CHAPTER 1 · GRAVITY, A FIELD OF PHYSICS

"What distinguishes the language of science from language as we ordinarily understand the word? How is it that scientific language is international? . . . The super-national character of scientific concepts and scientific language is due to the fact that they have been set up by the best brains of all countries and all times. In solitude and yet in cooperative effort as regards the final effect, they created the spiritual tools for the technical revolutions which have transformed the life of mankind in the last centuries. Their system of concepts has served as a guide in the bewildering chaos of perceptions, so that we learned to grasp general truths from particular observations." —A. EINSTEIN, *Out of My Later Years*

Introduction

In this book, and the course that goes with it, we shall study the nature and methods of physical science. We shall do that by studying some parts of physics thoroughly and leaving out other parts to gain time for discussion. In the samples we study, you will learn many scientific facts and principles, some useful for life in general, others important groundwork for discussions in the course. To gain much from the course, you need to learn this "subject matter" thoroughly. In itself it may seem unimportant—such factual knowledge is easily forgotten,[1] and we are concerned with a more general understanding which will be of lasting value to you as an educated person—but we shall use the factual knowledge as a means to more important ends. The better your grasp of that factual knowledge, the greater your insight into the science behind it. And this course is concerned with the ways and work of science and scientists.

To begin by discussing scientific methods or the structure of science would be like arguing about a foreign country before you have visited it. So we shall plunge at once into a sample of physics—gravity and falling bodies—and later discuss the general ideas involved.

What to Do about Footnotes

You are advised to read a chapter straight through first, omitting the footnotes. Then reread carefully, studying both text and footnotes. Some of the footnotes are trivial, but many contain important comments relevant to the work of the course. They are not minor details put there with a twinge of conscience to avoid their being omitted altogether. They are moved out to make the text more continuous for a first reading. Often the footnotes wander off on a side issue and would distract attention if placed in the main text. Yet this developing of new threads itself shows the complex texture of scientific work; so at a second reading you should include the footnotes.

Falling Bodies

Watch a falling stone and reflect on man's knowledge of falling objects. What knowledge have we? How did we obtain it? How is it codified into laws that are clearly remembered and easily used? What use is it? Why do we value scientific knowledge in the form of laws? Try the following experiment before you read further. Take two stones (or books or coins) of different sizes. Feel how much heavier the larger one is. Imagine how much faster it will fall if the two are released together. You might well expect them to fall with speeds proportional to their weights: a two-ounce stone twice as fast as a one-ounce stone. Now hold them high and release them together. . . . Which are you going to believe: what you saw, what you expected, or "what the book says"?

People must have noticed thousands of years ago that most things fall faster and faster—and that some do not. Yet they did not bother to find out carefully just *how* things fall. Why should primitive people want to find out how or why? If they speculated at all about causes or explanations, they were easily led by superstitious fear to ideas of good and evil spirits. We can imagine how such people living a dangerous life would classify most normal occurrences as "good" and many unusual ones as "bad"—today we use "natural" as a term of praise and "unnatural" with a flavor of dislike.

This liking for the usual seems wise: a haphazard unregulated world would be an insecure one to live

[1] Once learned, it is easily relearned if needed later. Much of the difficulty of learning a piece of physics lies in understanding its background. When you understand what physics is driving at, the rules or calculations will seem sensible and easy.

in. Children emerge from the sheltered life of a baby into a hard unrelenting world where brick walls make bruises, hot stoves make blisters. They want a secure well-ordered world, bound by definite rules, so they are glad to have its quirky behavior "explained" by reassuring statements. The pattern of seeking security in order, which we find in growing children today, probably applied to the slower growing-up of primitive savages into civilized men. As civilization developed, the great thinkers codified the world—inanimate nature and living things and even the thoughts of man—into sets of rules and reasons. Why they did this is a difficult question. Perhaps some were acting as priests and teachers for their simpler brethren. Perhaps others were driven by childish curiosity—again a need to know definitely, born of a sense of insecurity. Still others may have been inspired by some deeper senses of curiosity and enjoyment of thinking—senses rooted in intellectual delight rather than fear—and these men might be called true philosophers and scientists.

You yourself in growing up run through many stages of knowledge, from superstitious nonsense to scientific sense. What stage have you reached in the simple matter of knowledge of falling objects? Check your present knowledge by actually watching some things fall. Take two different stones (or coins) and let them fall, starting together. Then start them again together, this time throwing both outward horizontally (Fig. 1-1). Then throw one outward

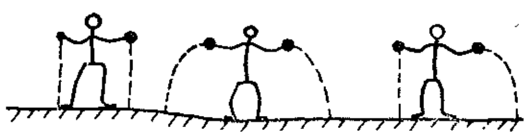

FIG. 1-1.

and at the same instant release the other to fall vertically. Watch these motions again and again. See how much information about nature you can extract from such trials. If this seems a childish waste of time, consider the following comments:

(i) This *is* experimenting. All science is built with information from direct experiments like yours.

(ii) To physicists the experiment of dropping light and heavy stones together is not just a fable of history; it shows an amazing simple fact that is a delight to see again and again. The physicist who does not enjoy watching a dime and a quarter drop together has no heart.

(iii) In the observed behavior of falling objects and projectiles lies the germ of a great scientific notion: the idea of *fields of force,* which plays an essential part in the development of modern mechanics in the theory of relativity.

(iv) And here is the practical taunt: if you use all your ingenuity and only household apparatus to try every relevant experiment you can think of, you will still miss some of the possible discoveries; this field of investigation is so wide and so rich that a neighbor with similar apparatus will find out something you have missed.

Mankind, of course, did not gather knowledge this way. Men did not say, "We will go into the laboratory and do experiments." The experimenting was done in daily life as they learned trades or developed new machines. You have been doing experiments of a sort all your life. When you were a baby, your bathtub and toys were the apparatus of your first physics laboratory. You made good use of them in learning about the real world; but rather poor use in extracting organized scientific knowledge. For instance, did your toys teach you what you have now learned by experimenting on falling objects?

Out of man's growing-up came some knowledge and some prejudices. Out of the secret traditions of craftsmen came organized knowledge of nature, taught with authority and preserved in prized books. That was the beginning of reliable science. If you experimented on falling objects you should have extracted some scientific knowledge. You found that the small stone and the big one, released together, fall together.[2] So do lumps of lead, gold, iron, glass, etc., of many sizes. From such experiments we infer a simple general rule: *the motion of free fall is universally the same, independent of size and material.* This is a remarkable, simple fact which people find surprising—in fact, some will not believe it when they are told it,[3] but yet are reluctant to try a simple experiment.[4]

[2] Yes; if you did not try the experiment, you now know the result of at least part of it. This is true of a book like this: by reading ahead you can find the answers to questions you are asked to solve. When you work on a crossword puzzle you would feel foolish to solve it by looking at the answers. In reading a detective story, is it much fun to turn to the end at once? Here you lose more still if you skip: you not only spoil the puzzle, but you lose a sense of the reality of science; you damage your own education. It is still not too late. If you have not tried the experiments, try them now. Drop a dime and a quarter together, and watch them fall. You are watching a great piece of simplicity in the structure of nature.

[3] Notice your own reaction to this statement: "A heavy boy and a light boy start coasting down a hill together on equal bicycles. In a short run they will reach the bottom together." The statement is based on the same general behavior of nature. See a demonstration. In a *long* run they gain high speeds and air resistance makes a difference.

The result *is* surprising. Would you expect a 2-pound stone to fall just as fast as a 10-pound one? Wouldn't it seem more reasonable for the 10-pound one to fall five times as fast? Yet direct trial shows that 1-pound, 2-pound, and 10-pound lumps of metal, stone, etc., all fall with the same motion.

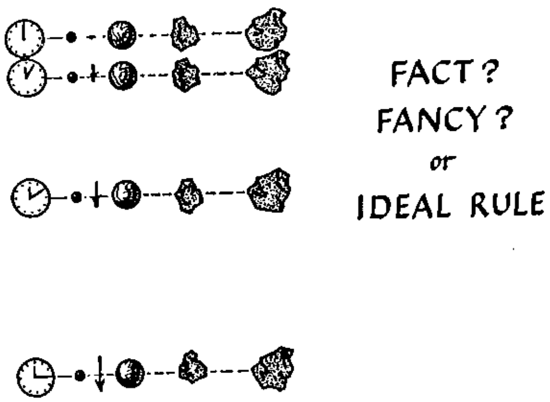

Fig. 1-2. Free Fall

Early Science of Falling Bodies

What is the history of this piece of scientific knowledge? There may have been a long gap between casual observation and careful experimenting. Interest in falling objects and projectiles grew with the development of weapons. Spears, arrows, catapults, and more ambitious "engines of war" favored a simple vague knowledge of ballistics. But that took the form of craftsmen's working rules rather than scientists' understanding—unspoken familiarity rather than extracted simplicity. Two thousand years ago the Greeks thought and wrote about nature with genuine scientific interest, possibly inspired by similar activity earlier still in Egypt and Babylon. They gave rules and reasons for falling, but not very reliable ones. Though some of the ancient scientists must have experimented sensibly with falling objects, medieval use of the Greek written tradition set down by Aristotle ($\sim$ 340 B.C.) clouded the matter rather than clarified it, and led to a muddle which lasted for many centuries. Gunpowder greatly increased the interest in projectiles, but early cannon were still chiefly used to frighten the enemy when Galileo ($\sim$ 1600) rewrote the science of ballistics in clear rules that agreed with experiment. Those were rules for heavy slow cannonballs that ignored air resistance. Since then, speeding up of projectiles has made air resistance more and more important, requiring modification of Galileo's simple treatment.

Aristotle and Philosophy

The great Greek philosopher and scientist Aristotle appears to have supported the popular idea that heavy things fall faster than light ones. Aristotle was a pupil of Plato and for a time the tutor of Alexander the Great. He founded a great school of philosophy and wrote many books. His writings were the authoritative sources of learning for centuries—through the dark ages when there were still no printed books but only handwritten ones copied and handed down by devout scholars in a rough and troubled world.

Why should philosophers be concerned with science? How is science related to philosophy? What *is* philosophy? Philosophy is not a weird high-brow scheme of impractical argument; it is *man's thinking about his own thoughts and knowledge*. Professional philosophy consists of criticizing knowledge, evolving systems of knowledge with rules of logic for critical argument. Philosophers are interested in questions of truth and nonsense, right and wrong, and in judgments of values. Just as professional physicians advise us on health, eating, sleeping, etc., so professional philosophers offer us advice on thinking and understanding, on all our intellectual activities. You and I indulge in amateur philosophy when we think intelligently about our life and its relation to the world around us, whenever we ask questions such as: "Is this really true?" "Does that really exist?" "What does it mean when I say something is right?" "Why is arithmetic right?" "Is happiness real or imaginary?" "Does a pin cause pain in the same sense that it causes a puncture?" Thinking about our place in the world is closely tied with our scientific knowledge of the world, so it is not surprising that the great philosophers studied science and influenced its progress. You cannot embark on science without a first step in philosophy. You need a philosophical assumption and a philosophical interest: you practically have to assume that there is an external world; and you have to wish to find out about it and "understand" it. And when you collect facts, formulate scientific laws, or invent theories, the philosopher in you will ask, "Are these *true*?" As you brood on that question you may change your view of science. When you have studied this course, you may not have settled a general philosophy, but you will have done some philosophical thinking and you will have started making your own Philosophy of Science.

[4] We all rely in many matters on authority embedded in home teaching or common sense; we are reluctant to risk disturbing our sense of security. If you do not believe this accusation, wait and you will presently catch yourself.

Aristotle inherited a general philosophical viewpoint from Plato. In trying to answer the question of ultimate truth and reality, Plato sheared away individual differences among the things we observe and extracted simple ideal forms. From dogs he extracted an ideal class, DOG; from all varieties of stones, an ideal STONE, and so on. Then he set forth the view that only these primary types or ideal forms really exist. These forms or essences remain universal and unchangeable, and individual examples of them are just shadows of the ideal. Aristotle used this insistence on classes of things as a basis for logical argument (If . . . , then . . . .). Yet, as a great observer and classifier of nature, he had to credit individual stones and dogs with some real existence; so his outlook was a compromise. Later stndents of his work gave increasing reality to ordinary objects, and came to treat the underlying classes as mental concepts or even mere names. This later view, that individual things are important and real, is a comfortable one for a scientist experimenting on objects and events in nature—he would like to feel that he is working with real things. Sir William Dampier seems to call some such view "nominalism" in the passage below, though modern philosophers use this name somewhat differently.

"Whatever be the truth of Plato's doctrine of ideas from a metaphysical point of view, the mental attitude which gave it birth is not adapted to further the cause of experimental science. It seems clear that, while philosophy still exerted a predominating influence on science, nominalism, whether conscious or unconscious, was more favourable to the growth of scientific methods. But Plato's search for the 'forms of intelligible things' may perhaps be regarded as a guess about the causes of visible phenomena. Science, we have now come to understand, cannot deal with ultimate reality; it can only draw a picture of nature as seen by the human mind. Our ideas are in a sense real in that ideal picture world, but the individual things represented are pictures and not realities. Hence it may prove that a modern form of [Plato's view of] ideas may be nearer the truth than is a crude nominalism. Nevertheless, the rough-and-ready suppositions which underlie most experiments assume that individual things are real, and most men of science talk nominalism without knowing it . . .

"The characteristic weakness of the inductive sciences among the Greeks is explicable when we examine their procedure. Aristotle, while dealing skilfully with the theory of the passage from particular instances to general propositions, in practice often failed lamentably. Taking the few available facts, he would rush at once to the widest generalizations. Naturally he failed. Enough facts were not available, and there was no adequate scientific background into which they could be fitted. Moreover, Aristotle regarded this work of induction as merely a necessary preliminary to true science of the deductive type, which, by logical reasoning, deduces consequences from the premises reached by the former process."[5]

While Aristotle may be regarded as having given *experimental* science a strong push forward, Plato was perhaps nearer to the modern *theoretical* physicist with his insistence on the importance of underlying general forms and principles. As a tool for his thinking, Aristotle developed a magnificent system of formal logic, that is, cast-iron argument that starts from admitted facts or agreed assumptions and draws a compelling conclusion. In treating science he first tried to extract some general scientific principles from observations, a process we call *induction*. Then he reasoned logically from these principles to deduce new scientific knowledge. His system of logic was itself a magnificent discovery, but it cramped the development of early experimental science by directing too much attention to argument. It has influenced the growth of our civilization profoundly. Most of us never realize how much our pattern of thinking has been influenced by the age-long tradition of Aristotelian logic, though many thinkers today question its rigid simplicity. It argued from one absolute yes or no to another absolute yes or no; argued with good logic to a valid conclusion, provided the starting-point was valid. "Is every man mortal?" "Do 4 times 3 make 14?" "Do 2 plus 2 make 4?" "Do all dogs have 7 legs?" We answer any of these with an absolute "yes" or "no" and then deduce answers to questions such as, "Is Jones mortal?" "Does my terrier have 7 legs?" But try the following:

"Is self-sacrifice good?"

"Was Lincoln a success?"

"Is my Boyle's law experiment right?"

These are important questions, but we can make fools of ourselves by insisting on a yes or no answer. If instead we spread our judgments over a wider scale of values, we may lose some "logic" but gain greatly in intellectual stature. It is well to beware of people who try to dissect every problem or dis-

[5] Sir William Dampier in *A History of Science* (4th edn., Cambridge University Press, Cambridge, Eng., 1949), pp. 34-35, from which some of this discussion is drawn.

cussion into components that have an absolute yes or no.

Aristotle's logic was safe as far as it went; modern logicians regard it as restricted and unfruitful but "true."[6] The damage to your thinking and mine comes from centuries of medieval scholarship drawing blindly and insistently on his writings—"the ingrown, argumentative, book-learned, world-ignorant atmosphere of medieval university learning." That medieval Aristotelian tradition is built into today's language and thought, and people often mistakenly require an absolute yes or no. For example, people trained to think they must choose between complete success and complete failure are heartbroken when they find they cannot attain the impossible goal of complete success. We are all in danger: students in college, athletes in contests, men and women in their careers, older people reviewing their life—all face terrible discouragement or worse if they demand absolute success as the only alternative to failure. Fortunately, many of us achieve a wiser balance; we stop judging ourselves by an absolute yes or no and enjoy our own measure of success. We then find the conflicting mixture of our record easier to live with.[7]

In science, where simple logic once seemed so safe, we are now more careful. Asked whether a beam of light is a wave, we no longer assume there is an absolute yes or no. We have to say that in some respects it is a wave and in others it is not. We are more cautious about our wording. Remembering that our modern scientific theory is more a way of regarding and understanding nature than a true portrait of it, we change our question, "*Is* it a wave?" to "Does it *behave as* a wave?" And then we can answer, "In some circumstances it does, in other circumstances it does not." Where an Aristotelian would say an electron must be either inside a certain box or not inside it, we have to say we would rather regard it as both! If you find such cautiousness irritating and paradoxical, remember two things: first, you have been brought up in the Aristotelian tradition (and perhaps you would be wise to question its strong authority); second, physicists themselves shared your dismay when experiments first forced some changes of view on them, but they would rather be true to experiment than loyal to a formality of logic.

