

Chapter 1

Preliminaries: Two-Sector Models

WE BEGIN our study of international trade with the classic Ricardian model, which has two goods and one factor (labor). The Ricardian model introduces us to the idea that technological differences across countries matter. In comparison, the Heckscher-Ohlin model dispenses with the notion of technological differences and instead shows how *factor endowments* form the basis for trade. While this may be fine in theory, the model performs very poorly in practice: as we show in the next chapter, the Heckscher-Ohlin model is hopelessly inadequate as an explanation for historical or modern trade patterns unless we allow for technological differences across countries. For this reason, the Ricardian model is as relevant today as it has always been. Our treatment of it in this chapter is a simple review of undergraduate material, but we will have the opportunity to refer to this model again at various places throughout the book.

After reviewing the Ricardian model, we turn to the two-good, two-factor model that occupies most of this chapter and forms the basis of the Heckscher-Ohlin model. We shall suppose that the two goods are traded on international markets, but we do not allow for any movements of factors across borders. This reflects the fact that the movement of labor and capital across countries is often subject to controls at the border and generally much less free than the movement of goods. Our goal in the next chapter will be to determine the pattern of international trade between countries. In this chapter, we simplify things by focusing primarily on *one* country, treating world prices as given, and examine the properties of this two-by-two model. The student who understands all the properties of this model has already come a long way in his or her study of international trade.

Ricardian Model

Indexing goods by the subscript i , let a_i denote the labor needed per unit of production of each good at home, while a_i^* is the labor need per unit of production in the foreign country, $i = 1, 2$. The total labor force at home is L and abroad is L^* . Labor is perfectly mobile between the industries in each

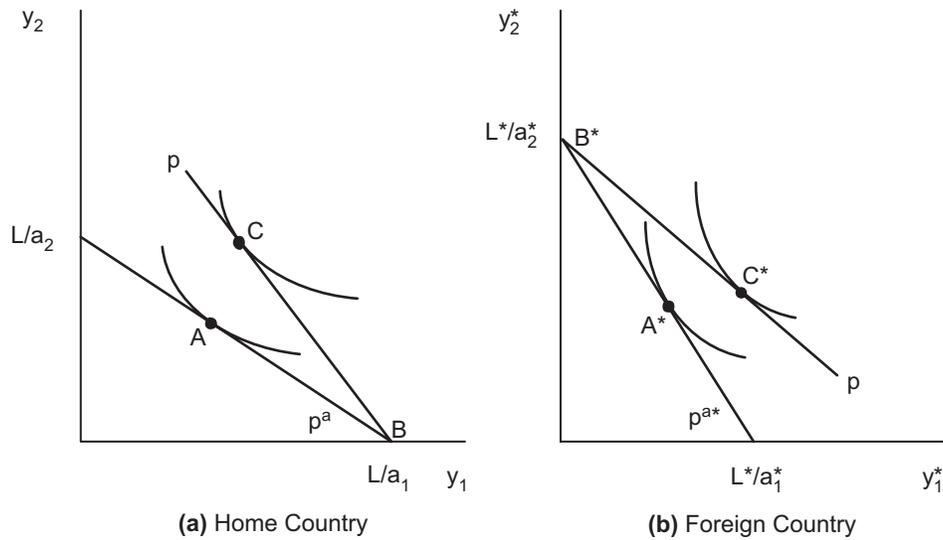


Figure 1.1

country, but immobile across countries. This means that both goods are produced in the home country only if the wages earned in the two industries are the same. Since the marginal product of labor in each industry is $1/a_i$, and workers are paid the value of their marginal products, wages are equalized across industries if and only if $p_1/a_1 = p_2/a_2$, where p_i is the price in each industry. Letting $p = p_1/p_2$ denote the *relative price* of good 1 (using good 2 as the numeraire), this condition is $p = a_1/a_2$.

These results are illustrated in Figure 1.1(a) and (b), where we graph the production possibility frontiers (PPFs) for the home and foreign countries. With all labor devoted to good i at home, it can produce L/a_i units, $i = 1, 2$, so this establishes the intercepts of the PPF, and similarly for the foreign country. The slope of the PPF in each country (ignoring the negative sign) is then a_1/a_2 and a_1^*/a_2^* . Under autarky (i.e., no international trade), the equilibrium relative prices p^a and p^{a*} must equal these slopes in order to have both goods produced in both countries, as argued above. Thus, the autarky equilibrium at home and abroad might occur at points A and A*. Suppose that the home country has a *comparative advantage* in producing good 1, meaning that $a_1/a_2 < a_1^*/a_2^*$. This implies that the home autarky relative price of good 1 is *lower* than that abroad.

