisa Crannell, Marc Frantz, and Fumiko Futamura is to say that they look at Dürer's perspective drawing from several different perspectives!

Vi Hart and Henry Segerman ask whether there are groups of symmetries that can be visualized using real-life objects but have never been represented as such—and propose a novel modeling of the quaternion group.

John Conway and Alex Ryba use wordplay and a humorous ingenuity to discuss the merits of several different proofs they give to an old geometry problems that looks deceptively easy (until you try to solve it).

Gila Hanna and John Mason discuss the many facets, relative merits, and theoretical pedigrees of various terms used by mathematicians or mathematics educators to qualify the worthiness of proofs—with the main reference to a similar attempt by Timothy Gowers.

Jim Fey, Sol Garfunkel, and their coauthors formulate five tenets they consider important to be taken as guiding principles for the mathematics education reform at high school level (in the United States).

Guili Zhang and Miguel A. Padilla compare multiple aspects of mathematics instruction in China and the United States, based on previous theoretical and empirical studies.

Against commonly held wisdom, Benoît Rittaud and Albrecht Heeffer argue that the pigeonhole principle, usually attributed to Dirichlet, was stated in writing at least two centuries earlier in *Selectae Propositiones*, a book by Jean Leurechon.

Lisa Rougetet traces the earliest written descriptions of the popular game of Nim to a treatise written at the beginning of the sixteenth century by Luca Pacioli and follows the subsequent European developments of the game since then.

Jan von Plato considers the context of mathematical ideas and the personalities that shaped German mathematician Gerhard Gentzen's ordinal proof theory—and how this work relates (or does not!) with a theorem by Reuben Goodstein.

James Franklin illustrates with several well-chosen examples the local-global synergy in mathematics, one of the many conceptual polarities that characterize mathematical thinking.

Carlo Cellucci reviews many opinions on what constitutes mathematical beauty and its role in mathematics; he concludes that aesthetic factors play an indirect epistemological role in discovery via their selective role in choosing what hypotheses to consider.

Mark Balaguer argues that philosophers of mathematics are mainly concerned with the meaning of mathematical discourse and that the semantic theories they adhere to can lead to claims hardly acceptable for the mathematicians.

Steven Strogatz discerns three broad types of rapport with mathematics in the general public and tells us how he honed his talent for writing about mathematics by paying attention to the writing qualities of masters in similar trades.

Domenico Napoletani, Marco Panza, and Daniele C. Struppa examine the methodological underpinnings and the philosophical implications of using ever-more powerful computing techniques in the modeling of complex phenomena.

Andrew Gelman and Eric Loken caution that evidential claims of statistical significance in research journals are often spurious because of multiple factors related to the gathering of data and its interpretation; they give several suggestive examples.

Jeffrey S. Rosenthal tells the true story of how his statistical expertise led to the discovery and prosecution of fraudulent lottery winnings in Ontario, Canada.

David Hand explains a bias of expectations that precludes us from perceiving the increased likelihood of coincidences following the rapid combinatorial growth of possibilities that comes with the increase of the number of simple events.

More Writings on Mathematics

Every year I started this section by naming one book outstanding among all others. This time I cannot decide on only one; I give two, both excellent and badly needed reference books: Lizhen Ji's *Great Mathematics Books of the Twentieth Century* and *Encyclopedia of Mathematics Education*, edited by Stephen Lerman.

Now, as usual, I roughly group the other titles by theme (full references are at the end of the introduction); some of the books listed here are not easy to categorize, but I made ad hoc choices for the sake of expediency.

A handful of books blend mathematical ideas with describing the world from nonmathematical viewpoints, sometimes with a strong historical perspective—a growing trend I signaled previously in these pages and picking up steam lately. Thus are Jeff Suzuki's *Constitutional Calculus*, Anders Engberg-Pedersen's *Empire of Chance*, Keith Tribe's *The Economy of the World*, along with *The Norm Chronicles* by Michael Blastland and David Spiegelhalter, and *How Reason Almost Lost Its Mind* by Paul Erickson and colleagues. Less shy with giving mathematics a prominent role in daily life are *Grapes of Math* by Alex Bellos and *Mathematics and the Real World* by Zvi Artstein.