Aristotle and Authority

Aristotle's chief interests lay in philosophy and logic, but he also wrote scientific treatises, summing up the knowledge available in his day, some 2000 years ago. His works on Biology were good because they were primarily descriptive. In his works on Physics he was too much concerned with laying down the law and then arguing "logically" from it. He and his followers wanted to explain *why* things happen and they did not always bother to observe *what* happens or *how* things happen. Aristotle explained why things fall quite simply: they seek their *natural place*, on the ground. In describing how things fall, he made statements such as these: ". . . just as the downward movement of a mass of lead or gold or of any other body endowed with weight is quicker in proportion to its size. . . ." ". . . a body is heavier than another which in equal bulk moves down more quickly. . . ." He was a very able man, discussing as a philosopher the *why* of falling, and he probably had in mind a more general survey of falling bodies, knowing that stones do fall faster than feathers, blocks of wood faster than sawdust. In the course of a long fall air-friction brings a falling body to a steady speed, and he probably referred to that.[8] But later generations of thinkers and teachers who used his books took his statements baldly and taught that "bodies fall with speeds proportional to their weights."

The philosophers of the Middle Ages grew more and more concerned with argument and disdained experimental tests. Most of the earlier writings on geometry and algebra had been lost and experimental physics had to wait until they were found and translated. For centuries, right on through the "dark ages," the authority of Aristotle's writings

[6] Roughly speaking, Aristotelian logic deals with classes of things, and its arguments can be carried out by machines, e.g., "electronic brains" in which "yes" or "no" is signified by an electron stream being switched "on" or "off." Modern logic deals with *relations* (such as ". . . larger than . . . ," ". . . better than . . .") as well as classes (such as "dogs," "mammals") and, nowadays, with implicational relations between complete propositions. Its arguments, too, can probably be carried out by machines, though that may be more difficult to arrange. But a machine cannot criticize the system of logic that it is asked to administer. Only man still thinks he can do that, making judgments of value.

For descriptions of machines see the following numbers of *Scientific American*:

Vol. 183, No. 5, "Simple Simon" (a small mechanical brain); Vol. 180, No. 4, "Mathematical Machines" (a detailed account of electronic calculators); Vol. 182, No. 5, "An Imitation of Life" (mechanical animals that learn); Vol. 185, No. 3, "Logic Machines"; Vol. 192, No. 4, "Man Viewed as a Machine" (excellent article by a philosopher); Vol. 197, No. 3, a complete issue on self-regulating machines.

[7] For a fuller discussion, see Ch. I of *People in Quandaries* by Wendell Johnson (Harper and Bros., New York, 1946).

[8] A denser body (or a bigger one) has to fall farther before approaching its limiting speed; and then that speed is much greater.

remained supreme, in a misinterpreted form at that. Simple people, like children, love security more than freedom; they will worship authority blindly, and swallow its teaching whole. You may smile at this and say, "We are civilized. We don't behave like that." But you may presently ask, "Why doesn't this book give us the facts and tell us the right laws, so that we can learn reliable science quickly?"; and that would be *your* demand for simple authority and easy security! We now condemn "Aristotelian dogmatism" as unscientific, yet there are still people who would rather argue from a book than go out and find what really does happen. The modern scientist is realistic; he tries experiments and abides by what he gets, even if it is not what he expected.

Logic and Modern Science

Wholesale appeal to Aristotle's logic may restrict our intellectual outlook, and medieval wrangling with it certainly hampered science; but logic itself is an essential tool of all good science. We have to reason inductively, as Aristotle did, from experiments to simple rules. Then we often assume such rules hold generally and reason deductively from them to predictions and explanations. Some of our reasoning is done in the shorthand logic of algebra, some of it follows the rules of formal logic, and some of it is argued more loosely.

In extracting scientific rules from old laws we trust the "Uniformity of Nature": we trust that what happens on Friday and Saturday will also happen on Sunday; or that a simple rule which holds for several different spiral springs will hold for other springs.[9] Above all, we rely on the agreement of other observers. That is what makes the difference between dreams and hallucinations on one hand and science on the other. Dreams belong to each of us alone, but scientific observations are common to many observers. In fact, scientists often refuse to accept a discovery until other experimenters have confirmed it.

Scientists do more than assume that nature is simple, that there are rules to be found; they also assume that they can apply logic to nature's ways. There lies the essential distinction that enabled science to emerge from superstition: a growing belief that *nature is reasonable.* As science grows, mathematics and simple logic play an essential part as faithful servants. The modern scientist puts them to more use than ever, but he goes back to nature for experimental checks. In a sense, the ideal scientist has his head in the clouds of speculation, his arms wielding the tools of mathematics and logic, and his feet on the ground of experimental fact.

[9] The obvious condition, "all other circumstances remain the same," is often difficult to maintain, and we blame many an exception to the Uniformity of Nature on some failure in that respect. Magnetic experiments in towns that have streetcars may give different results on Sundays, when fewer cars are running.

Greeks to Galileo

> "In studying the science of the past, students very easily make the mistake of thinking that people who lived in earlier times were rather more stupid than they are now."—I. BERNARD COHEN

Aristotle's authority grew and lasted until the 17th century when the Italian scientist Galileo attacked it with open ridicule. Meanwhile, many people must have privately doubted the Aristotelian views on gravity and motion. In the 14th century a group of philosophers in Paris revolted against traditional mechanics and devised a much more sensible scheme which was handed down and spread to Italy and influenced Galileo two centuries later. They talked of *accelerated motion,* and even of *uniform acceleration* (under archaic names), and they endowed moving objects with "impetus," meaning motion or momentum of their own, to carry them along without needing a force.

Galileo ($\sim$ 1600) was a great scientist. He started science advancing to a new level where critical thinking and imagination join with an experimental attitude—a partnership of theory and experiment. He gathered the available knowledge and ideas, subjected them to ruthless examination by thinking, experimenting, and arguing; and then taught and wrote what he believed to be true. He lost his temper with the Aristotelians when they disliked his teaching and disdained his telescope; and he wrote a scathing attack on their whole system of science, setting forth his own realistic mechanics instead. He cleared away cobwebs of muddled thinking and built his scheme on real experiment—not always his own experiments, more often those of earlier workers whose results he collected.

Thought-Experiments

In his books and lectures, Galileo often reasoned by drawing on common sense, quoting "thought-experiments." For example, he discussed the breaking-strength of ropes in this manner: suppose a rope 1 inch in diameter can just support 3 tons. Then a rope of double diameter, 2 inches, has four times the cross-section area (πr^2) and therefore four times as many fibers. Therefore, the rope of double diameter has four times the strength—it should support 12 tons. In general, STRENGTH must increase as DIAMETER². Galileo gave this argument and extended it to wooden beams, pillars, and bones of animals.[10] Some thought-experiments deal with simplified or idealized conditions, such as an object falling *in a vacuum.*[11]

Ideal Rules for Free Fall

Galileo realized that air resistance had entangled the Aristotelians. He pointed out that dense objects for which air resistance is relatively unimportant fall almost together. He wrote: ". . . the variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that in a fall of one hundred cubits a ball of gold would surely not outstrip one of copper by as much as four fingers. Having observed this, I came to the conclusion that, in a medium totally devoid of all resistance, all bodies would fall with the same speed."[12] By guessing what would happen in the imaginary case of objects falling freely in a vacuum, Galileo extracted ideal rules:

(1) All falling bodies fall with the same motion; started together, they fall together.

(2) The motion is one "with constant acceleration": the body gains speed at a steady rate; it gains the same addition of speed in each successive second.

Having guessed the rules for the ideal case, he could test them in real experiments by making allowances for the complications of friction.

Galileo's ? Experiment: Myth and Symbol

There is a fable that Galileo gave a great demonstration of dropping a light object and a heavy one from the top of the leaning tower of Pisa.[13] (Some say he dropped a steel ball and a wooden one, others say a 1-pound iron ball and a 100-pound iron ball.) There is no record of such a public performance, and Galileo certainly would not have used it to show his ideal rule. He knew that the wooden ball would be left far behind the iron one, but he said that a taller tower would be needed to show a difference for two unequal iron balls. He certainly tried rough experiments as a youth and knew as you do what does happen, but he did not suddenly turn the course of science with one fabulous experiment. He did accelerate the growth of real physics by refuting the Aristotelians' silly dogmatic statements. And he did start science on a new kind of growth by applying his simplifying imagination to experimental knowledge. These, and not the leaning tower, made him a landmark in scientific history. Many a myth is attached to great figures in history—stories about cherry trees, burning cakes, etc. Though scholars delight in debunking these anecdotes, they also use some of them to show how the people of the great man's day thought of him. The leaning tower story is not even credited with that advantage. Yet we might use it, quite apart from Galileo and the growth of science, as a symbol of a simple experiment. In your own experiment with unequal stones, they fell almost together, and not, as some people

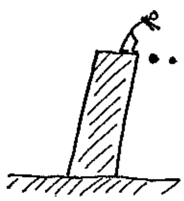

FIG. 1-3.

expect, the heavy one much faster. We shall use this Myth & Symbol in our course as a reminder of two things: the need for direct experiment, and a surprising, simple, important fact about gravity.

Honest Experimenting vs. Authority

Your own experiments did not show that all things fall together; they did not even show that large and small stones fall *exactly* together; and if in obedience to book or teacher you said, "They fall exactly together," you were cheating yourself of honest science. Small stones lag slightly behind big ones—the difference growing more noticeable the farther they fall. Nor is it simply a matter of different sizes: a wooden ball and a steel ball of the same size do not fall exactly together.

Once you accept Galileo's view that air resistance obscures a simple story, you can interpret your own observations easily—though that still leaves air re-

[10] See problems in Chapter 5.

[11] The Aristotelians had argued themselves into believing a vacuum to be impossible, so they cut themselves off from Galileo's satisfying simplification.

[12] From *Dialogues Concerning Two New Sciences*, by Galileo Galilei, English translation by H. Crew and H. de Salvio, Northwestern University Press, p. 72.

[13] Pisa. The leaning tower is a charming little building in a friendly Italian town. It is a round tower of white marble, built beside the cathedral. It began to lean as it was built, and it now has a remarkable tilt, about 5° from the vertical. The visitor who climbs its winding stair or walks around one of its open slanting balconies has strange sensations of shifting gravity. The tower was built long before Galileo's day, and he must have tried using it for some experiments. In his lifetime a pro-Aristotelian used the tower, to demonstrate *unequal* fall.

sistance to be investigated. Or you might pretend you had never heard Galileo's view, and proceed towards it yourself through a series of experiments with denser and denser objects. Finding the motion more and more nearly the same for larger or denser bodies, you might guess the rule for the ideal case. To examine the blame against air resistance, you might try streamlining, or reducing, using some object such as a sheet of paper.

Galileo's Guess: Newton's Crucial Experiment

Galileo could only *decrease* air resistance. He could not remove it completely, so he had to argue from real observations with less and less resistance to an ideal case with none. This intellectual jump, from real observations to an ideal case, was his great contribution. Then, looking back, he could "explain" the differences in real experiments by blaming air resistance. He could even study air resistance, codify its behavior, and learn how to make allowances for it. Not long after his time, air pumps were developed which enabled people to try free-fall experiments in a vacuum. Newton pumped the air out of a tall glass pipe and released a feather and a gold coin at the top. Even this extreme pair fell together. *There* was a crucial test of Galileo's guess.

Scientific Explanations

When we "explain" the differences of fall by air resistance, the term "explain" means, as so often in science, to point out a likeness between the thing under investigation and something else already known. We are saying essentially, "You know about wind resistance, when you move a thing along in the air. Well, the falling bodies experience wind resistance which depends in some way on their bulkiness. A wooden ball and a lead ball of the same size moving at the same speed would suffer the same air resistance—how could the air know or mind what is inside?—but the lead ball weighs more, is pulled harder by gravity, *so the air resistance matters less to it in comparison with the pull of the Earth.*"[14]

Further Investigation

The explanation leads to a whole new line of enquiry: wind resistance, fluid friction, streamlining—with applications to ballistics and airplane design—new science from more accurate study of some simple rule of behavior, from a study of its failures.

You could extend your series of experiments in the other direction, making more and more resistance, first with air, then with water, and find things of importance in the design of ships and planes. For simple experiments with fluid friction, try dropping small balls in water instead of in air. Balls of different sizes do not fall together. Moreover they fail to move any faster after a while in a long fall. Each ball seems to reach a fixed speed and then move steadily down at that speed. What is happening then? Investigations might lead you to Stokes' Law for fluid friction on a moving ball, a law which plays a vital part in measuring the electric charge of a single electron. If you investigated still smaller falling bodies, specks of dust or drops of mist, you would discover surprising *irregularities* in their fall, and these in turn could lead to useful information in atomic physics.

Galileo's experimenting and thinking, which you have been repeating, led to a simple rule that applies accurately to objects falling in a vacuum. For things falling in air, it applies with limited precision. In other words, the simple statement "all freely falling bodies fall together" is an artificial extract distilled by scientists out of the real happenings of nature. This is a good scientific procedure: first to extract a general rule, under simplified or restricted conditions, secondly to look for modifications or exceptions and then to use them to polish up the rule and to extend our knowledge to new things. In the case of falling bodies we can now test the extracted rule by dropping things in a vacuum. Ask to see Newton's "guinea and feather" experiment. In many cases in physics, however, we have to be content with knowing that our rule is an extracted simplification, believing in it as a sort of ideal statement, with only indirect evidence to justify our full belief.

Restricting the Number of Variables

Apart from ignoring air resistance, we have restricted our study of falling bodies in another way: we have concentrated attention on just one aspect of them, their comparative rate of fall. We have not observed what noise they make as they fall, or watched how they spin, or looked for temperature changes, etc. By narrowing our interests, temporarily, we have better hopes of finding a simple guiding rule. Again this is good scientific procedure. In many investigations we not only concentrate on a few aspects but even arrange to hold other aspects

[14] At this stage, the explanation ends in unsupported dogmatism that might be "straight out of Aristotle." Wait for studies of mass, force, and motion to make it good science.

constant so that they do not muddle the investigation. In physics we nearly always try to limit our investigation to one pair of variables at a time. For example, we compress a sample of air and measure its VOLUME at various PRESSURES, while we *keep the temperature constant*. Or we warm up the gas and measure the PRESSURE at various TEMPERATURES, while we *keep the volume constant*. From these experiments we can extract two useful "gas laws" that can be combined into one grand law. If we did not make restrictions but let TEMPERATURE and PRESSURE and VOLUME change during our experiments, we could still discover the grand law but our measurements would seem mixed and complicated—it would be harder to see the simple relationship connecting them. But other sciences such as biology and psychology, following the successful example of physics, have found this method very dangerous. While restricting attention to one aspect of growth or behavior, the investigator may lose sight of the body or mind as a whole. In attempts to apply the methods of natural sciences to social sciences such as economics this danger is even more severe.

Why Do Things Fall?

Aristotle was concerned with the answer to "Why?" Why *do* things fall? What is *your* answer to the question? If you say, "because of gravitation or gravity," are you not just taking refuge behind a long word? These words come from the Latin for heavy or weighty. You are saying, "things fall because they are heavy." Why, then, are things heavy? If you reply "because of gravity," you are talking in a circle. If you answer, "because the Earth pulls them," the next question is, "How do you know the Earth goes on pulling them *when they are falling*?" Any attempt to demonstrate this with a weighing machine during fall leads to disaster. You may have to answer, "I know the Earth pulls them because they fall"—and there you are back at the beginning. Argument like this can reduce a young physicist to tears. In fact, physics does not explain gravitation; it cannot state a cause for it, though it can tell you some useful things about it. The Theory of General Relativity offers to let you look at gravitation in a new light but still states no ultimate cause. We may say that things fall because the Earth pulls them, but when we wish to explain why the Earth pulls things all we can really say is, "Well it just *does*. Nature *is* like that."[15] This is disappointing to people who hope that science will explain everything, but we now consider such questions of ultimate cause outside the scope of science. They are in the province of philosophy and religion. Modern science asks *what?* and *how?* not the primary *why?* Scientists often explain why an event occurs, and you will be asked "why . . . ?" in this course; but that does not mean giving a first cause or ultimate explanation. It only means relating the event to other behavior already agreed to in our scientific knowledge. Science can give considerable reassurance and understanding by linking together seemingly different things. For example, while science can never tell us what electricity *is*, it can tell us that the boom of thunder and the crack of a man-made electric spark are much the same, thus removing one piece of fearful superstition.

Aristotle's explanation of falling was: "The natural place of things is on the ground, therefore they try to seek that place." People today call that a silly explanation. Yet it is in a way similar to our present attitude. He was just saying, "Things *do* fall. That's *natural*." He carried his scheme too far however. He explained why clouds float upward by saying that *their* natural place is up in the sky and thus he missed some simple discoveries of buoyancy.[16] Aristotle was much concerned with stating the "natural place" and "natural path" for things, and he distinguished between "natural motion" (of falling bodies) and "violent motion" (of projectiles). He might have produced good science of force and motion except for a mistake of applying common knowledge of horses pulling carts to all motion. If the horse exerts a constant pull, the cart keeps going with constant speed. This probably suggested Aristotle's general view that a constant force is needed to keep a body moving steadily; a larger force maintains larger speed in proportion. This is a sensible explanation for pulling things against an adjustable resisting force, but it is misleading for falling bodies and projectiles. In all cases it forgets the resisting force is there and prevents our seeing what happens when there is no resistance.

