APPENDIX 1

1. Pretend, for a moment, that $x$ has a fixed value. Let’s denote this fixed value by $x_i$ (“i” for “initial”). Let’s label the corresponding $y$-value $y_i$. The linear equation then reads

$$y_i = mx_i + b.$$ 

Now, let’s add 1 to the $x$-value. We do this by replacing $x_i$ in the equation by $x_i + 1$. Let’s call the new $y$-value $y_f$ (“f” for “final”). The new linear equation is

$$y_f = m(x_i + 1) + b.$$ 

We can distribute the $m$ on the right-hand side of this equation: $m(x_i + 1) = mx_i + m$. This simplifies the equation to

$$y_f = mx_i + m + b.$$ 

Now look closely: the right-hand side is just $m$ plus $mx_i + b$ (the order of the terms doesn’t matter). But since $y_i = mx_i + b$, we can substitute this in to get

$$y_f = y_i + m.$$ 

Okay, let’s recap what happened. After increasing the $x$-value by one unit (from $x_i$ to $x_i + 1$), the new $y$-value ($y_f$) ended up being the initial $y$-value ($y_i$) plus the slope $m$. If $m$ is positive, then $y_f$ is bigger than $y_i$ (the $y$-value has increased) whereas if it’s negative, $y_f$ is smaller than $y_i$ (the $y$-value has decreased). That’s the generalized slope interpretation I italicized on page 6.
2. Let’s solve $4x + 370 \leq 400$ using algebra. (Since there aren’t any negative numbers in our inequality the inequality sign gets treated the same as an equals sign.) Ready? Let’s begin.

1. Here’s the starting inequality: $4x + 370 \leq 400$
2. Now subtract 370 from both sides: $4x \leq 30$
3. Finally, divide both sides by 4: $x \leq \frac{30}{4} = 7.5$

3. Here’s how you would “mathematize” this problem. Let $p$ be the total grams of protein eaten in a day, $c$ the total grams of carbs, and $f$ the total grams of fat. The Atwater general factor factor system tells us that the protein contains $4p$ calories, the carbs $4c$ calories, and the fat $9f$ calories. The total calories eaten, $T$, is then

$$T = 4p + 4c + 9f.$$ 

This equation is an example of a multilinear function. We’ll discuss these in more detail in the next section. For now note that we can go through the same analysis of capping the total calories, $T$, to a certain number and then solving for the grams of each macronutrient. For example, capping $T$ at 1,000 yields the inequality

$$4p + 4c + 9f \leq 1,000.$$ 

If we know two of three variables in this equation we can solve for the remaining variable. For example, if you wanted to stick to a diet low in carbs (say, $c = 150$) and fat (say, $f = 20$), then your diet would have at most 55 grams of protein (i.e., $p \leq 55$).

4. The full $RMR_m$ equation involves four variables. To graph it would require a four-dimensional graph, which we can’t visualize. But if I plug in a height, say, $h = 67$, we get an equation with three variables:

$$RMR_m = 4.5w - 5a + 1070.3.$$  

(A1.1)

This equation requires a three-dimensional graph. But that’s okay; we graph in 3D just like we graph in 2D. We first draw the $xy$-plane on the
Appendix 1

Figure A1.1. The 3D graph of equation (A1.1) for weight values $w$ between 0 and 200 and age values $a$ between 0 and 60.

bottom (like a floor) and then add a third axis going up. Then we plot a bunch of points relative to the origin (defined to be where the upward axis intersects the plane) and connect the dots. Figure A1.1 shows the graph of (A1.1) (called a plane). Planes are multilinear functions (note the lines that make up the edges of the plane in the figure). To illustrate that, notice that setting $w = 0$ gives $\text{RMR}_m = -5a + 1070.3$. This is the downward sloping line (the slope is $-5$) connecting the points labeled $A$ and $B$ in the figure.

5. Starting from $20 = 0.15r - 8.85$, we…

a. Add 8.85 to both sides: $0.15r = 28.85$,

b. Divide both sides by 0.15: $r = \frac{28.85}{0.15} = 192.3 \approx 192$.

Here $\approx$ means “approximately.” (I’ve put a list of the mathematical symbols in the Glossary of Mathematical Symbols in Appendix A.)
Every polynomial has the form
\[ y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \]
for some numbers \(a_0, a_1, \ldots, a_n\) (we assume \(a_n \neq 0\)), and some non-negative whole number \(n\). The number \(n\) in this equation is called the degree of the polynomial; it’s the highest power of \(x\) present. Table A1.1 gives the general form of polynomials of degree 0 to 3, along with their names and concrete examples.

7. To find the answer we set \(MHR = MHR_{pop}\):

\[ 220 - a = 192 - 0.007a^2. \]

Adding 0.007\(a^2\) to both sides and subtracting 192 from both sides yields

\[ 0.007a^2 - a + 28 = 0. \]

The fastest way to solve this is to use the quadratic formula, which says that the solutions to \(Ax^2 + Bx + C = 0\) are

\[ x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \]

The \(\pm\) symbol means “plus or minus” (see the Glossary of Mathematical Symbols in Appendix A). It tells us to write down two solutions: one that uses the + sign and another that uses the − sign. Comparing \(Ax^2 + Bx + C = 0\) to 0.007\(a^2\) − \(a + 28 = 0\), we see that \(A = 0.007\),
Appendix 1  •  17

\( B = -1 \), and \( C = 28 \) (and \( x = a \)). The quadratic formula then gives the two solutions
\[
    a = \frac{20(25 - 3\sqrt{15})}{7} \approx 38.2, \quad a = \frac{20(25 + 3\sqrt{15})}{7} \approx 104.6.
\]
The first solution is the age (\( a \)-value) of the visible intersection point in Figure 1.2(b); the other solution corresponds to the other intersection point (not shown on the graph).

8. I’ll show you how to mathematize this using Jason’s ACB equation. Let \( t \) be the number of minutes it takes him to burn \( c \) calories. This means that
\[
    \text{ACB} = \frac{c}{t},
\]
since ACB is the aerobic caloric burn per minute. This, together with Jason’s ACB equation implies that
\[
    \frac{c}{t} = 0.15r - 8.85.
\]
Since Jason’s MHR is about 192 bpm, then \( x\% \) of that is \( \frac{192x}{100} \). (For example, to find 50\% of his MHR we’d first divide 50 by 100 and then multiply the result by 192.) Thus, Jason will be exercising at this heart rate:
\[
    r = \frac{192x}{100}.
\]
Inserting this into the previous equation yields
\[
    \frac{c}{t} = 0.15 \left( \frac{192x}{100} \right) - 8.85 \Rightarrow \frac{c}{t} = 0.228x - 8.85.
\]
To solve for \( t \) we take the reciprocal of both sides (the reciprocal of \( \frac{a}{b} \) is \( \frac{b}{a} \)) and then multiply both sides by \( c \):
\[
    t = \frac{c}{0.228x - 8.85}.
\]
For example, if Jason wanted to burn 400 calories (\( c = 400 \)) by exercising at 70\% (\( x = 70 \)) of his MHR, this analysis estimates it would take him about \( t \approx 46 \) minutes.