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**Paul J. Nahin: Digital Dice**

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### 1. *The Clumsy Dishwasher Problem*

We can calculate the theoretical probability as follows. There are  $4\binom{5}{4} + \binom{5}{5}$  ways to assign any four, or all five, of the broken dishes to “clumsy.” The factor of 4 in the term  $4\binom{5}{4}$  is there because, after we have “clumsy” breaking four dishes, the remaining fifth broken dish can be assigned to any of the other four dishwashers. There are a total of  $5^5$  different ways to assign the broken dishes among all five dishwashers. So, the answer to the question of the probability of “clumsy” breaking at least four of the five broken dishes *at random* is

$$\frac{4\binom{5}{4} + \binom{5}{5}}{5^5} = \frac{20+1}{3,125} = \frac{21}{3,125} = 0.00672.$$

The code `dish.m` simulates the problem, assuming the broken dishes occur at random, where the variable `brokendishes` is the number of dishes broken by “clumsy.” If the value of the variable `clumsy` is the total number of times “clumsy” breaks four or more dishes (that is, `clumsy` is the total number of simulations in which `brokendishes > 3`), then, with each such simulation, `clumsy` is incremented by one. When `dish.m` was run, line 14 produced an estimate for the probability that “clumsy” breaks at least four of the five broken dishes, due strictly to random chance, as 0.00676 (when run several times for just 10,000 simulations, the estimates varied from 0.0056 to 0.0083). Theory and experiment

are in pretty good agreement, and in my opinion, this probability is sufficiently small that a reasonable person could reasonably conclude, despite his denials, that “clumsy” really is clumsy! With some non-zero probability (see above), of course, that “reasonable” conclusion would actually be incorrect.

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**dish.m**

```
01 clumsy = 0;
02 for k = 1:1000000
03     brokendishes = 0;
04     for j = 1:5
05         r = rand;
06         if r < 0.2
07             brokendishes = brokendishes + 1;
08         end
09     end
10     if brokendishes > 3
11         clumsy = clumsy + 1;
12     end
13 end
14 clumsy/1000000
```

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