In the history of mathematics I note Joseph Mazur's *Enlightening Symbols*, Alexander Amir's *Infinitesimal*, Amir Aczel's *Finding Zero*, Jiri Hudecek's *Reviving Ancient Chinese Mathematics*, Vedveer Arya's *Indian Contributions to Mathematics and Astronomy*,David Reamer's *Count Like an Egyptian*, and, slightly more technical but with well-chosen historical vignettes, *From Mathematics to Generic Programming* by Alexander Stepanov and Daniel Rose. Books focused on personalities are *John Napier* by John Havil, *How Euler Did It Even More* by Edward Sandifer, *Beyond Banneker* by Erika Walker, and *Peter Lax, Mathematician* by Reuben Hersh. A broad swipe of history is covered in *Classical Mathematics from Al-Khwarīzmī to Descartes* by Roshdi Rashed. Theme-based histories are *Whatever Happened to the Metric System?* by John Marciano, *The New Math* by Christopher Phillips, and *The Palgrave Centenary Companion to "Principia Mathematica"* edited by Nicholas Griffin and Bernard Linsky. Focused on the formerly communist countries are *Mathematics across the Iron Curtain* by Christopher Hollings and *Pearls from a Lost City* by Roman Duda. Other (auto)biographical or celebratory volumes are *Gordon Welchman, Bletchley Park's Architect of Ultra Intelligence* by Joel Greenberg, *Wearing Gauss's Jersey* by Dean Hathout, *My Life and Functions* by Walter Hayman, as well as the collective volumes *Arnold* edited by Boris Khesin and Serge Tabachnikov, *Alexandre Grothendieck* edited by Leila Schneps, and *Four Lives [of Raymond Smullyan]* edited by Jason Rosenhouse.

Some books on the interactions between mathematics and other disciplines: *The Oxford Handbook of Computational and Mathematical Psychology* edited by Jerome Busemeyer and his collaborators; *Biographical Encyclopedia of Astronomers* with Thomas Hockey as editor-in-chief; *Scientific Visualization* edited by Charles Hansen et al.; *Bond Math* by Donald Smith; *Measuring and Reasoning [in Life Sciences]* by Fred Bookstein; *Mathematics* *for the Life Sciences* by Erin Bodine, Suzanne Lenhart, and Louis Gross; *Why You Hear What You Hear* by Eric Heller; *Classical Mechanics with Calculus of Variations and Optimal Control* by Mark Levi; and *The Sabermetric Revolution* by Benjamin Baumer and Andrew Zimbalist. Particularly on mathematics and art we have *Manifold Mirrors* by Felipe Cucker, *Mathematics in 20th Century Literature and Art* by Robert Tubs, *Explaining Beauty in Mathematics: An Esthetic Theory of Mathematics* by Ulianov Montano, *Crafting Conundrums* by Ellie Baker and Susan Goldstine, and *Geometrical Objects* edited by Anthony Gerbino.

Expository mathematical writing largely accessible to the general readers are *Mathematical Curiosities* by Alfred Posamentier and Ingmar Lehman; *How Not to Be Wrong* by Jordan Ellenberg; *Everyday Calculus* by Oscar Fernandez; *Beautiful Geometry* by Eli Maor and Eugen Jost; *Math Bytes* by Tim Chartier; *Explorations in Topology* by David Gay; *Mathematical Elegance* by Steven Goldberg; *Symmetry* by Ian Stewart; *The Fascinating World of Graph Theory* by Arthur Benjamin, Gary Chartrand, and Ping Zhang; *Things to Make and Do in the Fourth Dimension* by Matt Parter; *Mathematical Games, Abstract Games* by João Neto and Jorge Silva; *Paradoxes in Mathematics* by Stanley Farlow; and two books by Raymond Smullyan: *A Beginner's Guide to Mathematical Logic* and *The Gödelian Puzzle Book.* Well written and copiously illustrated are *Math in 100 Key Breakthroughs* by Richard Elwes, *The Mathematics Devotional* by Clifford Pickover, *A Curious History of Mathematics* by Joel Levy, and *Really Big Numbers* by Richard Schwartz.

Of the many books on mathematics education, I mention *Leaders in Mathematics Education* by Alexander Karp and David L. Roberts; *How to Study as a Mathematics Major* by Lara Alcock; *Effective Content Reading Strategies to Develop Mathematical and Scientific Literacy* by David Pugalee; the anthology *We Need Another Revolution* by Zalman Usiskin; *Questioning Numbers* by Karin Gwinn Wilkins; *Handbook on the History of Mathematics Education* edited by Alexander Karp and Gert Schubring; *Research Trends in Mathematics Teacher Education* edited by Jane-Jane Lo, Keith Leatham, and Laura Van Zoest; *Transforming Mathematics Instruction* edited by Yeping Li, Edward Silver, and Shiqi Li; *Exploring Mathematics through Play in the Early Childhood Classroom* by Amy Parks; *Mastering Basic Math Skills* by Bonnie Britt; *Putting Essential Understanding of Functions into Practice* by Robert Ronau and his collaborators; and the collective volumes *Using Research to Improve Instruction* edited by Karen Karp, *101*

Careers in Mathematics edited by Andrew Sterrett, *The Elementary Mathematics Specialist's Handbook* edited by Patricia Campbell et al., and *Raising Public Awareness of Mathematics* edited by Ehrhard Behrends, Nuno Crato, and José F. Rodriguez.

Great restitutions of ancient mathematics in new editions are *Conics: Books I–IV* by Apollonius of Perga and *The First Six Books of the Elements of Euclid* by Oliver Byrne.