To explain the motion of projectiles, the Greeks

[15] Parents often give answers such as, "Well it just *is*" or "because it *is*" to children's questions. Such answers are not so foolish as they sound. For a child they provide the reply that is really needed at that stage, an assurance that everything is normal, that the matter asked about is a part of a consistent world. When a child asks "Why is the grass green?" he does not want to have a lecture on chlorophyll. He merely wants to be reassured that it is o.k. for the grass to be green.

[16] Buoyancy affects falling objects. When a thing falls in water its effective weight is lessened by buoyancy, and this makes falling in water quite different for different objects. Even air buoyancy has some effect, trivial for cannon balls, overwhelming for balloons.

imagined a "rush of air" to keep them going; and even more mysterious agents were required to keep the stars and planets moving. On their view—shove is needed to maintain motion—an arrow was kept moving by the push of the bowstring until it left the bow. After that another pushing agent had to be invoked to keep the arrow moving. Aristotelian philosophers imagined a rush of air pushing the arrow, not just a gust of wind travelling with it, but a circulation of air, with the air ahead of the arrow being pushed aside and running around to shove the arrow from behind. This rush of air satisfactorily prevented the unthinkable vacuum forming behind the arrow.[17] So firmly established was the idea of a rush of air, with embellishments of initial commotions, that it was used as an argument to show that projectiles could not move in a vacuum. "In a vacuum with zero resistance any force would maintain infinite velocity," argued the Greeks, "therefore a vacuum is impossible." God could never make a vacuum. Aristotle himself understood that all things would fall equally in a vacuum, but he considered that too a proof that a vacuum could not exist.

Mass

Whatever gravity really is, falling bodies do fall together except for effects of air resistance. This hints at a useful idea which we shall meet again and again: the idea of mass. Suppose we have a 2-pound lump of lead and a 1-pound lump. When we hold them, we feel the big lump being pulled more; we feel its greater weight. That is why we expect it to fall faster. Yet it does not, so there must be some other factor involved, something that the doubled weight-pull has to contend with. The reason is: *there is twice as much lead to be moved.* The double chunk needs twice the pull to give the same motion to its double quantity of lead. Galileo felt his way towards this idea of quantity of matter, which we shall call mass, but it had to wait for Newton to state it clearly. Mass is not an easy idea to grasp, but we shall return to it many times, because it plays a very important part in physics. At this stage, the amazing thing about gravitation is this: whatever the materials, gravitational pulls are exactly proportional to the amounts of stuff being pulled. Gravity, the mysterious pulling agent, seems to be ready to pull indiscriminately on any body, whatever it is made of, ready to pull just twice as hard on two bricks as on one, four times as hard on 4 cubic feet of lead as on 1 cubic foot, so that the object with more stuff in it to be pulled on has just the bigger pull needed to make it fall with the same motion as a smaller object.

Gravitational Field

We give a name to this state of affairs all around us of gravity-prepared-to-pull. We say there is a *gravitational field.* We are not really explaining anything by inventing a new term,[18] but we shall find it useful later. For the moment you should think of a gravitational field as waiting to clutch (proportionally) on any piece of matter put there, to pull it down towards the Earth, to make it fall. Near a magnet there is a similar state of affairs for bits of iron, a *magnetic field* waiting to clutch them. In your television tube, *electric fields* and *magnetic fields* clutch the whizzing electrons, speed them up, and swing the beam to and fro to sketch the picture.

Here we have been letting our thoughts run away with new words and ideas, such as MASS and FIELD, arising from simple experimenting. If we just worship such new ideas and phrases we are liable to fall back to a state of witchcraft. But if we use them to develop our knowledge, and if we put our suggestions to experimental tests, they may help the progress of science.

Galileo's Argument

Galileo was a great arguer. The Aristotelians had woven a web of "scientific" arguments based on Aristotle's statements, but Galileo beat them with their own weapons. An argument would upset them more than an experimental demonstration. So he revived a thought-experiment which ran thus: Take three equal bricks, A, B, C. Release them together, to fall freely. Now chain A and B together (by an invisible chain which is not really there) so that they form one object A + B which is twice as heavy as C.

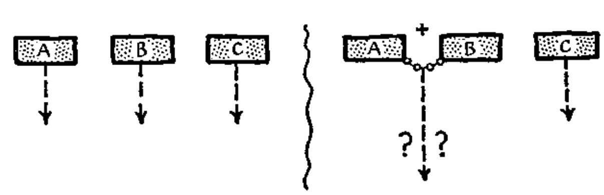

FIG. 1-4. GALILEO'S THOUGHT-EXPERIMENT

Again release them. The Aristotelians would now expect A + B to fall twice as fast as C; yet since it

[17] For a fuller and very interesting account of these views on motion, see H. Butterfield, *The Origins of Modern Science* (New York, 1952), Ch. 1.

[18] Cf. the great use in psychology or biology of special words such as "repression," "complex," "heliotropism," etc. Such new terms, coined or adopted for scientific use, cannot explain things, but they can aid clear thinking and discussion.

is really only two separate bricks it will fall just as before, at the same rate as C. Therefore, the double brick A + B and the single one C must fall together. "Ah no," says the Aristotelian in the argument, "there is the chain that joins A and B. One of the bricks will somehow get a little ahead of the other and then it will drag the other downward, making the combine fall faster." "I see," says the Galilean spokesman, "then the other, being a little behind, drags the first one back, making the combine fall slower!" Can you see in the comparison of A + B and C, the germ of the idea of mass?

The Motion of Free Fall

If all freely falling bodies have the same motion, that motion itself is worth detailed investigation. It might tell us something about nature in general, something common to all falling things. We can see that falling bodies move faster and faster; they accelerate. (This is merely a word meaning "move faster"—using it does not make our statement more scientific.) Just what kind of accelerated motion do they have?

(1) Does the SPEED increase by sudden jumps? Experiment says no.
(2) Does the SPEED increase in direct proportion to the DISTANCE TRAVELLED? Galileo devised an ingenious argument to show that this is very unlikely.[19]
(3) Does the SPEED increase in direct proportion to the TIME?
(4) or to the (TIME)^2,
(5) or in some other, more complicated manner?

Since we are asking a question about real nature, only experiments on real nature can answer it. (If you want to know how tall Abraham Lincoln was you must find out from someone who actually measured his height. Information from books is useless unless it came originally from real measurements. Algebra alone cannot possibly tell you.) We might go straight to the laboratory and experiment wildly and boldly, hoping to extract the essential story from a host of measurements. Or we might do some thinking first, guess cunningly at some simple types of motion, calculate the consequences of each, and then go into the laboratory and experiment on the consequences. Both methods have contributed to the growth of science.

Inductive and Deductive Methods

The first method is named the *inductive* method. We gather information either in a laboratory or from the accumulated lore of some trade; then we extract from it some simple rule, or story about nature. We call this extracting process "inductive inference" or simply "induction." We first gather experimental data and then infer some general rule or scientific law from the data. For example, after watching the Moon for some years an observer might extract the general rule that the Moon travels around the Earth regularly, about 13 times a year, and it might seem a safe inference by induction that this will continue. Again, from an extensive record of eclipses, we might *infer inductively* a rule that eclipses of the Moon run in several regular series, with a fixed time-interval of about 18 years between successive eclipses in any one series.

The second method is named the *deductive* method. We start with some general rules or ideas then derive particular consequences or predictions from them by logical argument. If we are scientists, we then test the predictions experimentally. If experiment confirms the predictions, we continue our scheme. If it disagrees with our predictions, we throw doubt on our original assumptions and try to modify them. For example, we might assume that eclipses of the Moon are due to the Earth getting in the Sun's light, casting a shadow on it; assume simple orbital motions for Sun and Moon, and then *infer deductively* (or *deduce*) that an eclipse must occur again after an interval of time sufficient for Sun *and* Moon to return to the same positions relative to the Earth. This interval must be the "lowest common multiple" of one Moon-month and one Sun-year. So, by combining simple observations with sensible assumptions we could make a striking deductive prediction of the 18-year repeat cycle of eclipses. (For a successful calculation, the "Sun-year" must be a special, short year geared to the Moon's changing orbit.)

As Lancelot Hogben points out:

"Readers of crime fiction will be familiar with two types of detectives. One adopts the card index method of Francis Bacon, collecting all rele-

[19] Galileo's argument was ingenious but not quite sound. It ran thus: "Compare two trips, each starting from rest, trip A of a certain distance, trip B of twice that distance. Then if speed increases in proportion to distance travelled, the speeds at corresponding stages (half way, ¾ way, etc.) of trip B are twice those of A. Then the double trip, B, is travelled with doubled speeds. Therefore B would take the same total time as A; which is absurd." But this argument supposes that the motion could start from rest. A sound version of the argument requires calculus to show that such a motion could never start from rest. Given a start, however, such a motion would continue in an ever increasing rush, its speed growing with compound interest.

vant information piece by piece. The other follows a hunch, like Newton, and, like Newton, abandons it at once when it comes into conflict with observed facts. From time to time the philosophers of science emphasize the merits of one or the other, and write as if one or the other were the true method of science. There is no one method of science. The unity of science resides in the nature of the result, the unity of theory with practice. Each type of detection has its use, and the best detective is one who combines both methods, letting his hunch lead him to test hypotheses and keeping alert to new facts while doing so."[20]

And here is an overall view, from a leading American physicist, P. W. Bridgman:

"I like to say that there is no scientific method as such, but that the most vital feature of the scientist's procedure has been merely to do his utmost with his mind, *no holds barred*."[21]

Accelerated Motion: Inductive and Deductive Treatment

Much of the *early* growth of science was made by induction; general laws were inferred from the knowledge gained in crafts and trades. In a simple way we have treated falling bodies inductively, inferring from many observations a general statement that all bodies falling freely in a vacuum fall together. When Galileo studied the details of this falling motion, he probably used a mixture of two approaches. He was good at making guesses, and he used geometry and reasoning powerfully.

We shall now follow the second method, deduction, in our study. We shall start by *assuming* a likely rule, and then we shall make a test comparing its consequences with real falling motion.

We choose guess (3) above and *assume that a falling body gains speed steadily, gaining equal amounts of speed in equal stretches of time.* We can express this more conveniently if we give a definite meaning to the word *acceleration*, so that we can say "the acceleration is constant." Therefore, we give the name ACCELERATION to

$$\frac{\text{GAIN OF SPEED}}{\text{TIME TAKEN}} \text{ or RATE-OF-CHANGE OF SPEED}$$

In making this definition of acceleration, we are really *choosing* the thing (GAIN OF SPEED)/(TIME TAKEN) to work with, and then giving it a name. We are not discovering some true meaning which the word acceleration possessed all along! We make this choice and assign it a name because it turns out to be useful in describing nature easily.

We shall start using the grander word *velocity* instead of *speed*, and presently we shall make a distinction between their meanings. Since we shall often deal with changing things, we want a short way of writing "change of . . ." or "gain of . . . ." We choose the symbol Δ, a capital Greek letter D, pronounced "delta." It was originally used to stand for the *d* of "difference." Then our definition[22] of acceleration states that:

$$\text{ACCELERATION} = \frac{\text{GAIN OF VELOCITY}}{\text{TIME TAKEN}} = \frac{\text{CHANGE OF VELOCITY}}{\text{CHANGE OF TIME-OF-DAY}}$$

$$a = \frac{\Delta v}{\Delta t}$$

where a, v, and t are obvious shorthand.

Deductive Treatment of Motion with Constant Acceleration

Now we express our assumption about falling bodies in this new terminology. We are *assuming* that:

$\frac{\Delta v}{\Delta t}$ is *constant*, for bodies freely falling (in vacuum). This states a huge assumption regarding real nature. Is it true? Is $\Delta v/\Delta t$ constant? To test this directly we should need an accelerometer to measure the acceleration of a body, $\Delta v/\Delta t$, at each stage of its fall. Such instruments are manufactured, but they are complicated gadgets which would not provide convincing proofs at this stage. Instead, we follow Galileo's example and ask mathematics, the logical machine, to grind out a consequence of our assumption, and then we test the consequence by experiment. The machine tells us that:

IF the acceleration a $(= \Delta v/\Delta t)$ is constant, and s is the distance travelled in time t with this constant acceleration, THEN

$s = \frac{1}{2}at^2$, if the motion starts from rest

$s = v_0t + \frac{1}{2}at^2$, if the motion starts with velocity v_0 at the instant $t = 0$, when the clock is started.

(The logical argument of this *IF . . . THEN . . .*

[20] *Science for the Citizen* (Allen and Unwin, London, 1938), p. 747.

[21] "New Vistas for Intelligence" in *Physical Science and Human Values*, ed. E. P. Wigner (Princeton, 1947), p. 144.

[22] In calculus, VELOCITY, v, at an instant, is defined by $v = \frac{ds}{dt}$ and ACCELERATION, a, at an instant, is $\frac{dv}{dt}$ or $\frac{d^2s}{dt^2}$.

is given in Appendix A of this chapter.) In these relations, ½ a is a constant number, since we are assuming a is constant; so, for motion starting from rest,

DISTANCE = (constant number) (TIME)^2
or DISTANCE increases in direct proportion to TIME^2
or DISTANCE varies directly as TIME^2
or DISTANCE $\propto$ TIME^2, this being shorthand for any of the versions above.

For example, if a body moving with fixed acceleration falls so far in one second from rest, then it will fall four times as far in two seconds from rest, nine times in three seconds, and so on.

★ PROBLEM 7. A CHART OF ACCELERATED MOTION

(a) Suppose a beetle crawls home with a motion for which it is true that *distance* $\propto$ *time*^{2}. Starting from rest he travels $\frac{1}{4}$ of an inch in the first second. How far will he travel in 2 seconds from his start? in 3 secs? in 4, 5, 6 secs?
(b) Draw a line across a sheet of paper; mark a starting-point near one end, and mark a rough scale of inches on it. Make marks to show the beetle's position at the end of each second.

★ PROBLEM 8. A SIMPLER RULE

Galileo announced the relation $s \propto t^2$ for uniformly accelerated motion, (where s is the *total* distance travelled in total time t from rest); but he stated another simple rule for such motion, relating the *distances* $d_1, d_2, \ldots$ *covered during 1 second,* in successive one-second intervals: (that is, the distance travelled in the first second, the distance travelled during the next period of one second, &c.) *Look for such a rule in the example of Problem 7, and state it. (Hint:* Calculate $d_1 = s_1 - 0$, $d_2 = s_2 - s_1, \ldots$ and look for some rule relating these one-second distances.)

★ PROBLEM 9. SCIENTIFIC THINKING

(a) You might have foreseen the rule of Problem 8, by common-sense thinking about accelerated motion, without using special algebra or studying an actual example. Why? (*Hint*: the distance travelled in any period of one second is a measure of . . ? . . in that period.)
(b) Is the rule of Problem 8 restricted (like $s \propto t^2$) to motion starting from rest when $t = 0$, or does it apply to any motion with constant acceleration?

★ PROBLEM 10. ANALYZING MOTIONS

Here are the records of four cyclists, moving with different motions. They all passed a post P at the instant the clock was started. Their distances from P after 1 second, 2, 3, 4, 5 seconds were as follows:

Time from Start	*1 sec*	*2 sec*	*3 sec*	*4 sec*	*5 sec*	
Cyclist A	1.8	7.2	16.2	28.8	45.0	feet
Cyclist B	1.8	3.6	5.4	7.2	9.0	feet
Cyclist C	1.8	5.2	10.2	16.8	25.0	feet
Cyclist D	1.8	14.4	48.6	115.2*	225.0*	feet

* These are distances Cyclist D would have travelled if he could have continued his motion.

(a) Try analyzing each of these motions, *looking for constant acceleration*, not by asking if $s \propto t^2$ but in the light of answers to Problems 8 and 9 above.
(b) Where the motion does not have constant acceleration describe its general nature if you can.

Experimental Investigations

The converse can be shown to be true. IF the distance s varies directly as t^2, THEN the acceleration is constant.[23] That gives us a relation to test in investigating real motions. We can arrange a clock to beat equal intervals of time, and measure the distances travelled from rest by a falling body, in total times with proportions 1:2:3: . . . If the total distances run in the proportions 1:4:9: . . . and so on, we may infer a fixed acceleration. Or, as in one form of laboratory experiment, we can measure the time t for various total distances s, and test the relation $s =$ (constant number) (t^2) by arithmetic, or by graph-plotting.

Over three centuries ago Galileo used this method, though he had neither a modern clock nor graph-plotting analysis. Galileo was one of the first to suggest an accurate pendulum clock, but he probably never made one. All he used to measure time was a large tank of water with a spout from which water ran into a cup. He estimated times by weighing the water that ran out—a crude method yet accurate enough to test his law. However, free fall from reasonable heights takes very little time—the experiment was too difficult with Galileo's apparatus.[24] So he "diluted" gravity by using a ball rolling down a sloping plank. He measured the times taken to roll distances such as 1 foot, 2 feet, etc., from rest.