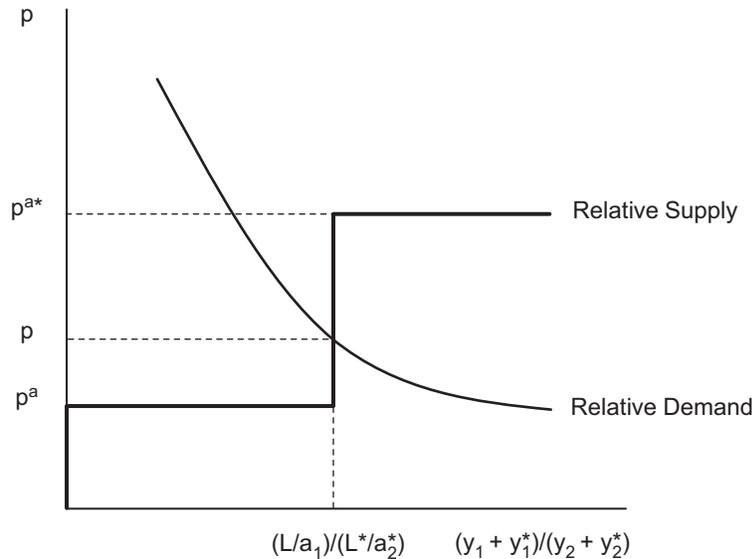


Figure 1.2

Now letting the two countries engage in international trade, what is the equilibrium price p at which world demand equals world supply? To answer this, it is helpful to graph the world relative supply and demand curves, as illustrated in Figure 1.2. For the relative price satisfying $p < p^a = a_1/a_2$ and $p < p^{a*} = a_1^*/a_2^*$ both countries are fully specialized in good 2 (since wages earned in that sector are higher), so the world relative supply of good 1 is zero. For $p^a < p < p^{a*}$, the home country is fully specialized in good 1, whereas the foreign country is still specialized in good 2, so that the world relative supply is $(L/a_1)/(L^*/a_2^*)$, as labeled in Figure 1.2. Finally, for $p > p^a$ and $p > p^{a*}$, both countries are specialized in good 1. So we see that the world relative supply curve has a “stair-step” shape, which reflects the linearity of the PPFs.

To obtain world relative demand, let us make the simplifying assumption that tastes are identical and homothetic across the countries. Then demand will be independent of the distribution of income *across* the countries. Demand being homothetic means that *relative* demand d_1/d_2 in either country is a downward-sloping function of the relative price p , as illustrated in Figure 1.2. In the case we have shown, relative demand intersects relative supply at the world price p that lies *between* p^a and p^{a*} , but this does not need to occur: instead, we can have relative demand intersect

one of the flat segments of relative supply, so that the equilibrium price with trade *equals* the autarky price in one country.¹

Focusing on the case where $p^a < p < p^{a*}$, we can go back to the PPF of each country and graph the production and consumption points with free trade. Since $p > p^a$, the home country is fully specialized in good 1 at point B, as illustrated in Figure 1.1(a), and then trades at the relative price p to obtain consumption at point C. Conversely, since $p < p^{a*}$, the foreign country is fully specialized in the production of good 2 at point B* in Figure 1.1(b), and then trades at the relative price p to obtain consumption at point C*. Clearly, *both* countries are better off under free trade than they were in autarky: trade has allowed them to obtain a consumption point that is above the PPF.

Notice that the home country exports good 1, which is in keeping with its comparative advantage in the production of that good, $a_1/a_2 < a_1^*/a_2^*$. Thus, *trade patterns are determined by comparative advantage*, which is a deep insight from the Ricardian model. This occurs even if one country has an *absolute disadvantage* in both goods, such as $a_1 > a_1^*$ and $a_2 > a_2^*$, so that more labor is needed per unit of production of *either* good at home than abroad. The reason that it is still possible for the home country to export is that its *wages* will adjust to reflect its productivities: under free trade, its wages are lower than those abroad.² Thus, while trade patterns in the Ricardian model are determined by *comparative advantage*, the level of wages across countries is determined by *absolute advantage*.