Now some books on the philosophy of mathematics or related to it: *The Consistency of Arithmetic and Other Essays* by Storrs McCall; *Space, Geometry, and Kant's Transcendental Deduction of the Categories* by Thomas Vinci; *Mathematics of the Transcendental* by Alain Badiou; *Pluralism in Mathematics* by Michéle Friend; *Philosophy of Mathematics in the Twentieth Century* (an anthology) by Charles Parsons; *Why Is There Philosophy of Mathematics At All?* by Ian Hacking; *A Mathematical Prelude to the Philosophy of Mathematics* by Stephen Pollard; *From Logic to Practice* edited by Gabriele Lolli, Marco Panza, and Giorgio Venturi; and *Being Realistic about Reasons* by T. M. Scanlon. Particularly on logic, are *Varieties of Logic* by Stewart Shapiro and *Church's Thesis: Logic, Mind, and Nature* edited by Adam Olszewski, Bartozsz Brożek, and Piotr Urbańczyk. Other books of essays or anthologies on the nature of mathematical thought are William Byers's *Deep Thinking*, Michael Harris's *Mathematics without Apologies*, V. I. Arnold's *Mathematical Understanding of Nature*, Max Tegmark's *Our Mathematical Universe*; also *Distilling Ideas* by Brian Katz and Michael Starbird, *50 Visions of Mathematics* edited by Sam Parc, and *Mathematicians on Creativity* edited by Peter Borwein, Peter Liljedahl, and Helen Zhai.

Accessible books on probabilities and/or statistics are *The Tao of Statistics* by Dana Keller, *Validity and Validation* by Catherine Taylor, *Will You Be Alive 10 Years from Now?* by Paul Nahin, *Standard Deviations* by Gary Smith, and the reference work *The SAGE Handbook of Qualitative Data Analysis* edited by Uwe Flick.

I found a few books difficult to categorize, so I list them apart: *Mathematical Modeling of Zombies* edited by Robert Smith? (the question mark is deliberate) and *Mathematics in Popular Culture* edited by Jessica and Elizabeth Sklar. *The Book of Trees* by Manuel Lima, *The Infographics History of the World* by Valentina D'Efilippo and James Ball, and *The Best American Infographics 2014* edited by Gareth Cook are nicely illustrated with mathematical visuals.

Finally, mathematicians and others who need to write mathematical script can pick up *Practical LATEX*, the latest book by George Grätzer.

Online Resources

A few months ago I started to use Twitter (@mpitici). This led me to many online resources I had not seen before. I will share them here (the links that follow were active as of May 2015; I try not to repeat sources I mentioned in the introductions to the previous volumes in this series). Before I start, I remind readers that abundant mathematical writing and writings on mathematics are hosted online by the Mathematical Association of America and the American Mathematical Society in their regular columns and frequently updated blogs. The same holds for some major newspapers, including the *Wall Street Journal* (Carl Bialik's column) and the *New York Times*. Simon Singh (http://simonsingh.net/) writes occasionally for the BBC and Alex Bellos for the *Guardian* (http://www.theguardian.com/science/alexs-adventures-in-numberland).

I am listing the addresses that follow in no order of preference.

Websites of general mathematical interest and/or maintained by mathematicians (either signed or anonymous): Alexander Bogomolny's extraordinary *Cut-the-Knot* (http://cut-the-knot.org/); *World of Numbers* (http://www.worldofnumbers.com/index.html); *Geometry and the Imagination* blog (http://lamington.wordpress.com/); *Popular Math* site (http://popularmath.strikingly.com./); Ben's blog, full of good humor and drawings (http://mathwithbaddrawings.com/); Presh Talwakar (http://mindyourdecisions.com/); Kalid Azad's blog (http://betterexplained.com/); Doron Zeilberg's web page (http://www.math.rutgers.edu/~zeilberg/) with his excellent collection of opinions (http://www.math.rutgers.edu/~zeilberg/OPINIONS.html); Max Roser's site (http://www.maxroser.com/); Edward Frenkel's site (http://www.edwardfrenkel.com/); Japheth Wood's *Math Wizard* (http://www.japheth.org/); Ron Knott's web pages on mathematics (http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/contactron.html); Stephen Wolfram's blog (http://blog.stephenwolfram.com/); Aatish Bhatia's webpages (http://www.aatishb.com/ and http://www.empiricalzeal.com/); *Emergent Math* blog (http://emergentmath.com/); Gil Kalai's blog on combinatorics (https://gilkalai.wordpress.com/); *Math Mama*