On the basis of rough experimenting and sturdy guessing, Galileo decided that a ball rolls down a sloping plank with *constant acceleration.* Believing that this would be true for *any* slope, and arguing from one slope to greater slopes and greater still, he expected it to hold for a vertical plank, that is for free fall.[25] The idea of constant acceleration had

[23] By calculus: if $s = kt^2$, then velocity $\frac{ds}{dt} = 2\,kt$;

and acceleration $\frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = 2k$, which is constant.

[24] Galileo's apparatus was rough. He used it to illustrate his argument rather than to measure acceleration.

[25] He convinced himself that the speed acquired by a body sliding down a frictionless incline depends only on the height, h, not on the length of slope, L. If so, a body falling freely through a vertical height, h, would acquire the same speed, since this would be like a vertical incline. Then he could argue safely from his experiments to vertical fall.

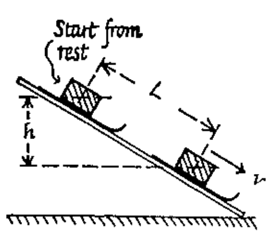

FIG. 1-5.

been suggested by earlier scientists—who were scorned for it. Galileo did his best to minimize friction, which threatened to complicate matters—though we now know that constant friction would not spoil the simple relationship. His results were rough, but seemed to convince him that his guess was right. It was the simplest kind of accelerated motion he could imagine, and he was probably influenced by the general faith, which has inspired scientists from the Greeks to Einstein, that nature is simple.

Later experiments, with improved apparatus, confirmed Galileo's conclusion: the motion *is* one with constant acceleration, i.e., with $\Delta v/\Delta t =$ constant, in all the following cases:

- for a ball or wheel rolling down a straight inclined plank;
- for a body sliding down a smooth inclined plank, or a truck with wheels running down it;
- for free fall.

Yet each such test has only shown that the acceleration is constant for that one set of apparatus, on that one occasion and within the limits of accuracy of that experiment. If as scientists we want to believe in a general rule inferred from these experiments, if we want to codify nature's behavior in a simple "law" as a starting point for new deductions, then we need a great body of consistent testimony as a foundation for our inference. The more the better, in quantity and variety, and no witness is unwelcome. If any experiment contradicts this general story—and some do—it thereby offers a searching test. "The exception proves the rule" is a fine scientific proverb—though often misunderstood—if "proves" means "tests" (as in "proving-grounds" for artillery, the "proving" of bank accounts). If "proves" had the modern common meaning of "shows it to be right" the proverb would be nonsense.[26] Exceptions do *not* show that the rule is correct. Exceptions do put a rule to fine tests and show its limitations. They raise the question "What is to blame?" and they lead either to limitations of the rule or to greater care in experimenting. Either way, the rule emerges more clearly established.

Experimental Tests in Lecture and Laboratory

Therefore you should see and make some tests of accelerated motion yourself. Not only will these make you feel that the experimental basis of science is more real, but they will enable you to add your assurance to the accumulated body of testimony. Galileo made little more than wise guesses; others have added careful measurements, and you should add your measurements and judgment.

Demonstration Experiment

We let a small truck run down a long sloping track, and make measurements to estimate its acceleration. It is not easy to measure speeds in a lecture experiment but rough estimates will suffice to show how the acceleration is derived.

We measure the truck's speed at some station, A, early in the run, and again at B farther down the track. The difference between these speeds gives us the gain of speed Δv. The time taken for this gain, Δt, is the time taken by the truck to travel from A to B. Then the acceleration is $\Delta v/\Delta t$. To measure Δt we equip the truck with a thin mast, M, and measure with a stopclock the time taken for M to travel from A to B.

To estimate the truck's speed at A we have to time it over a short run in the region of A. We might install a short billboard there, with its mid-point at

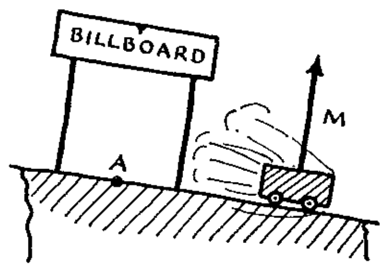

Fig. 1-6.

A, as in Fig. 1-6, and measure the time taken by the mast to run the length of the billboard. But human errors are inconveniently big for such short timings, so it is better to install the billboard on the moving truck and time its transit past A with the help of an electric eye (photocell). Fig. 1-7 shows a good arrangement. A lamp sends a beam of light across the track into the electric eye, where it produces a tiny electric current. The current is amplified and used to run an electromagnet. The electromagnet keeps an electric clock switched off. When the light is obstructed, the electromagnet releases the clock and lets it run. The truck carries a long strip of cardboard which obstructs the light while the truck carries it past. Thus the clock runs while the truck is passing the electric eye at A, and records the time

[26] The original legal meaning is amusing, but irrelevant here: "the quoting of an exception makes it clear that the rule exists."

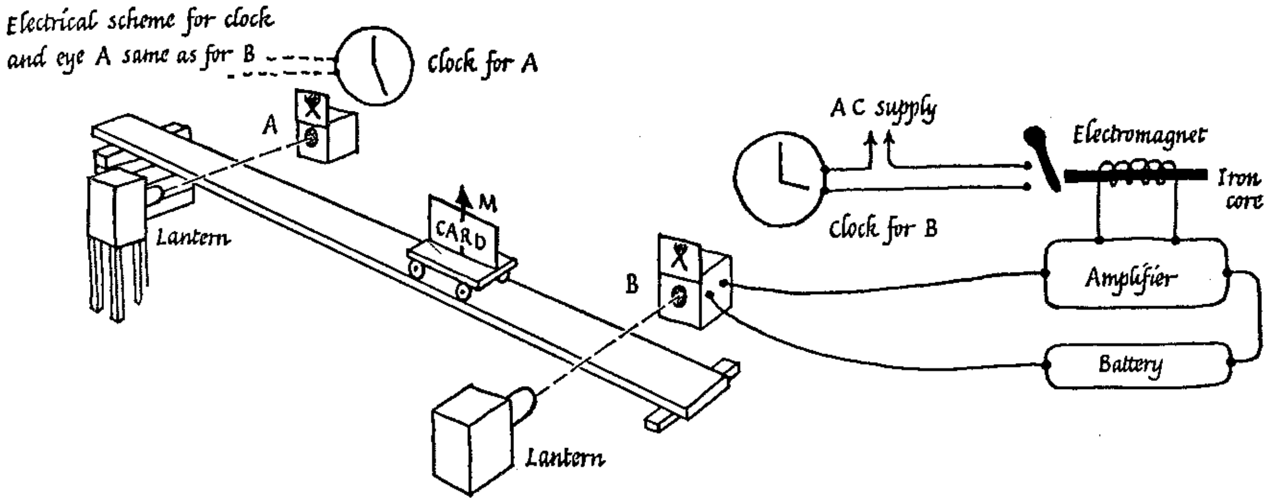

FIG. 1-7. EXPERIMENTAL ARRANGEMENT: measuring acceleration of model truck running down hill, with two electric eyes and three electric clocks. (Clock C for total travel-time not shown, operated by hand.)

taken for the card-length to travel past. In this time, the truck must travel the card-length. Then

CARD LENGTH/OBSTRUCTION TIME

gives the truck's speed.

We need three clocks, one to measure the total time between the two stations A and B where the speeds are estimated, one to measure obstruction time for the card passing A, and another for similar measurements at B. The following problem illustrates the calculation of the acceleration.

PROBLEM 11

Suppose the card on the truck is 2 feet long, and the obstruction at A takes 0.30 seconds. What is the truck's speed when running past A? If the obstruction at B takes 0.10 seconds, what is the truck's speed there? What is the gain of speed, Δv? If the truck takes 2.0 seconds to travel from A to B, what is its acceleration?

Nothing is said in Problem 11 about starting from rest. The truck is already moving when it passes A, and we can start it with any shove we like before that. So we can repeat the experiment with a variety of starting speeds. We can even start the car with an uphill shove so that it is moving backwards when it first passes A; but then we must be careful about + and − signs. The measurements tell us the acceleration whatever the starting speed. Whether the acceleration is the *same* for different starting speeds is a question about nature. To answer that you must see the real experiment.

In laboratory, you may experiment with a wheel rolling down sloping rails. You cannot easily measure the acceleration directly, or the (increasing) velocity. Instead you should measure DISTANCE TRAVELLED and TIME TAKEN, from rest, and then test whether they fit with the relation

DISTANCE varies directly as $(\text{TIME})^2$.

When you have collected reliable measurements, you should make the test both by arithmetic and by graph plotting.

Example of an Accelerated Motion Experiment

Meanwhile we shall proceed with a fictitious case of constant acceleration. Suppose measurements on a moving body gave the following results:

TABLE 1

DISTANCE TRAVELLED *from starting point* *(feet)*	TIME TAKEN FOR TRAVEL *(seconds)*			
0	0			
2	5.1	5.4	5.0	5.3
8	10.1	10.3	9.6	10.4
18	15.6	15.0	15.9	15.5

These measurements are too few and too poorly spaced for a good test, but will suffice for illustration. The four measurements 5.1, 5.4, 5.0, 5.3 are the result of four attempts to time the motion for 2 ft from start. Averaging is likely to remove some chance errors—though some errors may remain, such as the effect of impatient stopping of the watch too early. So we average these; we add them, and divide by 4:

$$\text{average time} = \frac{(5.1 + 5.4 + 5.0 + 5.3)}{4} = \frac{20.8}{4} = 5.2 \text{ secs}$$

Treating the other timings similarly we can make this table.[27]

TABLE 2

DISTANCE TRAVELLED *from starting point* *(feet)*	AVERAGE OF TIMINGS TIME OF TRAVEL *(secs)*
0	0
2	5.2
8	10.1
18	15.5

A glance at these numbers tells us that the times do not increase in proportion to the distances. Plotting the values on Graph (a) tells us the same thing. The graph shows clearly that the body is covering ground faster and faster, i.e., accelerating. It does not tell us whether the acceleration is constant.[28] To test that we plot a different graph, which will give a straight line *if* the acceleration is constant. We get a hint of what to plot by *assuming* constant acceleration and deducing DISTANCE $\propto$ TIME^2, which suggests we should plot DISTANCE against (TIME)^2. We make Table (3).

[27] A sensible experimenter in a real laboratory would save trouble by combining the two tables. He would leave a spare column for "average time" in his first table. If he foresaw the need for Table 3 he would leave another column for TIME^2. Even if he foresaw no need, an experienced experimenter would leave some blank colnmns, and blank lines below 18, for possible later use.

[28] We can make an indirect test by drawing tangents to the graph. See next section.

TABLE 3

DISTANCE TRAVELLED *from starting point,* *s* *(feet)*	AVERAGE OF TIMINGS, TIME OF TRAVEL *t* *(secs)*	(TIME OF TRAVEL)^2 t^2 *(secs)*^{2}
0	0	0
2	5.2	27
8	10.1	102
18	15.5	240

Then we plot Graph (b). To see whether the acceleration is constant, we draw a "best" straight line through the origin. We deliberately draw it straight, as a test, but we try to make it pass "as near as possible to as many as possible" of the plotted points. In this example, the points lie close to a straight line. If we think their displacements from the line are genuinely accountable by the incompetence of our apparatus, then we say that *so far as we can tell* from our measurements, the motion may well have constant acceleration.

Very Honest Graphing: Showing Likely Experimental Errors

If we wish to be more outspoken about our experimental uncertainties, we may spread each plotted point out into a patch to exhibit uncertainties of timing and distance measurement. Graph (c) in Fig. 1-10 shows this, with black points given by measurements surrounded by grey uncertainty patches. The timing is more risky than the distance measurement, so each patch is wider than it is tall.

Since we do not know how big our errors *are* but only how big they are *likely* to be, each patch should extend an indefinite distance out from its point; but we should show that the outer regions represent very unlikely errors. This might be done by shading

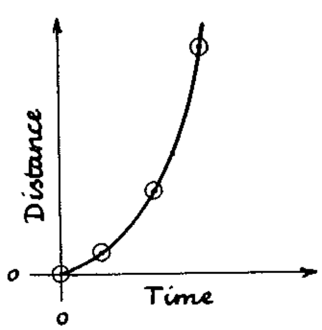

FIG. 1-8. GRAPH (a)

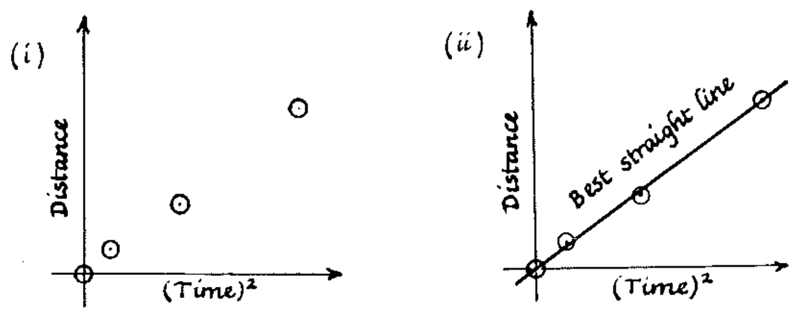

FIG. 1-9. GRAPH (b)

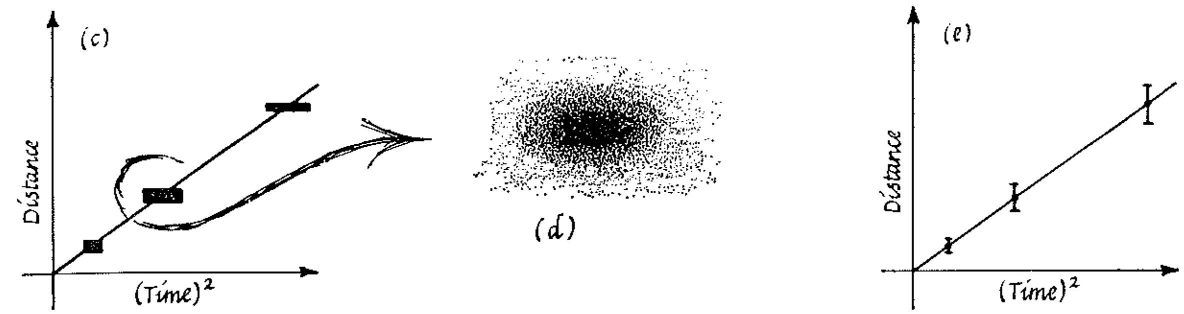

Fig. 1-10. Error boxes

the patch as in (d). Shading is too tedious, so the general custom is to mark a box of defiuite size such that the betting that the true value lies inside the box has some standard value—say 50-50 for and against. The dimensions of the box show the errors the experimenter is willing to admit to.

Professional scientists often show errors or uncertainties on graphs, but they lump them together and express horizontal and vertical uncertainties combined as an uncertainty of the thing plotted upward. The experimenter estimates his likely error, Δy, in the measurement plotted upward. He estimates his likely error in the horizontal plot, Δx, and then asks, "If I did make that error, Δx, how big an error in y at the same time would just compensate for it?" That tells him Δy^*, the equivalent of his Δx. He draws a vertical error-line running $(\Delta y + \Delta y^*)$ above and below his experimental point. Then each plotted point carries an uncertainty line as in (e).

Finding Velocity by Tangents

We could make a direct study of acceleration if we could plot a graph showing how *velocity* changes with time. For that we need estimates of velocities at various instants.