Two-Good, Two-Factor Model

While the Ricardian model focuses on technology, the Heckscher-Ohlin model, which we study in the next chapter, focuses on factors of production. So we now assume that there are two factor inputs—labor and capital. Restricting our attention to a single country, we will suppose that it produces two goods with the production functions $y_i = f_i(L_i, K_i)$, $i = 1, 2$, where y_i is the output produced using labor L_i and capital K_i . These production functions are assumed to be increasing, concave, and homogeneous

¹This occurs if one country is very large. Use Figures 1.1 and 1.2 to show that if the home country is very large, then $p = p^a$ and the home country does not gain from trade.

²The home country exports good 1, so wages earned with free trade are $w = p/a_1$. Conversely, the foreign country exports good 2 (the numeraire), so wages earned there are $w^* = 1/a_2^* > p/a_1^*$, where the inequality follows since $p < a_1^*/a_2^*$ in the equilibrium being considered. Then using $a_1 > a_1^*$, we obtain $w = p/a_1 < p/a_1^* < w^*$.

of degree one in the inputs (L_i, K_i) .³ The last assumption means that there are *constant returns to scale* in the production of each good. This will be a maintained assumption for the next several chapters, but we should point out that it is rather restrictive. It has long been thought that *increasing returns to scale* might be an important reason to have trade between countries: if a firm with increasing returns is able to sell in a foreign market, this expansion of output will bring a reduction in its average costs of production, which is an indication of greater efficiency. Indeed, this was a principal reason that Canada entered into a free-trade agreement with the United States in 1989: to give its firms free access to the large American market. We will return to these interesting issues in chapter 5, but for now, ignore increasing returns to scale.

We will assume that labor and capital are *fully mobile* between the two industries, so we are taking a “long run” point of view. Of course, the amount of factors employed in each industry is constrained by the endowments found in the economy. These resource constraints are stated as

$$\begin{aligned}L_1 + L_2 &\leq L, \\K_1 + K_2 &\leq K,\end{aligned}\tag{1.1}$$

where the endowments L and K are fixed. Maximizing the amount of good 2, $y_2 = f_2(L_2, K_2)$, subject to a given amount of good 1, $y_1 = f_1(L_1, K_1)$, and the resource constraints in (1.1) give us $y_2 = h(y_1, L, K)$. The graph of y_2 as a function of y_1 is shown as the PPF in Figure 1.3. As drawn, y_2 is a *concave* function of y_1 , $\partial^2 h(y_1, L, K)/\partial y_1^2 < 0$. This familiar result follows from the fact that the production functions $f_i(L_i, K_i)$ are assumed to be concave. Another way to express this is to consider all points $S = (y_1, y_2)$ that are feasible to produce given the resource constraints in (1.1). This production possibilities set S is *convex*, meaning that if $y^a = (y_1^a, y_2^a)$ and $y^b = (y_1^b, y_2^b)$ are both elements of S , then any point between them $\lambda y^a + (1 - \lambda)y^b$ is also in S , for $0 \leq \lambda \leq 1$.⁴

The production possibility frontier summarizes the technology of the economy, but in order to determine where the economy produces on the PPF we need to add some assumptions about the market structure. We will assume perfect competition in the product markets and factor markets. Furthermore, we will suppose that product prices are given *exogenously*: we can think of these prices as established on world markets, and outside the control of the “small” country being considered.

³Students not familiar with these terms are referred to problems 1.1 and 1.2.

⁴See problems 1.1 and 1.3 to prove the convexity of the production possibilities set, and to establish its slope.