Writes (http://mathmamawrites.blogspot.com/); *Math Blogging* blog (http://mathblogging.wordpress.com/); *Go Geometry* (http://www.gogeometry.com/index.html); Keith Devlin's websites (http://mooctalk.org/ and http://profkeithdevlin.org/); *The Center of Math* blog (http://centerofmathematics.blogspot.com/); Richard Elwes's blog (http://richardelwes.co.uk/blog/); Simon Singh's website (http://simonsingh.net/); Fredrik Johansson's web page (http://fredrikj.net/); *Math-Frolic!* blog (http://math-frolic.blogspot.com/); *Delta Scape* (http://deltascape.blogspot.com/); Jean-Pierre Merx's *Math Counterexamples* (http://www.mathcounterexamples.net/); *Fun with Num3ers* blog (http://benvitalenum3ers.wordpress.com/); *CTK Insights* (http://www.mathteacherctk.com/blog/); *Math with Bad Drawings* (http://mathwithbaddrawings.com/); Evelyn Lamb's class website (https://3010tangents.wordpress.com/); *SciLog International* blog (http://www.scilogs.com/hlf/); the *What's Special about This Number?* website (http://www2.stetson.edu/~efriedma/numbers.html); and Marcus du Sautoy's *Maths in the City* (http://www.mathsinthecity.com/).

Some blogs and websites oriented toward mathematics education: Michael Pershan's blog (http://rationalexpressions.blogspot.com/); *Good Math, Bad Math* blog (http://www.goodmath.org/blog/); *Solve My Maths* (http://solvemymaths.com/); *Teaching Photos* (http://www.teachingphotos.co.uk/gallery/maths/); Dave Richeson's *Division by Zero* blog (http://divisbyzero.com/); *Great Maths Teaching Ideas* (http://www.greatmathsteachingideas.com/); Joshua Bowman's blog (http://thalestriangles.blogspot.com/); James Tanton's website (http://www.jamestanton.com/); Gary Davis's *Republic of Math* site (http://www.republicofmath.com/); the excellent *Art of Mathematics* site hosted by a group of Westfield State University faculty (https://www.artofmathematics.org/); *Math Ed Matters* blog (http://maamathedmatters.blogspot.com/); *MathsPad* (http://www.mathspad.co.uk/myProfile.php); *Resourceaholic* (http://www.resourceaholic.com/); David Pleacher's page (http://www.pleacher.com/mp/mpframe.html); *The De Morgan Forum* of the London Mathematical Society (http://education.lms.ac.uk/); Grant Wiggins's blog (http://grantwiggins.wordpress.com/); *Finding Ways* blog (http://fawnnguyen.com/); *MatthewMaddux Education* blog (http://matthewmadduxeducation.com/); and Mrs. Peabody's *Inquiry Based Mathematics* (https://sites.google.com/a/ismanila.org/hl-math-peabody/home).

Websites rich in video and audio materials: James Tanton's *Curriculum Inspirations* (https://www.youtube.com/playlist?list=PLevtNOOa6SZXVJvtROAFCC0oYt0ySTSo4); Patrick Honner's page (http://mrhonner.com/); interviews with scientists and mathematicians (https://www.simonsfoundation.org/category/multimedia/science-lives/); world-famous Vi Hart productions (http://vihart.com/); *Relatively Prime* audio capsules (http://relprime.com/); the collection of videos by the quasi-mysterious math prof (https://www.pinterest.com/mathematicsprof/); and Mike's Maths Page (http://mikesmathpage.wordpress.com/).

Other visual websites and websites with visualizing apps: an app drawing graphs (https://phet.colorado.edu/sims/calculus-grapher/calculus-grapher_en.html); *Applet Magic* (http://www.applet-magic.com/mathematics.htm); Matt Henderson's collection of apps (http://blog.matthen.com/); *Geometric Loci* (http://geometricloci.tumblr.com/); Dave Whyte's *Bees & Bombs* (http://beesandbombs.tumblr.com/); *Desmos* (https://www.desmos.com/); the mathematical art exhibits side of the annual Bridges Conference and the Joint Mathematical Meeting (http://gallery.bridgesmathart.org/); *Curiosa Mathematica* (http://curiosamathematica.tumblr.com/); Martin Krzywinski's web page (http://mkweb.bcgsc.ca/); *Incredible Numbers* (http://incrediblenumbersapp.com/); *Khan Academy* (https://www.khanacademy.org/); and *Mathhombre* (http://mathhombre.tumblr.com/).

Philosophical, foundational, and other issues on the nature of mathematics: *Foundations of Mathematics* (http://sakharov.net/foundation.html); *M-Phi* (http://m-phi.blogspot.com/); *Art of Maths* Studio (https://artofmathstudio.wordpress.com/); and *The n-Category Café* (https://golem.ph.utexas.edu/category/).

Data, statistics, and other issues on mathematical applications: Tyler Viger's *Spurious Correlations* (http://www.tylervigen.com/); Cathy O'Neil's *MathBabe* blog (http://mathbabe.org/); a data visualization site (http://flowingdata.com/); another one, focused on global economic data (http://ourworldindata.org/); *Shell Method Demo Gallery* (http://mathdemos.org/mathdemos/shellmethod/gallery/gallery.html); *Understanding Uncertainty* site of the University of Cambridge (http://understandinguncertainty.org/); *Snappy Education* (http://www.snappyeducation.com/#!statistics/c1x46); John D. Cook's website (http://

www.johndcook.com/blog/twitter_page/); *Plus* Magazine (http://plus.maths.org/content/category/tags/understanding-uncertainty); Andrew Gelman's blog (http://andrewgelman.com/); and *Information Is Beautiful* (http://www.informationisbeautiful.net/).

