Chapter One

Observations of Exoplanetary Atmospheres:
A Theorist’s Review of Techniques in Astronomy

1.1 THE BIRTH OF EXOPLANETARY SCIENCE

It is no exaggeration that the close of the twentieth century was a time of great discovery. For centuries, if not millennia, humanity speculated upon the existence of other worlds—it is an ancient question that transcends cultures and nations. We debated the existence of eight or nine planets\(^1\) (depending on if one counts Pluto) orbiting our Sun: four rocky planets (Mercury, Venus, Earth and Mars) and four gas/ice giants (Jupiter, Saturn, Neptune and Uranus), flanked by the asteroid and Kuiper belts and encompassed by the Oort cloud. In the mid-1990s, this Solar System-centric view was shattered with the first discoveries of exoplanets—planets orbiting stars other than our Sun, beyond the Solar System—first around a pulsar \[253, 254\] and a few years later around a Sun-like star \[166\]. In the intervening years, the confirmed discoveries of exoplanets have increased exponentially and—at the time of publication of this textbook—number in the thousands.

It appears as if Nature is infinitely more creative than us at forming these new worlds \[92\]. The reported discovery of 51 Peg b in 1995 was the first example of a hot Jupiter, a Jupiter-sized exoplanet orbiting its star at a small fraction of the distance \((\sim 0.01-0.1\text{ AU})\) between Jupiter and our Sun \((\approx 5\text{ AU})\) and experiencing intense starlight that causes its temperatures to reach \(\sim 1000\text{ K}\). It was also the first instance of many surprises, including the discovery that super Earths—exoplanets with radii between that of our Earth and Neptune, which do not exist in the Solar System—are common and that Nature has a talent for making closely-packed, multi-exoplanet systems \[145\] and circumbinary exoplanets \[51\]—those orbiting a pair of stars. These discoveries have forced us to rethink our hypotheses and views on how the Solar System and other exoplanetary systems formed. If one may make a statement that will stand the test of time, it is that Nature will undoubtedly continue to surprise and challenge us in the future.

What is the myriad of worlds Nature has formed in the broader Universe? What is the chemical inventory of these worlds? And how many of them are capable of, and are, harboring life? Planetary science, which is the study of

\(^1\)Intriguingly, Batygin & Brown \[15\] have suggested the existence of a yet undiscovered planet in the outer Solar System.

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our Solar System, will continue to advance, but it is clear that the answers to these questions will ultimately come from the data gathered by astronomers harnessing telescopes to scan the heavens for worlds orbiting other stars. As exoplanet scientists, we have to develop a working knowledge of the nature and types of data gathered by astronomers in order to construct models that may be confronted by the data—or, at least, have some relevance to them. It is the purpose of this chapter to summarize the various techniques used by astronomers to observe and study exoplanets and their atmospheres.

1.2 TRANSITS AND OCCULTATIONS

The transit method is the workhorse of detecting and characterizing exoplanets and has been described as the “royal road” of exoplanetary science [252]. It exploits the fact that for a population of exoplanet-hosting stars, some fraction of them have exoplanets that reside on nearly edge-on orbits, which causes a diminution of the total light from the system when the exoplanet passes in front of its star. This event may be recorded by a distant observer, i.e., an astronomer. If a sufficient number of stars are monitored, then a harvest of exoplanet transits may be reaped.

1.2.1 Basics: transit probability and duration

A first, natural question to ask is: how likely is it for an exoplanet to transit its star [22]? If we visualize half the angle subtended by the star ($\theta$), then it is given by

$$\tan \theta = \frac{R_*}{a},$$  \hfill (1.1)

where $R_*$ is the stellar radius and $a$ is the exoplanet-star separation in distance. Since we expect this angle to be small, we have

$$\theta \approx \frac{R_*}{a}. \hfill (1.2)$$

If we integrate over the entire celestial sphere, then we obtain a “celestial band” within which a distant observer may record a transit,

$$\int \int d\theta \ d\phi \approx \frac{4\pi R_*}{a}. \hfill (1.3)$$

The transit probability obtains from dividing the coverage of the celestial band by the total solid angle subtended (4\pi steradians),

$$P_{\text{transit}} = \frac{1}{4\pi} \int \int d\theta \ d\phi \approx \frac{R_*}{a}. \hfill (1.4)$$

