HOW TO USE THIS BOOK

This book was written to help you learn and explore probability. You can use this as a supplement to almost any first text on the subject, or as a stand-alone introduction to the material (instructors wishing to use this as a textbook can e-mail me at the addresses at the end of this section for a partial solution key to exercises and exams). As you'll see in your studies, probability is a vast subject with numerous applications, techniques, and methods. This is both exciting and intimidating. It's exciting as you'll see so many strange connections and seemingly difficult problems fall from learning the right way to look at things, but it's also intimidating as there is so much material it can be overwhelming.

My purposes are to help you navigate this rich landscape, and to prepare you for future studies. The presentation has been greatly influenced by Adrian Banner's successful *The Calculus Lifesaver*. Like that book, the goals are to teach you the material and mathematical thinking through numerous worked out problems in a relaxed and informal style. While you'll find the standard statements and proofs, you'll also find a wealth of worked out examples and lots of discussions on how to approach theorems. The best way to learn a subject is to do it. This doesn't just mean doing worked out problems, though that's a major part and one which sadly, due to limited class time, is often cut in courses. It also means *understanding* the proofs.

Why are proofs so important? We'll see several examples in the book of reasonable statements that turn out to be wrong; the language and formalism of proofs are the mathematician's defense against making these errors. Furthermore, even if your class isn't going to test you on proofs, it's worth having a feel as to why something is true. You're not expected on Day One to be able to create the proofs from scratch, but that's not a bad goal for the end of the course. To help you, we spend a lot of time discussing *why* we prove things the way we do and what is it in the problem that suggests we try one approach over another. The hope is that by highlighting these ideas, you'll get a better sense of why the theorems are true, and be better prepared to not only use them, but be able to prove results on your own in future classes.

Here are some common questions and answers on this book and how to use it.

- **What do I need to know before I start reading?** You should be familiar and comfortable with algebra and pre-calculus. Unlike the companion book *The Calculus Lifesaver*, the courses this book is meant to supplement (or be!) are far more varied. Some probability courses don't assume any calculus, while others build on real analysis and measure theory or are half probability and half statistics. As much as possible, we've tried to minimize the need for calculus, especially in the introductory chapters. This doesn't mean these chapters are easier—far from it! It's often a lot easier to do some integrals than find the "right" way to look at a combinatorial probability problem. Calculus becomes indispensable when we reach continuous distributions, as the Fundamental Theorem of Calculus allows us to use anti-derivatives to compute areas, and we'll learn that areas often correspond to probabilities. In fact, continuous probabilities are often easier to study than discrete ones *precisely* because of calculus, as integrals are "easier" than sums. For the most part, we avoid advanced real analysis, save for some introductory comments in the beginning on how to put the subject on a very secure foundation, and some advanced chapters towards the end.

- **Why is this book so long?** An instructor has one tremendous advantage over an author: they can interact with the students, slowing down when the class is having trouble and choosing supplemental topics to fit the interest of who's enrolled in a given year. The author has only one recourse: length! This means that we'll have more explanations than you need in some areas. It also means we'll repeat explanations throughout the book, as many people won't read it in order (more on that later). Hopefully, though, we'll have a good discussion of any concept that's causing you trouble, and plenty of fun supplemental topics to explore, both in the book and online.
- **The topics are out of order from my class! What do I do?** One of my professors, Serge Lang, once remarked that it's a shame that a book has to be ordered along the page axis. There are lots of ways to teach probability, and lots of topics to choose. What you might not realize is that in choosing to do one topic your instructor is often choosing to ignore many others, as there's only so much time in the semester. Thus, while there will be lots of common material from school to school, there's a lot of flexibility in what a professor adds, in what tools they use, and in when they cover certain topics. To aid the reader, we occasionally repeat material to keep the different chapters and sections as self-contained as possible (you may notice we said something to this effect in answering the previous question!). You should be able to jump in at any point, and refer to the earlier chapters and appendixes for the background material as needed.
- **Do I really need to know the proofs?** Short answer: yes. Proofs are important. One of the reasons I went into mathematics is because I hate the phrase "because I told you so." Your professor isn't right just because they're a professor (nor am I right just because this book was published). Everything has to follow from a sound, logical chain. Knowing these justifications will help you understand the material, see connections, and hopefully make sure you never use a result inappropriately, as you'll be a master at knowing the needed conditions. By giving complete, rigorous arguments we try to cut down on the danger of subtly assuming something which may not be true. Probability generates a large number of reasonable, intuitively clear statements that end up being false; rigor is our best defense at avoiding these mistakes. Sadly, it becomes harder and harder to prove our results as the semester progresses. Often courses have some advanced applications, and due to time constraints it's impossible to prove all the

background material needed. The most common example of this in a probability course is in discussions of the proof of the Central Limit Theorem, where typically some results from complex analysis are just stated. We'll always state what we need and try to give a feeling of why it's true, either through informal discussions or an analysis of special cases, and end with references to the literature.