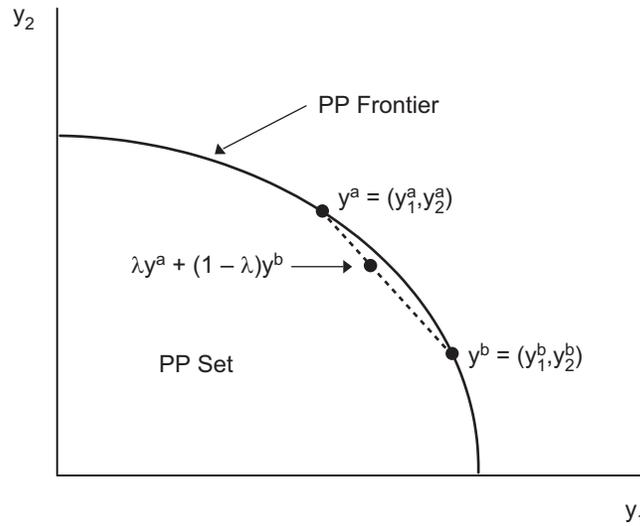


Figure 1.3

GDP Function

With the assumption of perfect competition, the amounts produced in each industry will maximize gross domestic product (GDP) for the economy: this is Adam Smith’s “invisible hand” in action. That is, the industry outputs of the competitive economy will be chosen to maximize GDP:

$$G(p_1, p_2, L, K) = \max_{y_1, y_2} p_1 y_1 + p_2 y_2 \quad \text{s.t.} \quad y_2 = h(y_1, L, K). \quad (1.2)$$

To solve this problem, we can substitute the constraint into the objective function and write it as choosing y_1 to maximize $p_1 y_1 + p_2 h(y_1, L, K)$. The first-order condition for this problem is $p_1 + p_2(\partial h / \partial y_1) = 0$, or

$$p = \frac{p_1}{p_2} = -\frac{\partial h}{\partial y_1} = -\frac{\partial y_2}{\partial y_1}. \quad (1.3)$$

Thus, the economy will produce where the relative price of good 1, $p = p_1/p_2$, is equal to the slope of the production possibility frontier.⁵ This is illustrated by the point A in Figure 1.4, where the line tangent through point A has the slope of (negative) p . An increase in this price will *raise* the

⁵Notice that the slope of the price line tangent to the PPF (in absolute value) equals the relative price of the good on the *horizontal* axis, or good 1 in Figure 1.4.

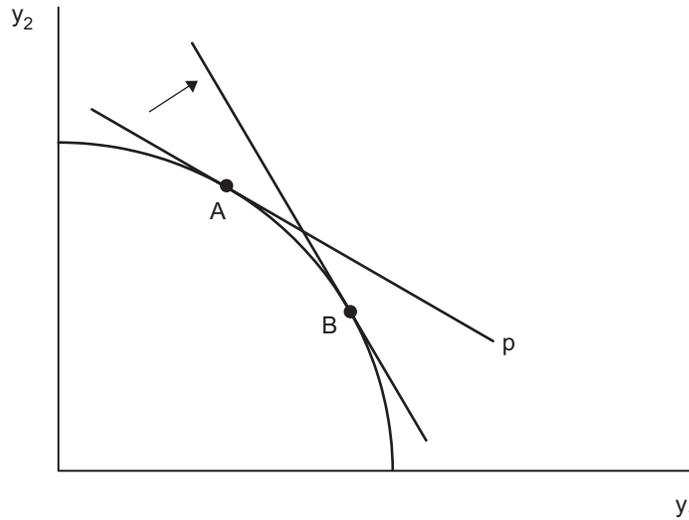


Figure 1.4

slope of this line, leading to a new tangency at point B. As illustrated, then, the economy will produce more of good 1 and less of good 2.

The GDP function introduced in (1.2) has many convenient properties, and we will make use of it throughout this book. To show just one property, suppose that we differentiate the GDP function with respect to the price of good i , obtaining

$$\frac{\partial G}{\partial p_i} = y_i + \left(p_1 \frac{\partial y_1}{\partial p_i} + p_2 \frac{\partial y_2}{\partial p_i} \right). \quad (1.4)$$

It turns out that the terms in parentheses on the right-hand side of (1.4) sum to zero, so that $\partial G/\partial p_i = y_i$. In other words, the derivative of the GDP function with respect to *prices* equals the *outputs* of the economy.