Rich lists of book reviews can be found at *Theorem of the Day* website (http://www.theoremoftheday.org/Resources/Bibliography.htm#compendia) and at the MAA website (http://www.maa.org/publications/maa-reviews).

Other websites: *Mathematics on the Web*, a website of information about mathematics journals (http://www.mathontheweb.org/mathweb/mi-journals5.html); and a rich (comprehensive?) collection of Paul Erdos papers (http://www.math.ucsd.edu/~fan/ep/ep.html).

I found almost all the links mentioned in this section thanks to Tweets by the following: America Mathematical Society (@amermathsoc), Alex Bellos (@alexbellos), Aatish Bhatia (@aatishb), Alexander Bogomolny (@CutTheKnotMath), Joshua Bowman (@Thalesdisciple), Center of Math (@centerofmath), Egan J. Chernoff (@MatthewMaddux), Federico Chialvo (@FedericoChialvo), Thony Christie (@mathematicus), David Coffey (@delta_dc), John D. Cook (@JohnDCook), Keith Devlin (@profkeithdevlin), Gary Ernest Davis (@republicofmath), Marcus du Sautoy (@MarcusduSautoy), Richard Elwes (@RichardElwes), Edward Frenkel (@edfrenkel), Andrew Gelman (@StatModeling), John Golden (@mathhombre), Antonio Gutierrez (@gogeometry), Vi Hart (@vihartvihart), Matt Henderson (@matthen2), Patrick Honner (@MrHonner), Ilana Horn (@tchmathculture), Martin Krzywinski (@MKrzywinski), Evelyn Lamb (@evelynjlamb), Mike Lawler (@mikeandallie), London Mathematical Society (@LondMathSoc), Mathematical Association of America (@maanow), Jean-Pierre Merx (@MathCounterexam), Joanne Morgan (@mathsjem), Fawn Nguyen (@fawnpnguyen), Jennifer Oullette (@JenLucPiquant), Michael Pershan (@mpershan), Ivars Peterson (@mathtourist), Cliff Pickover (@pickover), Dave Richeson (@divbyzero), Shecky R. (@SheckyR), Max Roser (@MaxCRoser), Peter Rowlett (@peterrowlett), Ed Southall (@edsouthall), Steven Strogatz (@stevenstrogatz), Presh Talwalkar (@preshtalwalkar), James Tanton (@jamestanton), Sue VanHattum (@suevanhattum), Benjamin Vitale (@BenVitale), Robin Whitty (@theoremoftheday), Stephen Wolfram

(@stephen_wolfram), Jahpeth Wood (@jahpethwood), and (@Inc Numbers), (@Snappyeducation), (@mathematicsprof), (@math blogging), (@aperiodical), (@plusmathsorg). Thanks to all.

I encourage you to send comments, suggestions, and materials I might consider for future volumes to Mircea Pitici, P.O. Box 4671, Ithaca, NY 14852; or electronic correspondence to mip7@cornell.edu.

Books Mentioned

Aczel, Amir D. *Finding Zero: A Mathematician's Odyssey to Uncover the Origins of the Numbers*. New York, NY: Palgrave MacMillan, 2014.

Amir, Alexander. *Infinitesimal: How a Dangerous Mathematical Theory Shaped the Modern World*. New York, NY: Farrar, Straus and Giroux, 2014.

Anderson, Britta L., and Jay Schulkin. (Eds.) *Numerical Reasoning in Judgments and Decision Making about Health*. Cambridge, UK: Cambridge University Press, 2014.

Alcock, Lara. *How to Study as a Mathematics Major*. Oxford, UK: Oxford University Press, 2013.

Apollonius of Perga. *Conics: Books I-IV*. Santa Fe, NM: Green Lion Press, 2014.

Arnold, V. I. *Mathematical Understanding of Nature*. Providence, RI: American Mathematical Society, 2014.

Artstein, Zvi. *Mathematics and the Real World: The Remarkable Role of Evolution in the Making of Mathematics*. Amherst, NY: Prometheus Books, 2014.

Aurenhammer, Franz, Rolf Klein, and Der-Tsai Lee. *Voronoi Diagrams and Delaunai Triangulations*. Singapore: World Scientific, 2014.

Badiou, Alain. *Mathematics of the Transcendental*. London, UK: Bloomsbury, 2014.

Baker, Ellie, and Susan Goldstine. *Crafting Conundrums: Puzzles and Patters for the Bead Crochet Artist*. Boca Raton, FL: A. K. Peters, 2015.

Baumer, Benjamin, and Andrew Zimbalist. *The Sabermetric Revolution: Assessing the Growth of Analytics in Baseball*. Philadelphia, PA: University of Pennsylvania Press, 2014.