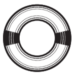
- **Why do you sometimes use "we," and other times use "I"?** Good observation. The convention in mathematics is to always use "we," but that makes it more formal and less friendly at times; those points argue for using "I." To complicate matters, parts of this book were written with various students of mine over the years. This was deliberate for many reasons, ranging from being a great experience for them to making sure the book truly is aimed at students. To continue the confusion, it's nice to use "we" as it gets you involved; we're in this together! I hope you can deal with the confusion our choice of language is causing us!
- **Could my school use this book as a textbook?** Absolutely! To assist we've provided many exercises at the end of each chapter that are perfect for homework; instructors can e-mail me at either Steven.Miller.MC.96@aya.yale.edu or sjm1@williams.edu for additional problems, exams, and their solutions.
- **Some of the methods you use are different from the methods I learned. Who is right—my instructor or you?** Hopefully we're both right! If in doubt, ask your instructor or shoot me an e-mail.
- **Help! There's so much material—how should I use the book?** I remember running review sessions at Princeton where one of the students was amazed that a math book has an index; if you're having trouble with specific concepts this is a great way to zero in on which parts of the book will be most helpful. That said, the goal is to read this book throughout the semester so you won't be rushed. To help you with your reading, on the book's home page is a document which summarizes the key points, terms, and ideas of each section, and has a few quick problems of varying levels of difficulty. I'm a strong believer in preparing for class and reading the material beforehand. I find it very difficult to process new math in real-time; it's a lot easier if I'm at least aware of the definitions and the main ideas before the lecture. These points led to the online summary sheet, which highlights what's going on in each section. Its goal is to help prepare you for exploring each topic, and provide you with a quick assessment of how well you learned it; it's online at http://web.williams.edu/Mathematics/sjmiller/public_html/probabilitylifesaver/problifesaver_comments.pdf.

  To assist you, important formulas and theorems are boxed—that's a strong signal that the result is important and should be learned! Some schools allow you to have one or two pages of notes for exams; even if your school doesn't it's a good idea to prepare such a summary. I've found as a student that the art of writing things down helps me learn the material better.

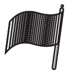
  Math isn't about memorization, but there are some important formulas and techniques that you should have at your fingertips. The act of making the summary is often enough to solidify your understanding. Take notes as you read each chapter; keep track of what you find important, and then check that against the summaries at the end of the chapter, and the document online highlighting the key points of each section.

Try to get your hands on similar exams—maybe your school makes previous years' finals available, for example—and take these exams under proper conditions. That means no breaks, no food, no books, no phone calls, no e-mails, no messaging, and so on. Then see if you can get a solution key and grade it, or ask someone (nicely!) to grade it for you. Another great technique is to write some practice exam problems, and trade lists with a friend. I often found that after an exam or two with a professor I had some sense of what they like, and frequently guessed some of the exam questions. Try some of the exercises at the end of each chapter, or get another book out of the library and try some problems where the solutions are given. The more practice you do the better. For theorems, remove a condition and see what happens. Normally it's no longer true, so find a counterexample (sometimes it is still true, and the proof is just harder). Every time you have a condition it should make an appearance somewhere in the proof—for each result try to know where that happens.

- **Are there any videos to help?** I've taught this class many times (at Brown, Mount Holyoke, and Williams). The last few times at Williams I recorded my lectures and posted them on YouTube with links on my home page, where there's also a lot of additional information from handouts to comments on the material. Please visit http://web.williams.edu/Mathematics/sjmiller/public_html/probabilitylifesaver/ for the course home pages, which include videos of all the lectures and additional comments from each day. As I've taught the course several times, there are a few years' worth of lectures; they're similar across the years but contain slight differences based in part on what my students were interested in. One advantage is that by recording the lectures I can have several special topics where some are presented as lectures during the semester, while others are given to the students to view at home.
- **Who are you, anyway?** I'm currently a math professor at Williams College. I earned my B.S. in math and physics from Yale, moved on and got a Ph.D. in math from Princeton, and have since been affiliated (in order) with Princeton, NYU, the American Institute of Mathematics, The Ohio State University, Boston University, Brown, Williams, Smith, and Mount Holyoke. My main research interests are in number theory and probability, though I do a lot of applied math projects in a variety of fields, especially sabermetrics (the art/science of applying math and stats to baseball). My wife is a professor of marketing; you can see a lot of her influence in what topics were included and how I chose to present them to you! We have two kids, Cam and Kayla, who help TA all my classes, from Probability to the Mathematics of Lego Bricks to Rubik's Cubes.
- **What's with those symbols in the margin?** Throughout the book, the following icons appear in the margin to allow you quickly to identify the thrust of the next few lines. This is the same notation as *The Calculus Lifesaver*.



– A worked out example begins on this line.



– Here's something really important.

– You should try this yourself.

– Beware: this part of the text is mostly for interest. If time is limited, skip to the next section.

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