For hot Jupiters, the transit probability is relatively high: about 5–50% (around Sun-like stars) [37]. This explains why these were among the first type of exoplanets found by the astronomers. By comparison, the transit probability of
Earth and Jupiter are 0.5% and 0.09%, respectively. Since the transit probability is linearly proportional to the stellar radius, it provides the motivation to hunt for smaller exoplanets around later-type stars [40, 41]. There is an understandable tension between planetary scientists, who are interested in the science of habitability, wishing to exclusively study Earth-like exoplanets and astronomers who worry about detecting high-probability events.

The second, natural question to ask is: if an exoplanet transits its star, how long does the event last? We expect that short transits are infeasible to observe, where “short” may mean that it is less than the typical cadence associated with an observing strategy or instrument. We may obtain a rough estimate by considering exoplanets that transit exactly at the equator of the star and reside on circular orbits. The circumference of the orbit is $2\pi a$. The diameter of the star is $2R_\star$, which implies that the fraction of the orbit during which the exoplanet transits is $R_\star/\pi a$. If the period of the orbit is given by $t_{\text{period}}$, then the transit duration is [252]

$$t_{\text{transit}} = \frac{R_\star t_{\text{period}}}{\pi a}. \quad (1.5)$$

We may eliminate $t_{\text{period}}$ for $a$ via Kepler’s third law (assuming the stellar mass far exceeds the mass of the exoplanet),

$$t_{\text{period}} = \frac{2\pi a^{3/2}}{(GM_\star)^{1/2}}, \quad (1.6)$$

and obtain

$$t_{\text{transit}} = 2R_\star \left(\frac{a}{GM_\star}\right)^{1/2}, \quad (1.7)$$

where $G$ is Newton’s gravitational constant and $M_\star$ is the mass of the star. It is worth noting that $\sqrt{GM_\star/a}$ is simply the circular speed. For a hot Jupiter, Earth and Jupiter, we have $t_{\text{transit}} \approx 1–4, 13$ and 30 hours, respectively. The weak scaling of $t_{\text{transit}}$ with $a$ means that even for a Jupiter-sized exoplanet located at $a = 100$ AU, one obtains $t_{\text{transit}} \approx 130$ hours, although this neglects the fact that $t_{\text{period}} \approx 10^3$ years and $P_{\text{transit}} \approx 0.005\%$.

Several considerations relegate these algebraic formulae to being accurate only at the order-of-magnitude level. If the exoplanet resides on an eccentric and/or inclined orbit, then correction factors on the order of unity need to be added to $P_{\text{transit}}$ and $t_{\text{transit}}$ [234]. More interestingly, if multiple exoplanets exist within a single system, then they exert mutual gravitational forces on one another and each orbit deviates from having a Keplerian period [60]. Specifically, these transit timing variations (TTVs) may be used to infer the masses of these exoplanets [3, 103].

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Astronomers have developed a spectral classification for stars, labeling them by the alphabets O, B, A, F, G, K and M, which inspired the mnemonic, “Oh be a fine girl/guy, kiss me.” O stars are early-type stars that live short lives ($\sim 10^6$ years), while M stars may last for the age of the Universe ($\sim 10^{10}$ years). Within each stellar type, there is a further subdivision by numbers. For example, an M0 star is an early-type red dwarf, while M9 is a late-type one.
1.2.2 Stellar density and limb darkening

The *ingress* and *egress* of the transit of an exoplanet are the moments when it first obscures the star and exits the transit, respectively. It turns out that the ingress and egress encode information about the density of the star. By assuming the star to be spherical,

\[ M_\star = \frac{4\pi}{3} \rho_\star R_\star^3 , \]

where \( \rho_\star \) is the stellar mass density, we may rewrite Kepler’s third law as [206]

\[ \rho_\star = \frac{3\pi}{G} \left( \frac{a}{R_\star} \right)^3 t_{\text{period}}^{-2} . \]

By observing repeated transits, one may measure the orbital period of the exoplanet. The quantity \( R_\star/a \) may be directly measured using transit photometry, as it is related to the duration of ingress/egress and the transit depth [206, 252].