Behrends, Ehrhard, Nuno Crato, and José F. Rodriguez. (Eds.) *Raising Public Awareness of Mathematics*. Heidelberg, Germany: Springer Verlag, 2012.

Bellos, Alex. *Grapes of Math: How Life Reflects Numbers and Numbers Reflect Math*. New York, NY: Simon and Schuster, 2014.

Benjamin, Arthur, Gary Chartrand, and Ping Zhang. *The Fascinating World of Graph Theory*. Princeton, NJ: Princeton University Press, 2015.

Blastland, Michael, and David Spiegelhalter. *The Norm Chronicles: Stories and Numbers about Danger and Death*. New York, NY: Basic Books, 2014.

Bodine, Erin N., Suzanne Lenhart, and Louis J. Gross. *Mathematics for the Life Sciences*. Princeton, NJ: Princeton University Press, 2014.

Bookstein, Fred L. *Measuring and Reasoning: Numerical Inference in the Sciences*. Cambridge, UK: Cambridge University Press, 2014.

Borwein, Peter, Peter Liljedahl, and Helen Zhai. (Eds.) *Mathematicians on Creativity*. Washington, DC: Mathematical Association of America, 2014.

Britt, Bonnie A. *Mastering Basic Math Skills*. Reston, VA: The National Council of Teachers of Mathematics, 2014.

Busemeyer, Jerome R., Zheng Wang, James T. Townsend, and Ami Eidels. (Eds.) *The Oxford Handbook of Computational and Mathematical Psychology*. Oxford, UK: Oxford University Press, 2015. *The Elementary Specialist's Handbook*

Byers, William. *Deep Thinking: What Mathematics Can Teach Us about the Mind*. Singapore: World Scientific, 2014.

Byrne, Oliver. *The First Six Books of the Elements of Euclid*. [Reproduces the 1847 Thomas Pickering edition.] Köln, Germany: Taschen, 2013.

Campbell, Patricia F., et al. (Eds.) *The Elementary Mathematics Specialist's Handbook*. Reston, VA: The National Council of Teachers of Mathematics, 2013.

Chartier, Tim. *Math Bytes: Google Bombs, Chocolate-Covered Pi, and Other Cool Bits in Computing*. Princeton, NJ: Princeton University Press, 2014.

Cucker, Felipe. *Manifold Mirrors: The Crossing Paths of the Arts and Mathematics*. Cambridge, UK: Cambridge University Press, 2014.

Danesi, Marcel. *Discovery in Mathematics: An Interdisciplinary Perspective*. Münich, Germany: Lincom Europa, 2013.

Davis, Gary E. *Coffee, Love, and Matrix Algebra*. Wilmington, DE: Republic of Mathematics, 2014.

D'Efilippo, Valentina, and James Ball. *The Infographics History of the World*. Richmond Hill, Canada: Firefly Books Ltd., 2014.

Duda, Roman. *Pearls form A Lost City: The Lvov school of Mathematics*. Providence, RI: American Mathematical Society, 2014.

Ellenberg, Jordan. *How Not To Be Wrong: The Power of Mathematical Thinking*. New York, NY: Penguin Press, 2014.

Elwes, Richard. *Math in 100 Key Breakthroughs*. New York, NY: Quercus, 2013.

Engberg-Pedersen, Anders. *Empire of Chance: The Napoleonic Wars and the Disorder of Things*. Cambridge, MA: Harvard University Press, 2015.

Erickson, Paul, et al. *How Reason Almost Lost Its Mind: The Strange Career of Cold War Rationality*. Chicago, IL: University of Chicago Press, 2014.

Farlow, Stanley J. *Paradoxes in Mathematics*. Mineola, NY: Dover Publications, 2014.

Fernandez, Oscar E. *Everyday Calculus: Discovering the Hidden Math All Around Us*. Princeton, NJ: Princeton University Press, 2014.

Flick, Uwe. (Ed.) *The SAGE Handbook of Qualitative Data Analysis*. Los Angeles, CA: Sage, 2014.

Friend, Michéle. *Pluralism in Mathematics: A New Position in Philosophy of Mathematics*.

Gay, David A. *Explorations in Topology: Map Coloring, Surfaces, and Knots*. 2nd ed. Amsterdam, Netherlands: Elsevier, 2014.

Gerbino, Anthony. (Ed.) *Geometrical Objects: Architecture and the Mathematical Sciences 1400-1800*. Heidelberg, Germany: Springer International, 2014.

Goldberg, Steven. *Mathematical Elegance: An Approachable Guide to Understanding the Basic Concepts*. New Brunswick, NJ: Transaction Publishers, 2014.

Grätzer, George. *Practical LATEX*. Heidelberg, Germany: Springer International, 2014.

Greenberg, Joel. *Gordon Welchman, Bletchley Park's Architect of Ultra Intelligence*. London, UK: Frontline Books, 2014.

Griffin, Nicholas, and Bernard Linsky. (Eds.) *The Palgrave Centenary Companion to 'Principia Mathematica'*. New York, NY: Palgrave MacMillan, 2014.