We can estimate velocities by drawing tangents to the curved graph of distance against time. If a tangent is drawn touching the cnrve at some point, the slope of the tangent gives the speed of the moving object at that time and place. To see why this is so, choose a point P on this curve, then move to a point Q a little farther up the curve corresponding to a time which is a little later. At P on the graph, the moving object has travelled a certain total distance in a certain total time. From P to Q it travels a small further distance, Δs, in a small further time Δt. Then the AVERAGE VELOCITY in the interval between stage P and stage Q is

$$\frac{\text{DISTANCE TRAVELED BETWEEN STAGE P AND STAGE Q}}{\text{TIME TAKEN BETWEEN STAGE P AND STAGE Q}}$$

$\therefore$ AVERAGE VELOCITY $= \dfrac{\Delta s_{PQ}}{\Delta t_{PQ}}$ see sketch (f)

= HEIGHT/BASE for the small triangle PQM

= HEIGHT/BASE for any similar big triangle

$= h/b$ in sketch (f)

= SLOPE, or HEIGHT/BASE, of chord joining PQ

If P and Q are very close, the line joining them is *very nearly* a tangent to the curve at a "point" PQ, and the velocity is still given by the slope of this "tangent." In the mathematical limit, when P moves up to coincide

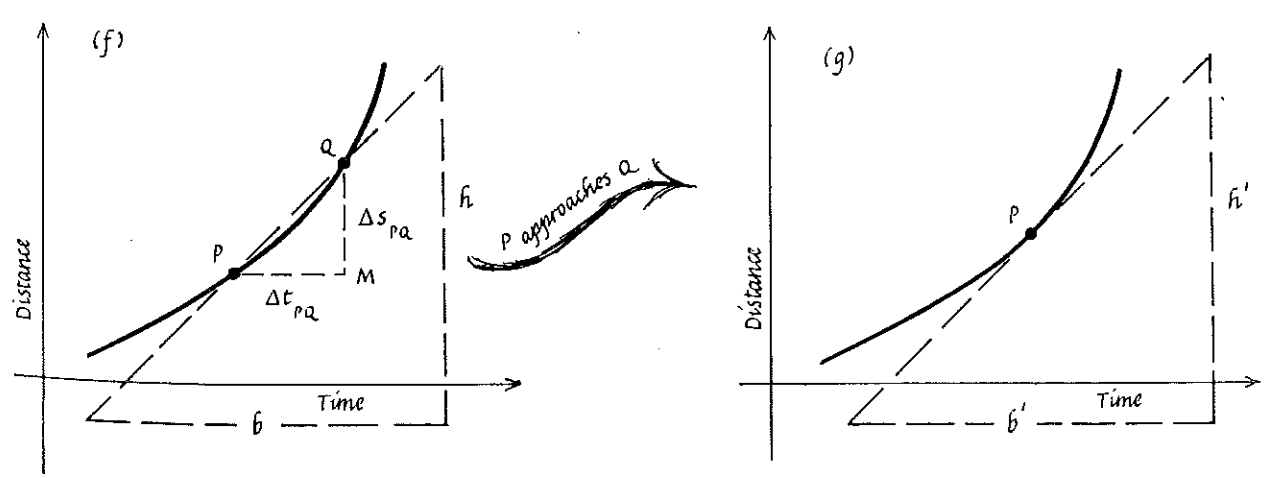

Fig. 1-11. Velocity = Slope of Tangent

with Q making a single point, the chord becomes a tangent at that point; Δs and Δt become zero, but the *ratio* $\Delta s/\Delta t$ still has a definite value, given by h'/b' for any large triangle having the tangent as sloping side, as in sketch (g). When PQ is a chord, the slope gives the *average* velocity in the motion from stage P to stage Q. In the limit, when P and Q coincide, the slope of the tangent gives the velocity at the instant of time represented by the single point P where the tangent is drawn. This is because the slope of the tangent is the same as the slope of an infinitely short piece of the curve which represents the motion at that point. By drawing tangents at a number of points on the curve and measuring their slopes we could obtain several velocities which could be plotted on a new graph showing VELOCITY *vs.* TIME. The shape of this new graph would tell us whether the acceleration is constant, but tangent-drawing is not easy and the original graph would have to be drawn very carefully, with many more plotted points to give a reliable set of tangents. So in practice we test for constant acceleration by plotting the second graph, DISTANCE against $(\text{TIME})^2$.

However, we can use this tangent-property to help us in drawing the original graph. Though our graph (a) passes through the origin, it is hard to see how the curve runs *near the origin*, since measurements of very short travels are difficult. We are not sure which of the possible curves in Fig. 1-12 is the true one. We can gain a

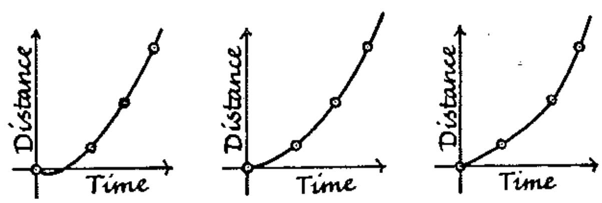

FIG. 1-12. VERSIONS OF GRAPH (a). Distance against Time

hint by arguing thus: the body started from rest, according to the record. Therefore, at the start its *speed* was zero. Therefore, at the origin of the graph the tangent-slope must be zero, the tangent horizontal. Of the three possible curves in Fig. 1-12, this argument tells us that the middle one is probably the true one.

Arithmetical Test of Constant Acceleration

We can also make a test in our fictitious experiment by arithmetic. If the acceleration is constant, then

$$\text{DISTANCE} = (\text{constant})\,(\text{TIME})^2.$$

Therefore, $\text{DISTANCE}/\text{TIME}^2 = \text{constant}$. And conversely, if $\text{DISTANCE}/\text{TIME}^2$ *is* constant, the acceleration is constant. So we enlarge our table with an extra column to investigate this.

To draw any fair conclusion from the numbers in the last column, we should need to know something about the accuracy of the measurements. Otherwise,

TABLE 4

DISTANCE TRAVELLED s (*feet*)	TIME OF TRAVEL t (*secs*)	(TIME OF TRAVEL)^2 t^2 (*secs*)^2	$\frac{\text{DISTANCE}}{\text{TIME}^2}$ $\frac{s}{t^2}$ *feet*/(*secs*)^2	
0	0	0	$\frac{0}{0}$	?
2	5.2	27	$\frac{2}{27}$	0.074
8	10.1	102	$\frac{8}{102}$	0.078
18	15.5	240	$\frac{18}{240}$	0.075

all we can say is "the motion seems to have a fairly constant acceleration."

Both the graph and the arithmetical test above suffer badly from having too few data. *However, this is only a fictitious example. The real test should be made in your own experiments.*

The work of many scientists, professional researchers and amateurs such as yourselves, has built up great faith in Galileo's discovery: *bodies falling freely under gravity, and bodies sliding or rolling down a straight slope, under diluted gravity, move with constant acceleration.*

Further experiments show that the acceleration has the same constant value even if the body does not start from rest but is given a push to start it. If it already has speed v_0 when the clock starts, then the simple relation $s = \frac{1}{2}at^2$ no longer holds; we must use $s = v_0 t + \frac{1}{2}at^2$. (See discussion in Appendix A.) But the acceleration, a, is the same. It could hardly be different: how could the ball know that it had started with a shove instead of rolling down an earlier piece of the same incline?

The Actual Acceleration

Experiments do more than just assure us the acceleration is constant: they tell us its actual value. If a is constant, then $\text{DISTANCE} = (\frac{1}{2}a)(\text{TIME})^2$, so $\text{DISTANCE}/\text{TIME}^2 = \frac{1}{2}\,\text{ACCELERATION}$. Thus, in the fictitious example, 0.076, etc., are estimates of $\frac{1}{2}a$. These give $a = 0.152$, or 2/13. But 2/13 is incomplete—two-thirteenths of what? Such a number is useless unless it carries a tag to show its units. We calculated this number by dividing distance, in feet, by time^2. Since the time is in seconds the answer must be feet/sec^2. (This is read "feet per second squared" or "feet per second per second.")

Units for Acceleration

Return to the definition of acceleration to look for its units directly;

$$a = \frac{\Delta v\text{, measured in velocity-units, e.g., feet/second}}{\Delta t\text{, measured in time-units, e.g., seconds}}$$

$$= \text{acceleration measured in acceleration-units, e.g., } \frac{\text{ft/sec}}{\text{sec}}.$$

Thus we expect to measure acceleration in units such as $\frac{\text{ft/sec}}{\text{sec}}$, which we write ft/sec/sec or ft/sec^2.

The Use of "per" in Science

The word "per" is of great use in science. We started using it above to mean "divided by" or "for each . . . ," as it does in ordinary arithmetic. Later we shall concentrate on a different aspect of its meaning, when it is used for ratio or proportion.

In arithmetic we divide 10 cents by 5 and get 2 cents. Or we divide 10 sheep by 5 sheep, and get 2 flocks. We feel doubtful about dividing 10 sheep by 5 cents—we object that they are different kinds of thing. But sometimes we do divide one kind of thing by another; such as 10 cents divided by 5 boys, which gives a pocket-money proportion of 2 cents per boy. Again, 60 cents divided by a dozen oranges gives a PRICE of 5 cents per orange. In science we often make divisions like these, and we preserve the truth by preserving the units as well as the number in the answer. If a beetle crawls steadily 10 feet in 2 hours, we can say "10 feet divided by 2 hours, or 10 feet/2 hours gives *5 feet per hour*." The answer shows the *distance* it crawls in *each hour*, but the statement does not restrict the beetle to one hour's travel. It applies to ¼ hour, ½ hour, 1½ hours, perhaps 2½ hours. But it also applies to very short time intervals; the beetle can still have a speed of *5 feet per hour* during a few seconds. We can, in imagination, shorten the time interval more and more, and still picture the beetle moving *5 feet per hour*. In the limit, we speak of the beetle having a *speed of 5 feet per hour at some particular instant*. This is a new idea, speed *at an instant* of time, at a certain mark on the clock. We can no longer divide a distance by a time—zero divided by zero is meaningless—yet a speedometer can register 5 feet per hour at an instant. True, a real beetle moves unevenly, but we can easily imagine an ideal one moving smoothly. Then the unit "one foot per hour" is no longer the result of division, but a thing-of-itself, a unit of rate; and the speed, 5 feet per hour, is a rate, ***a limiting value***, glimpsed at an instant.

Mathematical limits appear in physics as well as calculus—which *is* the algebra of calculating limits. To understand the essential idea of a ***limit*** look at the sum to many terms of the series: 1, ½, ¼, ⅛, 1/16, ... The sum of the first two terms is 1½; of three terms 1¾; of ten terms 1 511/512; &c. However far you go, the sum is never quite 2, but you can get as near as you like to 2 by taking enough terms. (Notice that the sum always falls short of 2 by just the last term that is included; so you can make that failure as small as you like.) So we say 2 is the ***limit*** of the sum of many terms. You met a limit in tangent-slope, the limit of the slope of a chord through two points on a graph.

Until this century, physics dealt with many smooth ratios, such as speed, density, illumination. But now, much as we find a real beetle's speed uneven, we find many physical quantities jumpy or chunky; we cannot reduce them smoothly to limiting values. As an obvious example consider the ratio mass/volume, which we call density. We can divide the mass of a large chunk of aluminum by its volume, or the mass of a small chunk by *its* volume, obtaining the same density. But if we try to push our determination of density to the limit of smaller and smaller samples we are stopped when we meet a single atom. What ratios in physics can be pushed to the mathematical limit? What things are not "atomic"? This is a question worth watching, to which we shall return at the very end of the course.

At present, you should take "per," or the sign / used for it, to mean "divided by" or "for each," but you should think about letting it take its place in the idea of a ratio.

Scientific Units

In ordinary life, we measure speeds in *feet per second* or in *miles per hour*, and engineers often use these units. We express accelerations in *feet/second per second*, or sometimes in stranger units such as *miles/hour per second*. But scientists all over the world have agreed to use the metric system of units in their measurements, and we shall use one version of this, the Meter-Kilogram-Second system. In this "MKS" system, lengths and distances are measured in meters instead of feet, masses of stuff in kilograms instead of pounds, and times in seconds. A meter is almost 10% longer than a yard, its exact length being defined by a bar of fireproof metal which is carefully preserved, with copies in standardizing laboratories throughout the world. A kilogram is roughly 2.2

TABLE OF UNITS AND ABBREVIATIONS

	Ordinary system used by householders and engineers (FPS system)	Metric system used by scientists: MKS system used in this course	Metric system used by scientists: CGS system (in common scientific use; not used in this course)
Length	foot (ft)	meter (m.)	centimeter (cm)
Mass	pound (lb)	kilogram (kg)	gram (gm)
Time	second (sec)	second (sec)	second (sec)

CONVERSION FACTORS

1 foot = 12 inches

1 meter = 100 centimeters = 1000 millimeters

1 inch = 2.540 centimeters = 0.02540 meters

1 foot = 0.3048 meter

1 meter = 39.37 inches ≈ 1.1 yards

1 pound = 454 grams = 0.454 kilogram

1 kilogram ≈ 2.2 pounds

pounds, 10% more than 2 pounds. It is defined by a standard lump of fireproof metal. A meter is subdivided into 100 centimeters, each about a fingerbreadth, and a kilogram is subdivided into 1000 grams, each about 1/28 ounce. Though many science courses use centimeters and grams, we shall follow the new fashion and use meters and kilograms, to make it easier to understand electrical units such as amps and volts. Scientists write m. as an abbreviation for meter or meters, but as this is easily confused with an algebra symbol m for mass, it is better to write it in full as meter(s). We write kg as an abbreviation for kilograms.

The gram was originally made of such size that one cubic centimeter of water weighs one gram. This gives the density of water, mass/volume, the useful value 1.00 gram per cubic centimeter (useful, but misleading because it can be left out so harmlessly). The density of water is *not* 1.00 kilogram per cubic meter. Nor is it 1 pound per cubic foot. If you make a hollow box with internal dimensions 1 ft × 1 ft × 1 ft, you will find it holds 62.4 pounds of water. The density of water is therefore:

62.4 pounds per cubic foot,
or 1.00 grams per cubic centimeter,
or . . ? . . kilograms per cubic meter.

In our scientific MKS system we measure speeds in *meters/second*, accelerations in *meters/second per second*. The acceleration in the example above, 0.076 *feet/sec per sec*, is the same as about

0.076 × 0.3 *meters/sec per sec*

since each foot is about 0.3 meter.

Acceleration of Free Fall

For free fall, the acceleration can be measured. To show that the acceleration is constant as a body falls faster and faster is difficult, though of course it can be done with modern timing apparatus, some of which can measure to one-millionth of a second. If we *assume* the acceleration is constant, then it is fairly easy to measure its value by timing free fall for one known distance from rest and using the relation $s = \frac{1}{2}at^2$. This leads to $a = 2s/t^2$. As a reminder that we are dealing with a characteristic constant acceleration "due to gravity," we label this particular acceleration "g" and write $g = 2s/t^2$. Using experimental values of s and t we can compute g. However, air friction limits the accuracy; it is difficult to make sure that we start the timing just when the falling body starts from rest, and the time of fall itself is a very short one; so such measurements do not give an accurate value of g. Yet we need to know g accurately for a number of uses in physics. Could we possibly eliminate the effects of friction? And could we lump together many falls, say several thousand, and measure the total time for the whole bunch to obtain the time for one fall with greater accuracy? These look like hopeless ambitions. Yet they can be achieved in a simple, easy experiment which Galileo foreshadowed, and which you will meet.

Measurements give a value about 9.8 meters/sec² for g, or 32.2 ft/sec². For ordinary calculations, 32 ft/sec² will suffice: accurate withiu 1%.

At the Equator, g is slightly smaller; and at the North Pole g is slightly greater.

Force and Acceleration

We think of a falling body as being pulled down by a force which we call its weight. To hold a body suspended we must support its full weight. If we cut the suspending cord we imagine the weight still acting, now unopposed by our supporting pull. If we suppose the body's weight remains constant while the body is falling, we may picture this constant force "causing" the constant acceleration of free fall. Trucks running down a slope have a smaller acceleration, a fraction of g; but only a fraction of their weight is available to pull them down along the slope. Later you will find what this fraction is. It depends on the slope of the hill. If you knew this fraction, you could follow Galileo in comparing downhill FORCE and downhill ACCELERATION. What kind of relation would you expect[29] to find between the force and the acceleration? You can see how early experimenters like Galileo could guess at it by studying falling and rolling bodies. That relation, to be discussed soon, is a very important piece of physics, a basic relation governing the motion of stars and the action of atoms, one of obvious importance in engineering.

While looking forward to discussing force and acceleration, we will end on a note of doubt. How do you know the weight of a body pulls it while it is falling freely? When you sit on a chair you feel the supporting force of the chair, and you believe you feel your own weight. But if you jump out of a window, do you feel your weight while you are falling? Suppose you jump out of a window with a lump of metal in your hand and try to weigh the lump as you fall. To make the temporary laboratory more comfortable, for a time, suppose you and the lump and the weighing apparatus are enclosed in a vast box which has been dropped from a tower and is falling freely. Suppose the box has no windows. When you release the lead lump inside the box, will it fall to the floor? If you think about this, you will see that gravity will seem to have disappeared. Can you possibly tell whether gravity has really disappeared or whether your laboratory is accelerating downwards? If you cannot tell the difference, *is* there any difference? Discussion of these questions would lead you towards the Theory of Relativity.

[29] Do we mean "expect" or "hope"? If *expect*, on what basis? If *hope*, is this scientific or not?

PROBLEMS FOR CHAPTER 1

1-6. These are at the beginning of the chapter.
7-11. These are in the text of Chapter 1.

★ 12. METHODS

Write a short note distinguishing between inductive and deductive methods.

★ 13. YOUR PRESENT VIEWS

Write a short note (¼ page to 2 pages) saying what you think are (or should be) the parts played by *experiment* and *theory* in a science like Physics. (*Note*: At this stage of the course we do not expect you to know all the answers to questions like this. Later you should know more of them. So we ask you now just to write some general comments stating *your present views*. Please do not extract some complicated statements from a book.

14. EXPLANATIONS

(a) How would Aristotelians explain the rising of a helium balloon?
(b) How would modern scientists explain it?

15. DENSITIES

(a) Look up the relative densities of gold, silver, aluminum, brass, stone, iron, and wood in reference tables (often at the end of physics books).
(b) Why did Newton use a gold guinea?

16. SCIENTIFIC WRITING

(a) Write a short essay, (half a page at most), giving your answers to the following questions:
  (i) Do you consider it good scientific writing to use long words wherever possible?
  (ii) Why do you suppose people who are trying to imitate a scientist tend to use long words?
  (iii) Do you consider it good scientific writing to avoid long technical words?
(b) Rewrite the following passage, replacing long words by suitable shorter ones wherever you can: "Henderson conducted considerable experimentation concerning the relationship between superficial area and electrical charge of aqueous solutions atomized into numerous spherical particles of microscopic dimensions. He theorized that the phenomenon of electrification was attributable to friction."
(c) Rewrite the following passage replacing a word by its technical equivalent wherever you feel that the change would make the passage more scientific: "When the r.p.m. of the fan is pepped up, the atoms of air whiz down the tube at a great rate of speed; and when they hit the thermometer its mercury rises and registers more degrees of heat."