As you will learn in Chapter 2, the *optical depth* (\( \tau \)) is the correct measure of whether any medium is transparent (\( \tau \ll 1 \)) or opaque (\( \tau \gg 1 \)). We see an “edge” to the Sun because we are detecting its \( \tau \sim 1 \) boundary. Since the optical depth is a wavelength-dependent quantity, the radius of the Sun varies with wavelength. Similarly, we expect the radii of stars to be wavelength-dependent. Furthermore, the two-dimensional projection, on the sky, of the star, which we term the *stellar disk*, does not have a sharp edge. A combination of these effects causes the phenomenon of *limb darkening*, which smooths the ingress and egress of the transit light curves and causes them to have rounded bottoms [158]. To correctly extract the radius of the exoplanet from the light curve requires that one account for limb darkening correctly.

1.2.3 Transmission and emission spectra

To lowest order, the radius of an exoplanet does not depend on wavelength. To the next order, it does because its atmosphere (if it has one) will tend to absorb starlight or its own thermal emission differently across wavelength [23, 205]. Let the transit radius of the exoplanet at some reference wavelength be denoted by \( R_0 \); let the transit radius at a general wavelength be \( R \), such that \( R = R_0 + \delta R \) and \( \delta R \) is the (small) deviation from the reference radius across wavelength. If we denote the change in transit depth by \( \delta_{\text{transit}} \), then it is [23]

\[ \delta_{\text{transit}} = \frac{\pi}{\pi R_\star^2} \left( \frac{R^2 - R_0^2}{R_\star^2} \right) \approx \frac{2R_0 \delta R}{R_\star^2} . \]

It is essentially the annulus of the atmosphere (\( 2\pi R_0 \delta R \)), seen in transit, divided by the area of the stellar disk (\( \pi R_\star^2 \)). The change in the transit radius is typically some multiple of the (isothermal) *pressure scale height* of the atmosphere, which is given by

\[ H = \frac{k_B T}{mg} , \]

where \( k_B \) is the Boltzmann constant, \( T \) is the temperature, \( m \) is the mass of a molecule, and \( g \) is the gravitational acceleration.
Figure 1.1: Transit light curves (top-left panel) of the hot Jupiter WASP-43b, measured at near-infrared wavelengths, and the corresponding transmission spectrum (top-right panel), taken using the Wide Field Camera 3 (WFC3) on the Hubble Space Telescope [132]. The spectral feature peaking at about 1.4 µm is attributed to the water molecule. Notice how the limb darkening of the star alters the widths of the transit light curves across wavelength, while the changing opaqueness of the atmosphere varies their depths. Also shown are the secondary-eclipse light curves (bottom-left panel) and the corresponding emission spectrum (bottom-right panel). Courtesy of Laura Kreidberg and Jacob Bean.
where \( k_B \) is Boltzmann’s constant, \( T \) is the temperature, \( m \) is the mean molecular mass and \( g \) is the surface gravity of the exoplanet. If we plug in typical numbers (\( T = 1000 \) K, \( g = 1000 \text{ cm s}^{-2} \)), then we obtain \( H \approx 400 \) km for a hydrogen-dominated atmosphere. For a hot Jupiter, we obtain \( \delta_{\text{transit}} \sim 10^{-4} \); for an Earth-sized exoplanet with the same pressure scale height, we obtain \( \delta_{\text{transit}} \sim 10^{-5} \). If we wish to characterize a twin of the Earth orbiting a Sun-like star, the change in transit depth drops to \( \delta_{\text{transit}} \sim 10^{-7} \), making it a formidable technological feat. A plausible way forward is to scrutinize exoplanets around smaller stars. For example, if the star had a radius 10\% that of the Sun, this would increase the transit depth by a factor of 100 for any atmosphere.