Hacking, Ian. *Why Is There Philosophy of Mathematics At All?* Cambridge, UK: Cambridge University Press, 2014.

Hansen, Charles D., et al. (Eds.) *Scientific Visualization: Uncertainty, Multifield, Biomedical, and Scalable Visualization.* Heidelberg, Germany: Springer International, 2014.

Harris, Michael. *Mathematics without Apologies: Portrait of a Problematic Vocation*. Princeton, NJ: Princeton University Press, 2014.

Hathout, Dean. *Wearing Gauss's Jersey*. Boca Raton, FL: CRC Press, 2014.

Havil, John. *John Napier: Life, Logarithms and Legacy*. Princeton, NJ: Princeton University Press, 2014.

Heller, Eric J. *Why You Hear What You Hear: An Experimental Approach to Sound, Music, and Psychoacustics*. Princeton, NJ: Princeton University Press, 2014.

Hersh, Reuben. *Peter Lax, Mathematician: An Illustrated Memoir*. Providence, RI: American Mathematical Society, 2014.

Hockey, Thomas. (Ed.) *Biographical Encyclopedia of Astronomers*. New York, NY: Springer Science+Business Media, 2014.

Hollings, Christopher. *Mathematics across the Iron Curtain: A History of the Algebraic Theory of Semigroups*. Providence, RI: American Mathematical Society, 2014.

Hudecek, Jiri. *Reviving Ancient Chinese Mathematics: Mathematics, History, and Politics in the Work of Wu Wen-Tsun*. Abingdon, UK: Routledge, 2014.

Ji, Lizhen. *Great Mathematics Books of the Twentieth Century*. Somerville, MA: International Press, 2014.

Karp, Alexander, and Gert Schubring. (Eds.) *Handbook on the History of Mathematics Education*. Heidelberg, Germany: Springer International, 2014.

Karp, Alexander, and David L. Roberts. *Leaders in Mathematics Education: Experience and Vision*. Rotterdam, Netherlands: Sense Publishers, 2014.

Karp, Karen. (Ed.) *Using Research to Improve Instruction*. Reston, VA: The National Council of Teachers of Mathematics, 2014.

Katz, Brian P., and Michael Starbird. *Distilling Ideas: An Introduction to Mathematical Thinking*. Washington, DC: Mathematical Association of America, 2013.

Keller, Dana K. *The Tao of Statistics: A Path to Understanding (with No Math).* 2nd ed. Los Angeles, CA: Sage, 2014.

Khesin, Boris A., and Serge L. Tabachnikov. (Eds.) *Arnold: Swimming against the Tide*. Providence, RI: American Mathematical Society, 2014.

Lerman, Stephen. (Ed.) *Encyclopedia of Mathematics Education*. Dordrecht, Germany: Springer Reference, 2014.

Levi, Mark. *Classical Mechanics with Calculus of Variations and Optimal Control: An Intuitive Introduction*. Providence, RI: American Mathematical Society, 2014.

Li, Yeping, Edward A. Silver, Shiqi Li. (Eds.) *Transforming Mathematics Instruction*. Heidelberg, Germany: Springer International, 2014.

Lima, Manuel. *The Book of Trees: Visualizing Branches of Knowledge*. New York, NY: Princeton Architectural Press, 2014.

Lo, Jane-Jane, Keith R. Leatham, and Laura R. Van Zoest. (Eds.) *Research Trends in Mathematics Teacher Education.* Heidelberg, Germany: Springer International, 2014.

Lolli, Gabriele, Marco Panza, and Giorgio Venturi. (Eds.) *From Logic to Practice: Italian studies in the Philosophy of Mathematics*. Heidelberg, Germany: Springer International, 2015.

Maor, Eli, and Eugen Jost. *Beautiful Geometry*. Princeton, NJ: Princeton University Press, 2014.

Marciano, John B. *Whatever Happened to the Metric System? How America Kept Its Feet*. New York, NY: Bloomsbury, 2014.

Mazur, Joseph. *Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Power*. Princeton, NJ: Princeton University Press, 2014.