17. Do you agree with Bernard Cohen's remark on page 8? Is it a mistake? Discuss briefly.

18. In the discussion of mass it is stated that ". . . gravitational pulls are exactly proportional to the amounts of stuff being pulled." On what piece of experimental knowledge is this statement based?

19. Suppose on a certain (fictitious) island it is a custom for each member of a family to give a small present to every other member of his family, and a present to him-

self as well, on New Year's Day. Suppose this custom is followed in every family, and that each present costs the same amount of money.

(a) How will the total expenditure of any one family be related to the number of members in it? (Find this empirically, that is, by trial and error, if you like.)
(b) Sketch a graph showing *total cost* plotted upward against *number of members in the family*.
(c) What graph do you suggest plotting, using these things, to obtain a straight line?

★ 20. IMPORTANT PUZZLE

When a ball is thrown vertically upward, it continues up until it reaches a certain point, then falls down again. At that highest point it stops momentarily and is not moving up or down.

(a) Is it accelerating at that point?
(b) Give reasons for your answer to (a). (*Hint*: See Problem 21.)
(c) Devise an experiment (given any apparatus you need) to find out whether it *is* accelerating at that point.

★ 21. PROBLEM TO HELP SOLVE PUZZLE

A man leans out of a window high above the ground, and throws a ball vertically up. The ball rises till it is about 30 feet above the man, then falls. (See Fig. 1-13.)

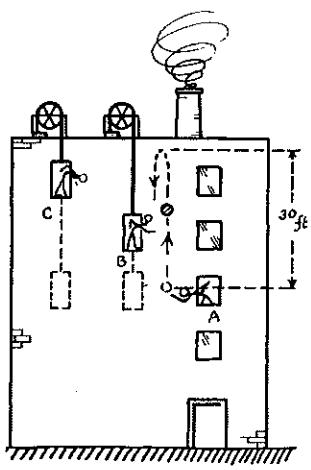

Fig. 1-13. Problem 21

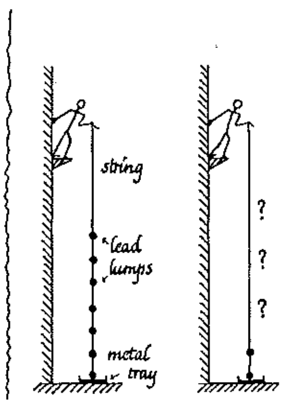

Fig. 1-14. Problem 22

(a) Give a short description of the motion of the ball, as seen by the man, A.
(b) At the instant that the man throws the ball, an elevator running up the outside of the building is passing the window with the same upward speed that the man gives the ball. The elevator continues upward with constant speed, carrying an observer, B, who watches the ball. Describe the motion of the ball as seen by B (who forgets that he is moving, and thinks that all the motion he observes belongs to the ball).
(c) Another elevator runs beside the first carrying an observer, C, steadily upward, with smaller constant speed. C is just passing the window when the man throws the ball. Describe C's observations of the motion of the ball.
(d) In the light of your answers to (a), (b), (c), comment on the puzzle of Problem 20. (Note that A, B, C all agree that the ball has the usual decelerated and accelerated motions, but they disagree in one respect.)

★ 22. DEMONSTRATING CONSTANT ACCELERATION

A lecturer wishing to demonstrate the constant acceleration of free fall drops a chain of lead lumps down a stairwell and asks his audience to listen to the sounds of them hitting a metal tray at the bottom. He makes one such chain by tying quarter-pound lumps of lead to a light string every foot along the string. Then holding the string so that the lowest lump is just on the ground, he has lumps 1 ft, 2, 3, 4 ft, and so on, from the ground. When he releases the string the lumps hit the ground with a tattoo of *increasing frequency*.

(a) What does this tell the audience about the motion of falling bodies?
(b) The lecturer wishes to test for constant acceleration by arranging the lumps *unevenly* on the string in such a way that if the acceleration is constant the audience will hear an *evenly spaced tattoo*. He ties one lump to the bottom of the string on the ground, the next 1 ft above the ground. Where should he tie the next five lumps? (See Fig. 1-14.)

23. HISTORY

Read Galileo's description of his own experiment on accelerated motion (available in Magie's *Source Book in Physics*, New York, 1935) and write a short account of it. Indicate the apparatus he used and the results he got.

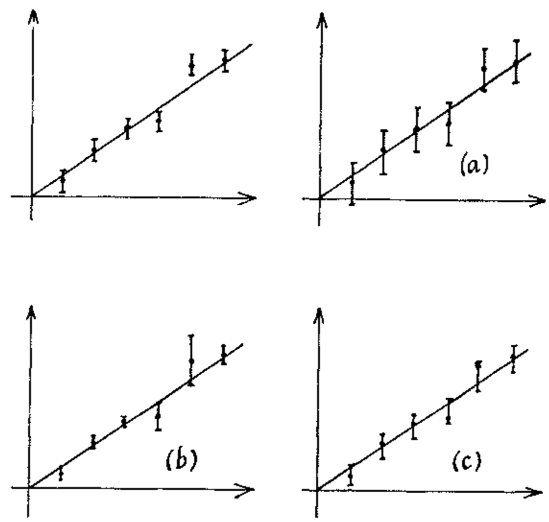

Fig. 1-15. Problem 24

★ 24. ERROR-BOXES ON GRAPH

A student, S, making an experimental investigation finds when he plots a graph of his measurements that his points do not lie on a straight line as he hoped. However, they are fairly near a "best straight line," and he knows his measurements are liable to some small errors. So he draws lines as "error-boxes" through each point on his graph. He finds that even the error-boxes miss his best line in several cases. He is, therefore, tempted to change his error-boxes (a) by making all of them taller or (b) by making some of them taller or (c) by sliding them up or down through each point until they hit the line.

Is any of the changes (a) or (b) or (c) a wise one for a scientist to make? Discuss with student S what he is really saying about his experiment in each case, (a), (b), (c).

★ 25. MKS UNITS

(a) Make a rough estimate of your height in meters.
(b) What do you weigh in kilograms?
(c) What is the width of this page in meters?
(d) What is the thickness of a page of this book in meters?
(e) Explain how you made the estimate asked for in (d) without using any special micrometer gauge.
(f) Atomic scientists find molecules and atoms so small that they like to use a much smaller unit than a meter for measuring them. They use an "Ångstrom Unit," which is one ten-billionth of a meter or 10^{-10} meter. What is the thickness of this page, in Å.U.?
(g) Most atoms are a few Å.U. in diameter, say 3 Å.U. How many atoms thick (roughly) is this page?

★ 26. DENSITIES IN MKS UNITS (Learn these values)

(a) What is the density of water in *kilograms/cubic meter*?
(b) Show the reasoning by which the answer to (a) can be obtained from the data in Chapter 1.
(c) Lead has a "specific gravity" of about 10. This means its density is 10 times that of water. What is the density of lead on the MKS system of units?
(d) Olive oil has a specific gravity of about 0.8. What does this mean?
(e) What is the density of olive oil in the MKS system?
(f) The specific gravity of mercury is 13.6. The atmosphere presses on each square inch of table, chair, our bodies, walls, . . . etc. with a force that can balance a column of mercury of cross-section 1 sq. inch and height about 30 inches. That is the "height of the barometer" in which mercury with a vacuum above it inside balances atmospheric pressure outside. What would be the height of a water barometer?

★ 27. A USEFUL CONVERSION FACTOR

Show that 60 miles/hour = 88 ft/sec.

28. A SPECIMEN ACCELERATED MOTION

The motor of a certain elevator gives it an upward acceleration of 150 ft/min/sec. The elevator starts from rest, accelerates thus for 2 secs, then continues steadily with constant speed.

(a) Explain what this statement of acceleration means.
(b) What is the final speed after 2 secs?
(c) Calculate the speed after 0 sec, 0.5 sec, 1 sec, 1.5 secs, 2, 3, 4, 5 secs. Sketch a rough graph showing speed (upward) against time from start (along), for the first 5 seconds.
(d) How far has the elevator risen 1 second from the start? How far has it risen 2 secs, 3 secs, 4 secs, from the start? Sketch a rough graph of distance against time.

★ 29. CALCULUS STATEMENTS

In this question, v is a symbol for speed or velocity; a is a symbol for acceleration, t for time.

(a) What does Δv mean?
(b) What does the statement "$\Delta v/\Delta t$ = constant" mean?
(c) What does the statement "$\Delta a/\Delta t$ = constant" mean? (Make an intelligent guess.)

30. FORMAL LOGIC*

Here is an example of a syllogism, a type of perfect deduction—too restricted to be much use in science but an important part of classical logic.

(1) All dogs have 4 legs. (the "major premise," a generalization)
(2) Fido is a dog. (the "minor premise")
(3) ∴ Fido has 4 legs. (the conclusion)

These three steps involve three "terms":

(a) 4-legged creatures (the major term, a large class)
(b) dogs (the middle term, a smaller class)
(c) Fido (the minor term, a member of a class)

The argument holds true if:
(c) falls wholly within the class (b) and (b) falls wholly within the class (a). Then, (c) must fall within the class (a). A corresponding argument can be carried out if (c) falls wholly within (b) but (b) falls wholly *outside* (a). Then (c) must fall *outside* (a).

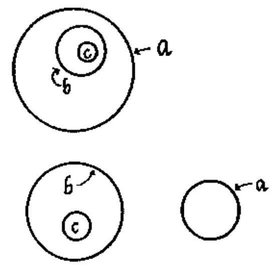

FIG. 1-16.

The inference or conclusion drawn in a "syllogism" may be untrue

(i) because the major or minor premise is not true,
(ii) because the reasoning is untrue, e.g., (b) falls partly within (a),
(iii) because there is confusion of language, e.g., ambiguous terms.

In each of the following examples there is something that makes the conclusion untrue or at least unjustified. Point out the defect in each case.

(A) All plums are vegetables.
This is a plum.
∴ This is a vegetable.
(B) All poisons are harmful.
Sugar is a poison.
∴ Sugar is harmful.
(C) All dogs are animals.
An antelope is an animal.
∴ An antelope is a dog.
(D) All salts dissolve in water.
George is an old salt.
∴ George dissolves in water.
(E) Caesar Augustus was a Roman Emperor.
Julius Caesar was a Roman general.
∴ Julius Caesar was the uncle of Augustus.

* Drawn from "Clear Thinking" by R. W. Jepson (Longmans, Green and Co., London, 1936).

APPENDIX A · THE ALGEBRA MACHINE

Grinding Out Useful Formulas for "Constant Acceleration"

In this appendix we shall neither discover new physics nor review old physics, but only do some intellectual machine-shop work. We shall start with a clear assumption, namely *motion with constant acceleration*, and make the algebra-machine grind out some logical consequences. The results, old information rehashed in new form, will be useful in examining the real world; but while we are deriving them we can sit in our ivory tower, confident that our work is necessarily true—true to its own assumptions—the work of perfect logic.

Definition. We choose to deal with (rate of change of speed), which is

$$\frac{\text{CHANGE OF SPEED}}{\text{TIME TAKEN FOR CHANGE}} \text{ or } \frac{\Delta v}{\Delta t}.$$

Since this turns out to be a useful thing-to-deal-with, or *concept*, we give it a name, ACCELERATION. Then the statement "ACCELERATION $= \Delta v/\Delta t$" is merely dictionary definition, explaining what we give that name to.

Assumption. We assume that acceleration is constant. (That is, we investigate the kind of motion which has constant $\Delta v/\Delta t$. There are many other types of motion common in nature; but this one is both simple and important, so we investigate it in detail.)

$$\frac{\Delta v}{\Delta t} = \text{constant, whose value we will call } a$$

In our elementary algebraic method, we shall assume that the *average* speed of an object moving with constant acceleration is just the average of the speeds at the beginning and end of the trip. We assume that

$$\text{AVERAGE SPEED} = \frac{\text{INITIAL SPEED} + \text{FINAL SPEED}}{2}$$

We also say

$$\text{DISTANCE TRAVELED} = \text{AVERAGE SPEED} \cdot \text{TIME}$$

$$\text{or } s = \bar{v} \cdot t$$

Note that we use · as a sign for multiplication, modern practice that we shall extend to *units* that are multiplied, such as man · hours, and we use v with a bar over it for "average v."

Terminology. Let:

(i) acceleration be a meters/second per second

(ii) moving object's speed be v_0 meters/second at the instant when the clock is started, that is, when $t = 0$. We shorten this description to:
initial speed $= v_0$ meters/second, at $t = 0$

(iii) moving object's speed be v meters/second when the clock reads t seconds; or:
final speed $= v$ meters/second.

(iv) distance traveled, in time t secs, be s meters.

These are merely descriptions of the letters we use. We can make a more connected statement, thus: the moving object starts with speed v_0, travels a distance s in time t, with acceleration a, reaching a final speed v.

Relationships. Now let the algebra machine grind out relationships.

(1) $v = v_0 + at$

$$\text{ACCELERATION, } a = \frac{\Delta v}{\Delta t} \quad \text{(dictionary definition)}$$

$$= \frac{\text{GAIN OF SPEED}}{\text{TIME TAKEN}}$$

$$= \frac{\text{FINAL SPEED} - \text{INITIAL SPEED}}{\text{TIME TAKEN}}$$

(if acceleration is constant)

$$= \frac{(v - v_0)}{t}$$

The last line gives only an average acceleration, *unless* the acceleration is constant, as we assume here. To obtain a useful expression for final speed, v, we must now rearrange the algebra. We start with the equality-truth

$$a = \frac{v - v_0}{t}$$

and arrive at an equally true statement by multiplying both sides by t.

Then: $a \cdot t = v - v_0$

We arrive at another equally true statement by adding v_0 to both sides.

Then: $v_0 + at = v - v_0 + v_0 = v$

$\therefore\ v_0 + at = v$

And, reversing this,

$$\underline{v = v_0 + at.}$$

All the changes we have made from $a = \dfrac{v - v_0}{t}$ are merely changes allowed by the rules of logic. The result, $v = v_0 + at$, is just as true or untrue as the starting-point, $a = (v - v_0)/t$. In this case we can see that the new "formula" is just a new version of the old starting-point, because it says

FINAL SPEED = INITIAL SPEED + RATE OF GAIN · TIME (and this must be the gain of speed)

which says,

FINAL SPEED = INITIAL SPEED + GAIN OF SPEED = FINAL SPEED.

This discussion must have seemed nnnecessarily long if you are familiar with algebra and trust it. You could just say,

$$a = \frac{v - v_0}{t} \qquad \therefore\ at = v - v_0 \qquad \therefore\ v = v_0 + at$$

the essence of the logic being coded in the ∴ sigus. But if you regard formula-making as a mystery, you should read the detailed discussion carefully. The novice might take our word for the algebra, but he needs to grow out of mistaken ideas about the "truth" of formulas or about the mystery of their derivation.

(2) $s = \frac{1}{2}(v + v_0)\, t$

Our experimental tests will deal with DISTANCE, not SPEED. To see what our assumption of constant acceleration predicts for the relation between DISTANCE TRAVELED and TIME TAKEN, we need some way of computing distance when the speed is changing. We make a common-sense guess that we should use the AVERAGE SPEED, $\bar{v}$, got by adding the initial and final speeds and dividing by 2. Thus,

$$\text{AVERAGE SPEED, } \bar{v} = \frac{v_0 + v}{2}.$$

We use this AVERAGE SPEED as a steady speed to replace the real changing speed and we find the DISTANCE TRAVELED by multiplying AVERAGE SPEED by TIME.

Then, DISTANCE $s = \bar{v} \cdot t$

$$\text{or } s = \tfrac{1}{2}(v + v_0)\, t$$

In this relation, acceleration, a, and its definition do not appear. Yet the relation is not true unless the motion is one with constant acceleration (see Problem A-1 at the end of this appendix). This statement is not just a rearrangement of the earlier one. It contains the assumption about average speed. That assumption, so far only a guess from "common sense," can be checked by calculus (special algebra) or by Galileo's clever geometrical treatment (see Problem A-1). These show that this use of average speed is correct for motion with fixed acceleration. For other types of motion, a different kind of average, not the arithmetic mean, is needed.[30] So our assumption is a lucky one, correct for constant acceleration—we use it in teaching only because we know from calculus that it is a safe one. Thus the elementary presentation has been tailored to give the right results. Though this is sometimes unavoidable it gives a regrettable impression of glib plausible assumptions and fails to show the careful feeling-of-the-way and honest testing which are characteristic of science. Therefore, you should study Problem A-1.

(3) $s = v_0 t + \frac{1}{2} a t^2$

We still want to express DISTANCE TRAVELED in terms of TIME and ACCELERATION, without using the FINAL SPEED. We obtain this from the other two relations, (1) and (2), by using one of them to provide an expression for v which can then be inserted instead of v in the other.

Thus, $s = \frac{(v_0 + v)}{2} t$ and $v = v_0 + at$

$$\therefore\ s = \frac{(v_0 + v_0 + at)}{2} t = \frac{(2v_0 + at)}{2} t = \frac{2v_0 \cdot t}{2} + \frac{at \cdot t}{2}$$

$$\therefore\ s = v_0 t + \tfrac{1}{2} a t^2$$

This gives a relation, belonging with constant acceleration, which is useful in experimental tests.