Generally, starlight filters through the atmosphere of a transiting exoplanet along a chord [62, 97]. The location of this chord, and thus the size of the transit radius, depends on wavelength. Specifically, \( \delta R \) and \( \delta_{\text{transit}} \) are wavelength-dependent quantities. By measuring \( \delta_{\text{transit}} \) across a range of wavelengths, one may constrain the composition of the atmosphere (Figure 1.4) [38]. A thought experiment illustrates this: imagine an exoplanetary atmosphere that is composed purely of water vapor. At some wavelengths, the water molecule absorbs radiation strongly and the transit radius of the exoplanet becomes larger. At other wavelengths, it is transparent to radiation and the transit radius becomes smaller. Scanning the transit radius across wavelength produces a transmission spectrum, which encodes the opacity function of the atmosphere and allows us to identify the constituent atoms and molecules. Figure 1.1 provides an example of such observations by astronomers. In the ultraviolet, transmission spectra probe the upper atmospheres of exoplanets and provide constraints on atmospheric escape [246].

A complementary and harder way of scrutinizing the atmosphere of an exoplanet is to measure its occultation or secondary eclipse (Figure 1.1). This occurs when the exoplanet is obscured by its star, which is usually a smaller effect than a transit. At this moment in time, the system shines only in starlight. Just before secondary eclipse, one may measure photons from the dayside of the atmosphere by substracting out the stellar contribution [39, 46]. The secondary eclipse depth is

\[
D_{\text{eclipse}} = \frac{F}{F + F_*} \approx \frac{F}{F_*},
\]

where \( F \) and \( F_* \) are the wavelength-dependent fluxes from the exoplanet and the star, respectively. To extract the secondary-eclipse spectrum (\( F \) as a function of wavelength) requires that we understand the stellar spectrum (\( F_* \) as a function of wavelength) and the peculiarities of the star (activity, flares, starspots, etc).

### 1.2.4 Geometric albedos

If one records the secondary eclipse depth in the range of wavelengths dominated by reflected starlight (usually in the visible or optical), then the geometric albedo
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Figure 1.2: Infrared phase curves of the hot Jupiters HD 189733b [121, 122] and HD 209458b [262]. Courtesy of Heather Knutson and Robert Zellem.

may be measured [208],

\[ A_g = D_{\text{eclipse}} \left( \frac{a}{R} \right)^2. \]  

(1.13)

It is the albedo of the exoplanet at zero phase angle. Chapter 2 describes its relationship to other types of albedo in more detail.

1.2.5 Phase curves

As the exoplanet orbits its star, it exposes different sides of itself to the astronomer, who may measure its light as a function of the orbital phase. Such a light curve is known as the phase curve and was first\(^3\) measured for the hot Jupiter HD 189733b [121]. It is a one-dimensional “map,” since the phase curve contains both longitudinal (east-west) and geometric information on the exoplanetary atmosphere (Figure 1.2). If phase curves are measured at different wavelengths, then the atmosphere is being probed at slightly different altitudes or pressures, thus providing limited two-dimensional information [236]. In rare cases, one may obtain both latitudinal and longitudinal information by sampling the transit light curve with high cadence at ingress and egress, a technique known as eclipse mapping [156]. The method requires high precision also in the

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\(^3\)To be fair, Harrington et al. [82] detected phase variations for the non-transiting hot Jupiter \(\nu\) Andromedae b in 2006.
Figure 1.3: Eclipse mapping of the hot Jupiter HD 189733b [50]. The phase curve and ingress/egress of a transit provide complementary information in two dimensions. Courtesy of Julien de Wit and Sara Seager.

measured orbital parameters (e.g., eccentricity), since their uncertainties may easily mimic the timing offsets associated with atmospheric flux variations [50].

