# Chapter 1

## Introductory discussion of quantum macrophysics

Quantum theory began with Planck's [Pl] derivation of the thermodynamics of black body radiation from the hypothesis that the action of his oscillator model of matter was quantised in integral multiples of a fundamental constant, $h$. This result provided a microscopic theory of a macroscopic phenomenon that was incompatible with the assumption of underlying classical laws. In the century following Planck's discovery, it became abundantly clear that quantum theory is essential to natural phenomena on both the microscopic and macroscopic scales. Its crucial role in determining the gross properties of matter is evident from the following considerations.

(1) The stability of matter against electromagnetic collapse is effected only by the combined action of the Heisenberg and Pauli principles [DL, LT, LLS, BFG].

(2) The third law of thermodynamics is quintessentially quantum mechanical and, arguably, so too is the second law.[1]

(3) The mechanisms governing a vast variety of cooperative phenomena, including magnetic ordering [Ma], superfluidity [La1, BCS] and optical and biological coherence [Ha1, Fr1], are of quantum origin.

As a first step towards contemplating the quantum mechanical basis of macrophysics, we note the empirical fact that macroscopic systems enjoy properties that are radically different from those of their constituent particles. Thus, unlike systems of few particles, they exhibit irreversible dynamics, phase transitions and various ordered structures, including those characteristic of life. These and other macroscopic phenomena signify that complex systems, that is, ones consisting of enormous numbers of interacting particles, are qualitatively different from the sums of their constituent parts. Correspondingly, theories of such phenomena must be based not only on quantum mechanics *per se* but also on conceptual structures that serve to represent the characteristic features of highly complex systems. Among the key general

[1] The essential point here is that the classical statistical mechanical formulation of entropy depends on an arbitrary subdivision of phase space into microcells (cf. [Fe, Chapter 8]).

concepts involved here are ones representing various types of order, or organisation, disorder, or chaos, and different levels of macroscopicality. Moreover, the particular concepts required to describe the ordered structures of superfluids and laser light are represented by macroscopic wave functions [PO, Ya, GH, Se1] that are strictly quantum mechanical, although radically different from the Schrödinger wave functions of microphysics.

To provide a mathematical framework for the conceptual structures required for quantum macrophysics, it is clear that one needs to go beyond the traditional form of quantum mechanics [Di, VN1], since that does not discriminate *qualitatively* between microscopic and macroscopic systems. This may be seen from the fact that the traditional theory serves to represent a system of $N$ particles within the standard Hilbert space scheme, which takes the same form regardless of whether $N$ is 'small' or 'large'. In fact, it was this very lack of a sharp characterisation of macroscopicality that forced Bohr [Bo] into a dualistic treatment of the measuring process, in which the microscopic system under observation was taken to be quantum mechanical, whereas the macroscopic measuring apparatus was treated as classical, even though it too was presumably subject to quantum laws.

However, a generalised version of quantum mechanics that provides the required qualitative distinctions between different grades of macroscopicality has been devised over the last three decades, on the basis of an *idealisation* of macroscopic systems as ones possessing infinite numbers of degrees of freedom. This kind of idealisation has, of course, long been essential to statistical thermodynamics, where, for example, the characterisation of phase transitions by singularities in thermodynamical potentials necessitates a passage to the mathematical limit in which both the volume and the number of particles of a system tend to infinity in such a way that the density remains finite [YL, LY, Ru1]. Its extension to the full description of the observables and states of macroscopic systems [AW, HHW, Ru1, Em1] has served to replace the merely quantitative difference between systems of 'few' and 'many' (typically $10^{24}$) particles by the qualitative distinction between finite and infinite ones, and has thereby brought new, physically relevant structures into the theory of collective phenomena [Th, Se2].

The key element of the generalisation of quantum mechanics to infinite systems is that it is based on the algebraic structure of the observables, rather than on the underlying Hilbert space [Seg, HK]. The radical significance of this is that, whereas the algebra of observables of a finite system, as governed by the canonical commutation relations, admits only one irreducible Hilbert space representation [VN2], that of an infinite system has infinitely many inequivalent such representations [GW]. Thus, for a finite system, the algebraic and Hilbert space descriptions are equivalent, while, for an infinite one, the algebraic picture is richer than that provided by any irreducible representation of its observables.

Moreover, the algebraic quantum theory of infinite systems, as cast in a form designed for the treatment of fundamental problems in statistical mechanics and quantum field theory [Em1, BR, Th, Se2, Haa1], admits just the structures required for the treatment of macroscopic phenomena. In particular, it permits clear definitions of various kinds of order, as well as sharp distinctions between global and local variables, which may naturally be identified with macroscopic and microscopic ones. Furthermore, the wealth of inequivalent representations of the observables permits a natural classification of the states in both microscopic and macroscopic terms. To be specific, the vectors in a representation space[2] correspond to states that are macroscopically equivalent but microscopically different, while those carried by different representations are macroscopically distinct. Hence, the macrostate corresponds to a representation and the microstate to a vector in the representation space. This is of crucial significance not only for the description of the various phases of matter, but also for the quantum theory of measurement. The specification of the states of a measuring apparatus in microscopic and macroscopic terms has provided a key element of a fully quantum treatment [He, WE] of the measurement process that liberates the theory from Bohr's dualism.

Our approach to the basic problem of how macrophysics emerges from quantum mechanics will be centred on macroscopic observables, our main objective being to obtain the properties imposed on them by general demands of quantum theory and many-particle statistics. This approach has classic precedents in Onsager's [On] irreversible thermodynamics and Landau's fluctuating hydrodynamics [LL1], and is at the opposite pole from the many-body-theoretic computations of condensed matter physics [Pi, Tho]. Our motivation for pursuing this approach stems from the following two considerations. Firstly, since the observed laws of macrophysics have relatively simple structures, which do not depend on microscopic details, it is natural to seek derivations of these laws that are based on general quantum macrostatistical arguments. Secondly, by contrast, the microscopic properties of complex systems are dominated by the molecular chaos that is at the heart of statistical physics; and presumably, this chaos would render unintelligible any solutions of the microscopic equations of motion of realistic models of such systems, even if these could be obtained with the aid of supercomputers.

Thus, we base this treatise on macroscopic observables and certain general structures of complex systems, as formulated within the terms of the algebraic framework of quantum theory. The next three chapters are devoted to a concise formulation of this framework, for both conservative and open systems (Chapter 2), and of the descriptions that it admits of symmetry, order and disorder (Chapter 3), and of irreversibility (Chapter 4).

[2] To be precise (cf. Section 2.6.3), this is true for primary representations.