If the timing starts from the instant when the moving thing is at rest, the initial speed is zero, $v_0 = 0$, and the relation becomes

$$s = \tfrac{1}{2} a t^2$$

Since a is constant, $\frac{1}{2} a$ is constant, so we can say

$$s = (\text{constant}) \cdot t^2 \text{ or } s \propto t^2.$$

Thus we can say, "theory predicts that $s \propto t^2$ for constant acceleration from rest." When we say "theory predicts," we mean that starting from some assumptions and using reasoning-machinery (which includes mathematics) we have recast those assumptions in what looks like a different form. *If* experiment agrees with this new form, we may decide our assumptions (and our machinery) are "true" or "justified." Yet often we cannot be sure that our chosen assumptions give the only possible true underlying story. We should be safer to say that they fit the facts so far. If you found in experi-

[30] For example, if the acceleration is not constant but starts with a large value and soon dwindles to zero, the moving object makes most of its gain of speed quite early in its trip, and then the proper average speed is not $(v_0 + v)/2$ but greater than that.

ments on falling bodies that distances and timings agreed closely with the relatiou $s \propto t^2$, then you could say that they agree with the relation predicted for constant acceleration. You could say that falling bodies seem to move with constant acceleration. In experimeuts on balls rolling down a plank, Galileo found that distauces and timings fitted fairly well with the relation $s \propto t^2$. So they agreed with his prediction for constant acceleration.

Notice that the experiments do not prove the formula is the right one for constant acceleration. The formula itself is necessarily, logically, true for any motion which does have fixed acceleration. Experiments only show that the rolling motion, in agreeing with the formula (probably) has constant acceleration. When we compare experimental data with the formula we can discover something about nature.

Arriving at the formula involved the following stages:

Definition of acceleration: We invented it, chose a name, then used it.

Decision to think about motion with constant acceleration. This is one of the many choices we might have to try for real falling bodies. But, once made, the decision enables us to proceed with algebra. In making this decision we are not discovering anything about nature.

Algebra: A logical sausage-making machine. Mathematics cannot manufacture scientific facts, though it may help us to discover them.

Common-sense assumption that the proper $\bar{v}$ to use is $(v_0 + v)/2$. The risk in this can be avoided by Galileo's geometry (Problem 1), or by a calculus investigation, which would justify it for fixed acceleration.

Algebra again

Result: A useful relationship, deduced from our assumptions, useful in experimental tests.

(4) $v^2 = v_0^2 + 2\ as$ [This is a form which we shall not need for a long time yet. This section may be postponed till it is needed.]

We can use further algebra, a few more turns of the sausage machine, to change the formulas to other forms. We already have three relations:

(1) involving v, v_0, a, t, but not distance, s;
(2) involving s, v, v_0, t, but not acceleration, a;
(3) involving s, v_0, a, t, but not final speed, v.

Later we shall want a relation expressing v in terms of v_0, a, s, but not involving the time t explicitly. Since we want it without t, we obtain it from any two of the earlier relations by eliminating t. For example, we can use (1) and (3). Then $v = v_0 + at$ gives $t = \dfrac{(v - v_0)}{a}$ and we substitute this in $s = v_0 t + \frac{1}{2} at^2$.

$$\text{Then:}\quad s = v_0 \left[\frac{(v - v_0)}{a} \right] + \tfrac{1}{2}\, a \left[\frac{(v - v_0)}{a} \right]^2$$

Will this lead to the formula (4) quoted above? Yes, if you use courage and algebra. You will have to square and cross-multiply and rearrange and simplify. The work will be clumsy and messy, but the final expression for v^2 will be $v_0^2 + 2\,as$. Try it, if you like.

The professional mathematician has a strong poetic sense of form in his own language of mathematics and he would consider the method above horribly clumsy. He would say, "Here is a more elegant derivation . . ." and would produce the answer quickly and neatly. Non-mathematicians who see him do this are mystified by his superior knowledge, and may be annoyed by the magical atmosphere. The real story is a sordid one. The mathematician is quite human, and feels his way in several trials, like any other explorer—though in simple problems his exploring may have all been done before and stored in his mind as "mathematical common sense." When he has found the answer by *any* method, clumsy or not, he may try working *backwards* from it to find a neat method of deriving it, like a mountaineer seeking a better path. There is no sin in this, but then he often forgets to tell the layman about the previous work, and startles him by producing the elegant method out of his hat. Let us try such an analytical search, thinking aloud as we go. The answer we want is $v^2 = v_0^2 + 2\,as$, so far obtained by algebraic drudgery. Try to undo it. Does it look as if it could be twisted or changed easily by algebra? Does it simplify or split up in any obvious way? No. Then we must push it around. Try shifting something across the =. Then we can have $v^2 - v_0^2 = 2\,as$. Is *this* easily attacked by algebra? Yes, the left hand side is an old friend, with factors $(v + v_0)\,(v - v_0)$. We could manufacture it from those factors if we could obtain them separately from somewhere. Where have we seen $(v + v_0)$ before? In the relation (2), $s = \frac{1}{2}(v + v_0)t$. Then $v + v_0 = 2s/t$. Where have we seen $(v - v_0)$? In the definition of acceleration, which we wrote $a = (v - v_0)/t$. Therefore, $(v - v_0) = at$. Now we want $v^2 - v_0^2$, which we can get by multiplying $(v + v_0)$ and $(v - v_0)$. We do this, using $(v + v_0) = 2\,s/t$ and $(v - v_0) = at$.

$$(v + v_0)\,(v - v_0) = (2\,s/t)\,(at)$$

$\therefore\ v^2 - v_0^2 = 2\,as$, which leads to the form we want. Now, having found the method by analysis, we erase the details of our search and start afresh, thus:

To derive $v^2 = v_0^2 + 2\,as$ by an elegant method, start with the definition of acceleration,

$$a = (v - v_0)/t,$$

and with the formula for distance travelled in terms of average speed, $s = \frac{1}{2}\,(v + v_0)t$, and just multiply these two equations together, obtaining $a \cdot s = \frac{1}{2}\,(v^2 - v_0^2)$ which reduces to

$$\underline{v^2 = v_0^2 + 2\,as}$$

Here, then, are four relations between v, v_0, a, s, and t.

$$v = v_0 + at \qquad s = \tfrac{1}{2}\,(v + v_0)\,t \qquad s = v_0t + \tfrac{1}{2}\,at^2$$

$$v^2 = v_0^2 + 2\,as$$

They provide a quick way of calculating the value of any one of these quantities, given the values of three others.

Algebra Yields Net *Distance*

The numerical values must be given appropriate + and — signs. For example, if the initial velocity is 6 ft/sec eastward and the acceleration 2 ft/sec/sec eastward, we can say $v_0 = +6$ and $a = +2$. However, if v_0 is 6 ft/sec eastward but the acceleration is in the opposite direction, 2 ft/sec/sec westward, then one of them must have a *minus* value. If we say $v_0 = +6$ we must say $a = -2$, using + signs for eastward velocities, accelerations and travel-distances, and — signs for westward ones. Then s is the *net* distance travelled in time t, not the arithmetic sum of westward and eastward travels. This is because in calculating each part of the trip the algebra will give + sign to eastward travels and — sign to westward ones and in adding up these + and — parts to find s the algebra will give the net difference. With $v_0 = +6$ and $a = -2$ the motion is decelerated: slower and slower forward for 3 secs, then at rest, then faster and faster backward. In 5 seconds it will show a path like Fig. 1-17, with 9 ft forward travel, then 4 ft backward, giving a net travel 5 ft.

Algebra gives:

$$s = v_0t + \tfrac{1}{2}\,at^2 = (+6)(5) + \tfrac{1}{2}\,(-2)(5)^2$$
$$= 30 - 25 = 5\,\text{ft}.$$

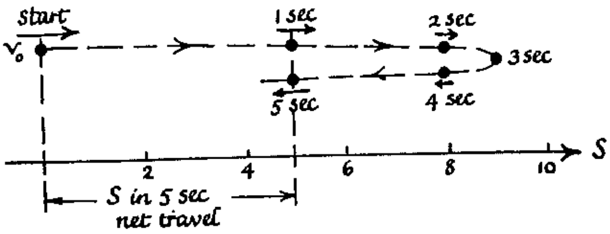

FIG. 1-17. S IS NET DISTANCE

Thus s always gives the *net* distance from start to finish.

These useful relations are tools, not vital pieces of science. They are absolutely true for motion with constant acceleration, and they are not reliable for other motions. Only experiment can tell us where they apply in the real world.

PROBLEMS FOR APPENDIX A

★ A-1. NON CALCULUS PROOF

Galileo, lacking the help of calculus and preferring geometry to algebra, dealt with uniformly accelerated motion as follows: Imagine a graph with time plotted along and velocity of a moving body plotted upwards. *If* the body has constant acceleration, its velocity must increase *steadily* as time goes on. The graph must be a straight line. It will not necessarily pass through the origin, but will start at the initial velocity, v_0 when time is zero, and run up to some value v at time t.

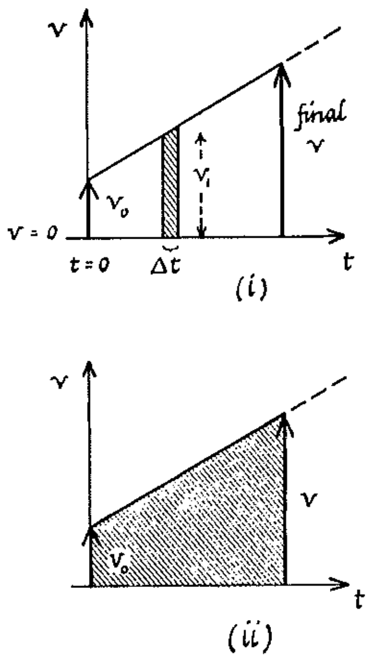

FIG. 1-18. GALILEO'S PROOF

Now consider what happens in some very short interval of time Δt, when the velocity is, say, v_1. (Of course v is increasing, but we can take v_1 as the average during short Δt.) Then the body moves a distance $[(v_1) \cdot (\Delta t)]$ in that time. But on the graph $[(v_1) \cdot (\Delta t)]$ is the [*height* · *width*] of the small pillar resting on Δt and running up to the graph-line. It is the *area* of that pillar, shaded in sketch (i).

Therefore, the *total* distance covered is given by the total area of all such pillars—i.e., the shaded area in sketch (ii).

(a) If in sketch (ii) the heights of this patch at its edges are v_0 and v as marked, and the base is time t, what expression gives the *area*? (Outline your geometrical argument briefly.)

(b) If the heights at the edges are v_0 and $v_0 + at$ (which follows from the definition of acceleration), what expression gives the area? (Outline your argument briefly.)

(c) Write the results of (a) and (b) as expressions for s the distance covered by the body in time *t*.

(d) Now suppose the acceleration is *not* constant but starts with a smaller value, rising to a greater one, so that the velocity still changes from v_0 to v in time *t*, but not steadily. (i) Sketch the new graph picture. (ii) Will the expressions from (a) and (b) apply now? (iii) What weakness in the earlier algebraic discussion in Appendix A has now been removed?

★ A-2. CALCULUS PROOF

In the limit, velocity, v, is rate-of-change of distance, *ds/dt*, and acceleration, *a*, is rate-of-change of velocity $\frac{dv}{dt}$ or $\frac{d}{dt}\left(\frac{ds}{dt}\right)$ or $\frac{d^2s}{dt^2}$ · Show that if *a* is constant, each of the following is true:

(i) $dv/dt = a$ integrates to $v = v_0 + at$ (where v_0 is a constant, the value of v at time $t = 0$)

(ii) $v = v_0 + at$ integrates to $s = v_0t + \frac{1}{2}at^2$ (*Hint*: remember $v = ds/dt$.)

(iii) $dv/dt = a$ integrates to $v^2 = v_0^2 + 2as$ (*Hint*: try multiplying both sides by v.)

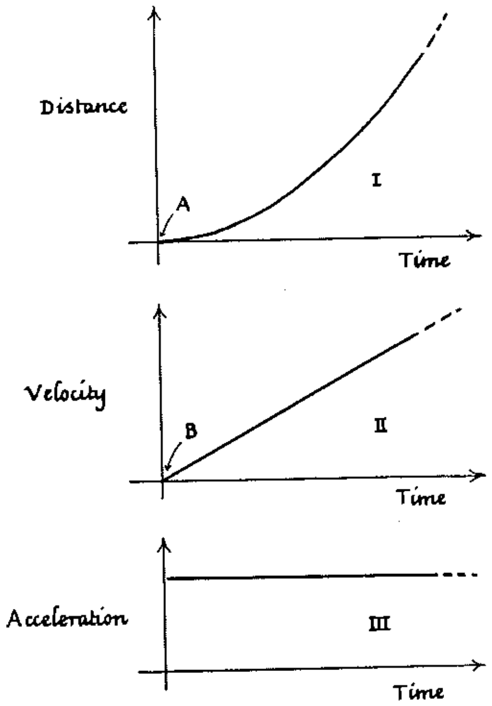

FIG. 1-19. PROBLEM A-3, parts a, b, and c

A-3. GRAPHS OF MOTION

Fig. 1-19 shows an arrangement of three time-graphs for the motion of an object along a straight track. Graph I shows *distance* plotted against *time*; graph II *velocity* against *time*; graph III *acceleration* against *time*. They are drawn with matching time-scales. The graphs sketched relate to an object moving with constant acceleration, starting at $s = 0$ (shown by A) and velocity $v = 0$ (shown by B) at $t = 0$. In graphs for more complicated motions, all three lines may be curved.

(a) In the general case of any motion, one or more of the graphs can be derived from another of the three by tangent slopes. Which one(s)? Explain why.

(b) In the general case, one or more of the graphs can be derived from another of the three by measuring areas under the curve. Which one(s)? Explain why.

(c) A motorcycle policeman starts from rest, accelerates 15 ft/sec² for 6 secs; runs at constant velocity for 10 secs; then skids to a stop in 4 secs, with constant deceleration. Sketch a trio of graphs I, II, III, for his motion.

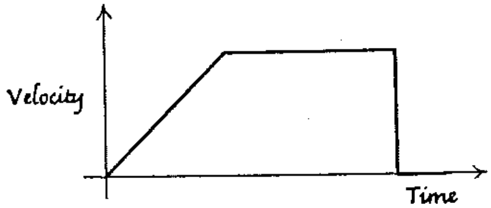

FIG. 1-20. PROBLEM A-3, part d

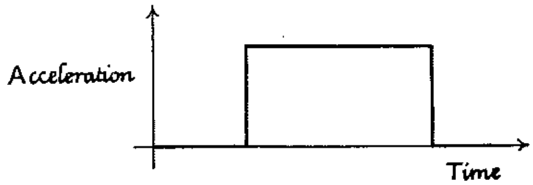

FIG. 1-21. PROBLEM A-3, part e

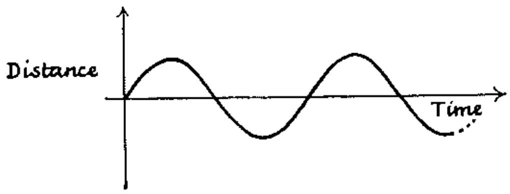

FIG. 1-22. PROBLEM A-3, part f

(d) Fig. 1-20 shows graph II for the motion of a car. Copy it and add sketches of graphs I and III.

(e) Fig. 1-21 shows graph III for the motion of a truck. Copy it and add sketches of graphs I and II.

(f) Fig. 1-22 shows graph I for the motion of the bob of a long pendulum along its almost-straight path. Copy it and add sketches of graphs II and III. (*Difficult*: Deserves careful guessing.)

APPENDIX B · "g"

Measurement of "g"

We have glibly announced the value of "g" as 9.8 meters/sec² (or 32 ft/sec²), but this came from laboratory measurements. You will use it for simple calculations concerning falling bodies, and for important calculations of forces when you treat "g" as gravitational field-strength. "g" is such a useful quantity that you should see its value measured before you use it. You could make a very rough estimate with a stone and a stopwatch and a meter-stick.

PROBLEM B-1. ROUGH MEASUREMENT OF "g"

An experimenter drops a big stone from a 14th-story window and finds it takes "just over" 3 seconds to reach the ground. If the window is 150 ft from the ground,

(a) Make an estimate of "g."

(b) Taking 150 ft to be about 46 meters, estimate "g" in meters/sec².

A better measurement can be made with an electric clock, as illustrated in Fig. 1-23, and you should see some such demonstration. For very accurate measurements you must wait for the promised scheme which avoids friction and takes a group of falls.

PROBLEM B-2. MORE ACCURATE MEASUREMENT OF "g"

A metal ball is allowed to fall from ceiling to floor. At the ceiling it is held against two metal pins so that it makes an electrical connection which *prevents* the electric clock from starting. The ball is released abruptly, and the clock starts.