1.2.6 Putting it all together: A wealth of information from transits and occultations

Figure 1.4 unifies the information we have discussed regarding transits and secondary eclipses into a single schematic. It focuses on exoplanets that are tidally locked and have permanent daysides and nightsides, but the principles highlighted in the schematic apply generally. Specifically, it describes how the peak offset of the phase curve is expected to occur at secondary eclipse in the absence of atmospheric circulation. Measuring a peak offset in the phase curve thus implies the presence of atmospheric winds [216].

1.3 RADIAL VELOCITY MEASUREMENTS

One often visualizes the planets of our Solar System orbiting a static Sun. In reality, the Sun and the planets orbit a common center of gravity. As the star wobbles about this center of gravity, its light is blue- or redshifted relative to the astronomer, enabling its radial velocity signal to be measured. This was how the first exoplanet orbiting a Sun-like star was discovered [166]. Specifically, the
Figure 1.4: Schematic depicting the phases of a transiting exoplanet, how this maps onto a one-dimensional phase curve and the influence of atmospheric dynamics [96].
velocity semi-amplitude is measured [150, 252],

\[ K_\star = \left( \frac{2\pi G}{t_{\text{period}}} \right)^{1/3} \frac{M \sin i}{(M_\star + M)^{2/3}} (1 - e^2)^{-1/2}, \]  

(1.14)

where \( M \) is the mass of the exoplanet and \( i \) and \( e \) are the inclination and eccentricity of its orbit, respectively.

The preceding expression describes the well-known degeneracy associated with radial-velocity measurements, which is that it is \( M \sin i \), and not \( M \), that is being measured. The stellar mass (\( M_\star \)) is usually inferred from models of stellar evolution. If the exoplanet transits its star, then \( \sin i \approx 1 \) and \( M \) may be measured. For a hot Jupiter, we have \( K_\star \sim 0.1 \text{ km s}^{-1} \), nearly an order of magnitude faster than a professional sprinter. For the 12-year orbit of Jupiter, this drops to \( K_\star \sim 10 \text{ m s}^{-1} \). For the Earth, we have \( K_\star \sim 10 \text{ cm s}^{-1} \). By comparison, humans tend to walk at speeds \( \sim 1 \text{ m s}^{-1} \). One may imagine that the technology needed to make these measurements is demanding as one moves towards lower masses.

Precise radial-velocity measurements may also be used to infer the tilt, between the rotational axis of the star and the orbital axis of the exoplanet, projected onto the plane of the sky. A star that rotates produces blueshifted light in the hemisphere rotating towards the line of sight of the astronomer; in the other hemisphere that is rotating away, the light is redshifted. If the axes of the star and the exoplanet are aligned, then as the exoplanet transits the stellar disk it first produces a deficit of blueshifted light, followed by an equal deficit of redshifted light (or vice versa). If the orbit of the exoplanet is misaligned with respect to the rotational axis of the star, then these deficits of blue- and redshifted light are unequal, which is known as the Rossiter-McLaughlin effect [167, 200]. It has been used to measure the spin-orbit misalignments of hot Jupiters [197, 241].

More recently, astronomers figured out that high-resolution spectrographs may also be used to study the atmospheres of exoplanets. Since these spectrographs are typically mounted on ground-based telescopes, astronomers have to apply clever techniques to subtract both the stellar and telluric lines from the total spectrum. The resulting ultra-high-resolution spectrum may be used to produce the transmission spectrum [257] or be cross-correlated with theoretical templates of various molecules to identify their presence or absence in the atmosphere of the exoplanet [227, 228].

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4Meaning the spectral lines associated with the Earth’s atmosphere.