McCall, Storrs. *The Consistency of Arithmetic and Other Essays*. New York, NY: Oxford University Press, 2014.
Montano, Ulianov. *Explaining Beauty in Mathematics: An Esthetic Theory of Mathematics*. Heidelberg, Germany: Springer International, 2014.
Nahin, Paul J. *Will You Be Alive 10 Years from Now? And Numerous Other Questions in Probability.* Princeton, NJ: Princeton University Press, 2014.
Neto, João P., and Jorge N. Silva. *Mathematical Games, Abstract Games*. Mineola, NY: Dover Publications, 2014.
Olszewski, Adam, Bartozsz Brożek, and Piotr Urbańczyk. (Eds.) *Church's Thesis: Logic, Mind, and Nature*. Kraców, Poland: Copernicus Center Press, 2014.
Parc, Sam. (Ed.) *50 Visions of Mathematics*. Oxford, UK: Oxford University Press, 2014.
Parter, Matt. *Things to Make and Do in the Fourth Dimension*. New York, NY: Farrar, Straus, and Giroux, 2014.
Parks, Amy N. *Exploring Mathematics through Play in the Early Childhood Classroom*. Reston, VA: The National Council of Teachers of Mathematics, 2014.
Parsons, Charles. *Philosophy of Mathematics in the Twentieth Century: Selected Essays*. Cambridge, MA: Harvard University Press, 2014.
Phillips, Christopher J. *The New Math: A Political History*. Chicago, IL: Chicago University Press, 2014.
Pickover, Clifford A. *The Mathematics Devotional*. New York, NY: Sterling, 2014.
Pollard, Stephen. A Mathematical Prelude to the Philosophy of Mathematics. Heidelberg, Germany: Springer International, 2014.
Posamentier, Alfred S., and Ingmar Lehman. *Mathematical Curiosities: A Treasure Trove of Unexpected Entertainments*. Amherst, NY: Prometheus Books, 2014.
Pugalee, David K. *Effective Content Reading Strategies to Develop Mathematical and Scientific Literacy*. Lanham, MD: Rowman & Littlefield, 2014.
Rashed, Roshdi. *Classical Mathematics from Al-Khwarīzmī to Descartes*. London, UK: Routledge, 2015.
Reamer, David. *Count Like an Egyptian: A Hands-On Introduction to Ancient Mathematics*. Princeton, NJ: Princeton University Press, 2014.
Ronau, Robert N., et al. *Putting Essential Understanding of Functions into Practice*. Reston, VA: The National Council of Teachers of Mathematics, 2014.
Rosenhouse, Jason. (Ed.) *Four Lives: A Celebration of Raymond Smullyan*. Mineola, NY: Dover Publications, 2014.
Sandifer, Edward C. *How Euler Did It Even More*. Washington, DC: Mathematical Association of America, 2015.
Scanlon, T, M. *Being Realistic about Reasons*. Oxford, UK: Oxford University Press, 2014.
Schwartz, Richard E. *Really Big Numbers*. Providence, RI: American Mathematical Society, 2014.
Shapiro, Stewart. *Varieties of Logic*. Oxford, UK: Oxford University Press, 2014.
Sklar, Jessica K., and Elizabeth S. Sklar. (Eds.) *Mathematics in Popular Culture: Essays on Appearances in Film, Fiction, Games, Television, and Other Media*. Jefferson, NC: McFarland & Co., 2012.
Smith, Donald J. *Bond Math: The Theory behind the Formulas*. 2nd ed. Hoboken, NJ: Wiley and Sons, 2014.
Smith, Gary. *Standard Deviations: Flawed Assumptions, Tortured Data, and Other Ways to Lie with Statistics*. New York, NY: Overlook Duckworth, 2014.
Smith?, Robert. (Ed.) *Mathematical Modeling of Zombies*. Ottawa, ON: University of Ottawa Press, 2014.

Smullyan, Raymond M. *A Beginner's Guide to Mathematical Logic*. Mineola, NY: Dover Publications, 2014.

Smullyan, Raymond M. *The Gödelian Puzzle Book: Puzzles, Paradoxes, and Proofs*. Mineola, NY: Dover Publications, 2014.

Stepanov, Alexander A., and Daniel E. Rose. *From Mathematics to Generic Programming*. Upper Saddle River, NJ: Addison-Wesley, 2014.

Sterrett, Andrew. (Ed.) *101 Careers in Mathematics*. Washington, DC: Mathematical Association of America, 2014.

Stewart, Ian. *Symmetry: A Very Short Introduction*. Oxford, UK: Oxford University Press, 2013.

Suzuki, Jeff. *Constitutional Calculus: The Math of Justice and the Myth of Common Sense*. Baltimore, MD: The Johns Hopkins University Press, 2014.

Taylor, Catherine S. *Validity and Validation: Understanding Statistics*. Oxford, UK: Oxford University Press, 2014.

Tribe, Keith. *The Economy of the World: Language, History, and Economics*. Oxford, UK: Oxford University Press, 2014.

Tubs, Robert. *Mathematics in 20th Century Literature and Art: Meaning, Form, Content*. Baltimore, MD: The Johns Hopkins University Press, 2014.

Usiskin, Zalman. *We Need Another Revolution: Five Decades of Mathematics Curriculum Papers*. Reston, VA: The National Council of Teachers of Mathematics, 2014.

Vinci, Thomas C. *Space, Geometry, and Kant's Transcendental Deduction of the Categories*. Oxford, UK: Oxford University Press, 2015.

Walker, Erica N. *Beyond Banneker: Black Mathematicians and the Paths to Excellence*. Albany, NY: SUNY Press, 2014.

Wilkins, Karin Gwinn. *Questioning Numbers: How to Read and Critique Research*. Oxford, UK: Oxford University Press, 2011.