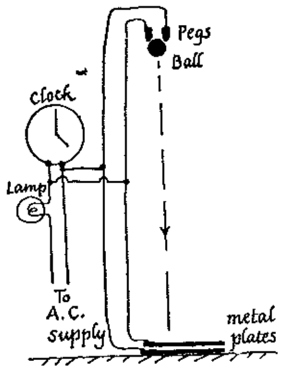

FIG. 1-23. MEASURING "g"

As it reaches the floor, the ball pushes two light metal plates together, making another electrical connection which stops the clock. In an actual experiment, the height of the fall was 7.00 meters from ceiling pegs to floor contacts, and the clock recorded a time of 1.20 secs.

(a) Estimate the value of "g," using these data.

(b) Say what assumptions you made in (a) concerning the type of motion; the apparatus; the conduct of the experiment. (Give details; avoid prim generalities such as "apparatus accurate" or "avoided personal error.")

Values of "g" in various localities

"g" has been measured very precisely at a few standard laboratories. Comparative measurements have then provided accurate values of "g" at many places all over the world.

	New York	Equator	Pole
Value in meters/sec/sec	9.80267	9.780	9.832
Value in feet/sec/sec	32.16	32.09	32.26

For ordinary calculations, in problems or in experiment-design, you should use the rough values $g = 9.8$ meters/sec/sec, $g = 32$ ft/sec/sec.

Arithmetical Problems on Free Fall: Dissected Problems

When you know the value of "g," you can make simple calculations about dropping stones, arrows shot at monkeys, etc. Such calculations are occasionally used by physicists in designing apparatus or in dealing with some experiment, but they are not important physics. Elementary textbooks and examinations make much of them "because they make accelerated motion clearer." Students trained to solve them mechanically may gain little but a damaging prejudice that "physics consists of putting numbers in the formulas." We wish to avoid that foolish picture of science, and we would not give you such problems in this course except for two reasons: (1) You may meet similar calculations, that are important, in atomic physics; (2) They will show you something important about the place of mathematics in physics. For these two reasons you should work through Problems B-3, 4, 5, and 6. Even so, if earlier studies have made you a convinced formula-monger you had better omit these problems unless you are prepared to start with an open mind.

Problems B-3 to B-6 have been dissected. You should answer them step by step, on question sheets reproduced from the small ones printed here. This scheme—which you will meet several times in the course—is intended to give you preliminary help and teaching towards later problems to be done on your own. Note that this insulting simplicity is meant to help you with the mathematics but not to save you from thinking out the physics for yourself. As you work such problems you should stop to notice that you are learning a method of solving them, but you should then concentrate on the physical results that emerge.

The problems here are intended to be answered on typewritten copies of these sheets. Work through the problems on the enlarged copies, filling in the blanks, (______), that are left for answers.

PROBLEMS ON ACCELERATED MOTION   NAME ________________

Work through these problems, filling in the blanks which are left for answers. Doing that will show you how to solve such problems and will, it is hoped, give you a pleasant understanding of the use of mathematics in physics. You will see that mathematics is a faithful servant, but sometimes a slightly dumb one, carrying out instructions relentlessly.

PROBLEM B-3. A rock falls from rest, with constant acceleration 32 ft/sec per sec, downward. (GIVEN: The acceleration is constant, and air friction is negligible, in this case.)

(a) What will be its velocity after 3 seconds of fall?

(b) How far will it fall in 3 seconds?

A. ARITHMETICAL METHOD.

(a) Acceleration 32 ft/sec per sec means that the rock gains in velocity

by ________ . ________ (units) in each second.

∴ In 3 secs of fall it gains in velocity. . . . . ________ ft/sec

∴ Since it starts from rest, its final velocity is. . . . ________ ft/sec

(b) The velocity grows from ________ ft/sec (at start) to. . . ________ ft/sec

∴ Average velocity is $\frac{1}{2}$(________ + ________) or. . . ________ ft/sec

∴ Distance travelled, with this average velocity,

in 3 sec is (________) · (________) or. . ________ ft

B. ALGEBRAIC METHOD.

(a) The acceleration, $a = 32$ ft/sec/sec   Time, $t = 3$ secs

Initial velocity, $v_o = 0$

Substituting these values in $v = v_o + at$

we have: final v = ________ + ________ = ________ ft/sec

(b) Substituting the values above in $s = v_o t + \frac{1}{2}at^2$

we have: s = ________ + $\frac{1}{2}$(________) = ________ ft

(NOTE: In using algebra, always state the "formula" clearly first, as above. Also state the values you are going to substitute, attaching units to them. For example, "t = 3 secs", not "t = 3".)

PROBLEMS ON ACCELERATED MOTION   SHEET 2

PROBLEM B-4. A ball is thrown downward, and released with velocity 10 ft/sec, to fall freely at the instant when the clock is started.

(a) What will be its velocity after 3 seconds of fall?

(b) How far will it fall in 3 seconds?

A. ARITHMETICAL METHOD.

(a) Acceleration 32 ft/sec per sec means that

the ball gains in velocity by ________ ________ (units) in each second.

∴ in 3 secs of fall it gains in velocity . . . . ________ ft/sec

∴ since it starts with velocity 10 ft/sec downward,

its final velocity will be ________ ft/sec

(b) The velocity grows from ________ ft/sec at start to

the final value of ________ ft/sec

∴ average velocity is $\frac{(\_\_\_\_\_\_)+(\_\_\_\_\_\_)}{2}$ or . . ________ ft/sec

Distance ball would travel in 3 sec with this

average velocity is. . . . ________ ft

B. ALGEBRAIC METHOD.

(a) The acceleration, $a = 32$ ft/sec/sec, downward

Initial velocity, $v_o = 10$ ft/sec, downward   Time of travel, $t = 3$ secs

Substituting these values in $v = v_o + at$, we have

Final velocity, v = ________ + ________ = ________ ft/sec

(b) Substituting the values above in $s = v_o t + \frac{1}{2}at^2$, we have

Distance s = ________ + $\frac{1}{2}$(________) = ________ ft

PROBLEM B-5. From a place on the top of a tower, a ball is thrown upward, released with velocity 10 ft/sec upward at the instant the clock is started.

(a) What will its velocity be after 3 seconds?

(b) How far below its starting point will it have fallen after 3 secs?

PRELIMINARY DISCUSSION

In this case, the ball moves upward at first, but more and more slowly, having a downward acceleration 32 ft/sec per sec (equivalent to an upward deceleration) all the while. It reaches a highest point or vertex, (still with the same downward acceleration); and then it falls, (still with the same downward acceleration). Questions (a) and (b) do not inquire about the highest point; and, if acceleration and velocity are given + and - signs for down and up, the algebra should carry right through the highest point and yield the net distance, s. (So, although you may have been shown methods which make you calculate first the trip up to the vertex, then the trip down, avoid such methods -- see the discussion below.)

A. ARITHMETICAL METHOD.

(a) Acceleration 32 ft/sec per sec down means that

in every second ball gains in downward velocity by ________ ________

∴ In 3 secs, its gain in downward velocity is ________ ft/sec

But it started with 10 ft/sec upwards; so, with the

gain of downward velocity, the final velocity must be ________ ft/sec down

(b) To calculate the net distance fallen we need the average downward velocity. The velocity is upwards at first and downwards at the finish. To find the proper average velocity, we cannot just add the two numbers for the velocities and then halve the total, because that would be the procedure for a ball thrown downwards, as in the previous problem.

PROBLEM B-5 CONTINUES ON NEXT SHEET

PROBLEM B-5 (ARITHMETICAL METHOD continued)

This is like the problem of averaging a bank balance which changes steadily from $10 in the red to $40 in the black.

The average is not $\frac{1}{2}$($10 + $40) or $25; it is $\frac{1}{2}$(-$10 + $40) or $15.

To see that this is right, try making the change from $10 in the red to $40 in the black by a series of daily jumps, each of $5. The daily balance runs:

red	red		black	black . . .						. . . black
$10	$5	zero	$5	$10	$15	$20	$25	$30	$35	$40

The balance on the middle day is $15. The series is better shown thus:

-$10	-$5	0	+$5	+$10	+$15	+$20	+$25	+$30	+$35	+$40

So, in this problem, we attach + signs to all downward velocities and accelerations, but call the upward velocity at the start -10 ft/sec.

Then average velocity is given by $\bar{v} = \frac{-10 + (\_\_\_\_\_\_)}{2}$ = ________ ft/sec

Ball falling for 3 secs with this steady average velocity would travel

a downward distance given by (average velocity)·(time) = ________ ft

THIS GIVES THE NET DOWNWARD TRAVEL, NOT THE TOTAL LENGTH OF PATH UP AND DOWN. It gives the direct distance down from the start to the finish point. This is because we have used the kind of average that will yield this net distance.

B. ALGEBRAIC METHOD.

The acceleration, a = +32 ft/sec/sec (plus meaning "downward")

Initial velocity, v_o = -10 ft/sec (minus because upward)

Time of travel, t = 3 seconds

(a) Substituting in $v = v_o + at$, we have

final v = ________ + ________ = ________ ft/sec

(b) Substituting in $s = v_o t + \frac{1}{2}at^2$, we have

distance s = ________ + $\frac{1}{2}$(________) = ________ ft (up?/down?)

PROBLEMS ON ACCELERATED MOTION SHEET 5

PROBLEM B-6. (THE MOST IMPORTANT PROBLEM) A bird sits in a tree, 48 feet above the ground. A man on the ground vertically below shoots a projectile vertically up at the bird with initial velocity 64 ft/sec upward. How long will the projectile take to reach the bird?

A. ARITHMETICAL METHOD. Here the methods of arithmetic and/or common-sense become almost impossibly clumsy. You could find out where and when the "vertex" is, and work the problem from there, but the algebraic method is neater and more interesting. The difficulty is you do not know the velocity of the projectile when it reaches the bird.

B. ALGEBRAIC METHOD. Here we must distinguish between up and down. It does not matter which you label + so long as you keep to your choice. (Try choosing each way, and you will reach the same equations and the same answers in both cases.) It feels more comfortable to call upward distances and velocities and accelerations all + here. We will work the problem with that choice.

In that case, the (downward) acceleration must be called -32 ft/sec/sec.

Then v_o = +64 ft/sec; s = +48 feet; a = -32 ft/sec/sec

We wish to find the time, t, taken to reach the bird, 48 ft above the ground.

Substituting in the relation $s = v_o t + \frac{1}{2}at^2$, we have

$$\_\_\_\_\_\_ = (\_\_\_\_\_\_)t + \tfrac{1}{2}(\_\_\_\_\_\_)t^2$$

This is an ordinary quadratic equation. Like any quadratic, it has two answers. Simplify it and solve it by any method, in the space below.

Answers: t = ________ seconds or t = ________ seconds

One answer gives the time taken to fly up and hit the bird. Suggest below a meaning for the other answer. ________________________________

____

With his limited instructions, how could the faithful servant, mathematics, do otherwise than yield both answers?

PROBLEMS FOR APPENDIX B

Problems B-1 and B-2 are in the text.

B-3, -4, -5, -6. Dissected problems, to be worked on special sheets.

B-7. DOUBLE ANSWERS. (Another problem, like Problem B-6. Try this problem, using the methods shown in dissected problems, B-3 to B-6, and the hints given here. Leave it if you find it too hard, but try it anyway.)

A man standing on the top of a tower throws a stone up into the air with initial velocity 32 ft/sec upward. The man's hand is 48 ft above the ground.

(a) How long will the stone take to reach the ground? [*Hint*: Once again you must use + and — signs. If you choose + for upward, the *acceleration must have a negative value and the distance s from hand to ground being downward must have a negative value; but the initial* velocity will be +. If, disliking negative signs, you choose to use + for downwards, then you will still get the same equations and answers. Try both if you like, *but do not mix the two in one set of calculations.*

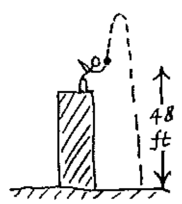

FIG. 1-24. PROBLEM B-7

(b) Once again, you will have a quadratic with two answers. Try to state a meaning for "the other answer." In doing this, ask yourself, "Was the mathematical machine ever told that the man actually threw the stone?"

EASY PROBLEMS ON FREE FALL MOTION* (Neglect air resistance)

v	?
v_0	−5 ft/sec
a	+32 ft/sec/sec
s	X
t	2 sec
$\bar{v}$	X

* In working problems on accelerated motion, you will find it pays to organize your information clearly, like a good engineer. A table like this is worth making. Write your data in the table, with ? where you seek information, and X where you neither have it nor want it. (This specimen shows the data and question in Problem B-10(a).) Then you can see which algebraic relation will be useful. (In this example it must be the one that does not contain s.)

B-8. A helicopter, remaining still above the ground, drops a small mailbag. When the bag has fallen for 2 seconds:
(a) What is its speed?
(b) How far has it fallen?

★ B-9. FREE FALL FROM MOVING OBJECT

A helicopter, falling steadily 5 ft/sec without acceleration, releases a small mailbag. After 2 seconds:
(a) What is the speed of the bag?
(b) How far has it fallen?
(c) How far is it below the helicopter?

★ B-10. A helicopter, rising steadily 5 ft/sec, releases a small mailbag. After 2 seconds:
(a) What is the speed of the bag?
(b) How far has it fallen?
(c) How far is it below the helicopter?

★ B-11. FREE FALL FROM MOVING OBJECT

What common property is shown by the answers to Problems 8, 9, 10?

★ B-12. IMPORTANT PROBLEM (Answer needed for later problems)

A man standing on a shelf 4 ft above the floor steps off and falls to the floor.
(a) How long does he take to fall?
(b) What is his speed just before landing?

★ B-13. CAR BRAKES

A certain car with smooth tires on a wet road can have an acceleration of 1/5 of "g" but not more. (To accelerate, the car must be pushed by some real, external, agent. The agent is the road, pushing the car by friction. With these tires, friction can provide up to $g/5$ acceleration but, if asked to provide more, the wheels begin to slip and friction falls to an even lower value, giving a smaller acceleration.)
(a) What speed will the car gain in 4 secs, with this maximum acceleration?
(b) How far can it travel from rest in 4 secs?

★ B-14. CAR BRAKES AND SAFETY

A car with good brakes but smooth tires on a wet road can have a deceleration of 1/5 of "g" but not more (see Problem B-13). Discuss the stopping of this car by answering the following questions:
(a) Driving at 30 miles/hour (= 44 ft/sec) the driver takes 1 sec to react to danger, decide to stop and get the brakes working; then he makes the brakes give maximum deceleration.
  (i) How far does he travel in the 1 second before braking?
  (ii) How much time do the brakes then take to reduce the speed from 30 miles/hour to zero?
  (iii) How far does the car travel in the braking time?
  (iv) How far does the car travel in the *total* time from seeing the danger until stopped?
(b) If the car is travelling twice as fast, 60 miles/hour, how far does it travel in the *total* time, as in (iv) above?
(c) The car is travelling 30 miles/hour and the driver (after 1 second of thought, etc.) jams the brakes on so that the tires skid, commanding less friction, giving a deceleration of only $g/8$. How far does the car travel in coming to rest? (Sliding friction in a skid is unable to provide such a large maximum force as non-slip friction.)
(d) With new tires on dry concrete, the car has maximum deceleration $g/2$. (Friction of rubber on concrete can do much better than that, but many a brake mechanism can not.) Again calculate the total distance of stopping, from 30 mi/hr.

B-15. "g" IN A MOVING LABORATORY

A portable timing apparatus can now be made to time free fall of a few feet from rest accurately enough to give a value of "g" reliable within 1% or better. Suppose such an apparatus gave $g = 32$ feet/sec². What would you expect it to give:
(a) If used in a railroad train running smoothly at fixed speed along a level track? (Think what happens when you drop something, say an orange, in a moving train.)
(b) In an elevator moving downward at constant speed? (*Hint*: Think . . .)
(c) In an elevator falling freely after its cable has broken?
(d) In elevator accelerating downward 16 ft/sec²? (Make a bold guess.)
(e) In an elevator accelerating up 16 ft/sec²?

MORE SIMPLE PROBLEMS ON FREE FALL

B-16. How long would it take a freely falling body to fall 400 feet from rest?

B-17. A ball is thrown upward with speed 80 ft/sec. How high will it rise?

B-18. An explorer discovers a deep crevasse in a rocky mountain. He drops a stone into it and 4 seconds later he hears the sound of the stone hitting the bottom of the crevasse.
(a) Estimate the depth.
(b) Comment on the accuracy of this method.

B-19. A stone thrown vertically upward with initial velocity 40 ft/sec takes 1 second to reach a bird.
(a) What is the vertical height of the bird above the thrower?
(b) A time of 1.5 seconds gives the same answer for the bird's height. Give a physical reason for this duplicity.

PROBLEMS ON APPARATUS OF PROBLEM B-2

B-20. Why is the lamp (or some other resistance) necessary in the arrangement sketched in Problem B-2?

B-21. In the experiment of Problem B-2, the following troubles may occur:
(a) The clock may lag a few tenths of a second in starting.
(b) The clock may lag a few tenths of a second in stopping.
(c) The pegs at the top, being compressed when the ball is held there, may give the ball a small downward shove when it is released.
(d) Air friction may have an appreciable effect.
  (i) For each of the troubles (a)-(d), say whether it, operating alone, would make the estimated value of "g" too big or too small; and give a brief reason for your answer.
  (ii) What would happen if (a) and (b) operated together, about equally?
  (iii) Suggest experiments to test for each trouble, (a)-(d). Describe them with sketches where possible.