5The resolution is defined as \( \lambda/\Delta \lambda \), where \( \lambda \) is the wavelength and \( \Delta \lambda \) is the increment in wavelength. In this instance, we are discussing resolutions \( \sim 10^5 \) compared to the \( \sim 10^2-10^3 \) usually encountered.
1.4 DIRECT IMAGING

As its name suggests, direct imaging aims to directly take a photograph (and spectrum) of an exoplanet and its atmosphere. This is a formidable challenge, since the light from the exoplanet versus the star is typically between one part in a million to a billion, depending on whether one observes in the visible or infrared range of wavelengths. It is easier to directly image an exoplanet when it is young and hot, flushed with its remnant heat of formation, in which case the contrast between it and its star may be as high as one part in ten thousand. A technique is needed to block out the light from the star such that the light of the exoplanet may be isolated and the emission spectrum $(F)$ may be measured. Such a feat was achieved for the four gas giants orbiting the HR 8799 star (Figure 1.5) [163, 164], which allowed water and carbon monoxide to be identified in its atmosphere.

Figure 1.5: The iconic image of the four young ($\sim 10^7$–$10^8$ years), gas-giant exoplanets orbiting the HR 8799 star, whose light has been suppressed [163, 164]. Courtesy of the National Research Council of Canada, Christian Marois and Keck Observatory.
atmosphere via spectroscopy [13, 128]. The disadvantage of direct imaging is that the radius and mass of the exoplanet cannot be directly measured, as they need to be inferred from spectral and evolutionary models and thus have model-dependent values [13, 154, 161].

1.5 GRAVITATIONAL MICROLENSING

The Polish and Princeton astronomer Bohdan Paczyński was the first to propose that unseen objects in the halo of our Galaxy may act as gravitational lenses, which cause the transient brightening of a background star [183]. He estimated the probability of such an event occurring to be one in a million, implying that a million stars need to be monitored in order for it to be detected. Such reasoning was later applied to the detection of exoplanets [159]. A foreground star lensing a background or source star would produce a smoothly rising and falling light curve, as the light from the latter becomes magnified when the stars move past each other. If an exoplanet is orbiting the foreground star, then it acts to distort this smooth curve in a manner analogous to astigmatism. The relatively high probability (∼ 10%) of detecting this effect arises from the numerical coincidence between the orbital distance of Jupiter-like exoplanets (a ≈ 5 AU) and the Einstein radius of Sun-like stars [75]. The Einstein radius is the characteristic length scale at which gravitational lensing occurs,

\[ l_{\text{Einstein}} \sim \left( \frac{GM_*l_{\text{lens}}}{c} \right)^{1/2}, \]  

and is the geometric mean of the Schwarzschild radius,

\[ l_{\text{Schwarzschild}} \sim \frac{GM_*}{c^2}, \]  

and the distance to the lens or foreground star (l_{lens}). Despite being an elegant technique, gravitational lensing is of utility only in detecting exoplanets and not in characterizing their atmospheres. Furthermore, the detection is unrepeatable and it is typically challenging to photometrically distinguish the lens star from the source star.

1.6 FUTURE MISSIONS AND TELESCOPES

Given the dynamism of the field of astronomy, it is somewhat pointless to try and exactly predict the types of space missions and telescopes that will be built and mobilized in the future, but one may perhaps describe the general investments that will be made by astronomers in their hunt for exoplanets. Regardless of its exact configuration, there is clearly a need for a space-based telescope that

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6Yes, you read it correctly: stars in our Galaxy are not static and appear to move with speeds ∼ 10–100 km s⁻¹.
will record spectra of exoplanetary atmospheres, from the ultraviolet to the infrared range of wavelengths, and perhaps build a statistical sample of them in order to search for trends in the data. From the ground, telescopes will feature ever bigger mirrors (or arrays of mirrors) and more sophisticated spectrographs with the goal of measuring light from ever fainter sources. Eventually, when the technology is available and mature, humanity may mobilize a fleet of space telescopes, flying in formation, to image a twin of Earth, using a technique known as interferometry.

\[7\] At the time of writing, a trio of space-based telescopes dedicated to exoplanet detection (CHEOPS, TESS and PLATO) were being designed and built for the main purpose of greatly increasing the sample size of confirmed exoplanets orbiting nearby, bright stars [98].