The fact that the terms in parentheses sum to zero is an application of the “envelope theorem,” which states that when we differentiate a function that has been maximized (such as GDP) with respect to an exogenous variable (such as p_i), then we can *ignore* the changes in the endogenous variables (y_1 and y_2) in this derivative. To prove that these terms sum to zero, totally differentiate $y_2 = b(y_1, L, K)$ with respect to y_1 and y_2 and use (1.3) to obtain $p_1 dy_1 = -p_2 dy_2$, or $p_1 dy_1 + p_2 dy_2 = 0$. This equality must hold for any small movement in y_1 and y_2 around the PPF, and in particular, for the small movement in outputs *induced* by the change in p_i . In other words, $p_1(\partial y_1/\partial p_i) + p_2(\partial y_2/\partial p_i) = 0$, so the

terms in parentheses on the right of (1.4) vanish and it follows that $\partial G/\partial p_i = y_i$.⁶

Equilibrium Conditions

We now want to state succinctly the equilibrium conditions to determine factor prices and outputs. It will be convenient to work with the *unit-cost functions* that are dual to the production functions $f_i(L_i, K_i)$. These are defined by

$$c_i(w, r) = \min_{L_i, K_i \geq 0} \{wL_i + rK_i \mid f_i(L_i, K_i) \geq 1\}. \quad (1.5)$$

In words, $c_i(w, r)$ is the minimum cost to produce one unit of output. Because of our assumption of constant returns to scale, these unit-costs are equal to both marginal costs and average costs. It is easily demonstrated that the unit-cost functions $c_i(w, r)$ are nondecreasing and concave in (w, r) . We will write the *solution* to the minimization in (1.5) as $c_i(w, r) = wa_{iL} + ra_{iK}$, where a_{iL} is optimal choice for L_i , and a_{iK} is optimal choice for K_i . It should be stressed that these optimal choices for labor and capital *depend* on the factor prices, so that they should be written in full as $a_{iL}(w, r)$ and $a_{iK}(w, r)$. However, we will usually not make these arguments explicit.

Differentiating the unit-cost function with respect to the wage, we obtain

$$\frac{\partial c_i}{\partial w} = a_{iL} + \left(w \frac{\partial a_{iL}}{\partial w} + r \frac{\partial a_{iK}}{\partial w} \right). \quad (1.6)$$

As we found with differentiating the GDP function, it turns out that the terms in parentheses on the right of (1.6) sum to zero, which is again an application of the “envelope theorem.” It follows that the derivative of the unit-costs with respect to the wage equals the labor needed for one unit of production, $\partial c_i/\partial w = a_{iL}$. Similarly, $\partial c_i/\partial r = a_{iK}$.

To prove this result, notice that the constraint in the cost-minimization problem can be written as the isoquant $f_i(a_{iL}, a_{iK}) = 1$. Totally differentiate this to obtain $f_{iL} da_{iL} + f_{iK} da_{iK} = 0$, where $f_{iL} \equiv \partial f_i/\partial L_i$ and $f_{iK} \equiv \partial f_i/\partial K_i$. This equality must hold for any small movement of labor da_{iL} and capital da_{iK} around the isoquant, and in particular, for the change in labor and capital *induced* by a change in wages. Therefore, $f_{iL}(\partial a_{iL}/\partial w) + f_{iK}(\partial a_{iK}/\partial w) = 0$. Now multiply this through by the product price p_i , noting that $p_i f_{iL} = w$ and $p_i f_{iK} = r$ from the profit-

⁶Other convenient properties of the GDP function are explored in problem 1.4.

maximization conditions for a competitive firm. Then we see that the terms in parentheses on the right of (1.6) sum to zero.

The first set of equilibrium conditions for the two-by-two economy is that *profits equal zero*. This follows from free entry under perfect competition. The zero-profit conditions are stated as

$$\begin{aligned} p_1 &= c_1(w, r), \\ p_2 &= c_2(w, r). \end{aligned} \tag{1.7}$$

The second set of equilibrium conditions is full employment of both resources. These are the same as the resource constraints (1.1), except that now we express them as equalities. In addition, we will rewrite the labor and capital used in each industry in terms of the derivatives of the unit-cost function. Since $\partial c_i / \partial w = a_{iL}$ is the labor used for *one unit* of production, it follows that the total labor used in $L_i = y_i a_{iL}$, and similarly the total capital used is $K_i = y_i a_{iK}$. Substituting these into (1.1), the full-employment conditions for the economy are written as

$$\begin{aligned} \underbrace{a_{1L}y_1}_{L_1} + \underbrace{a_{2L}y_2}_{L_2} &= L, \\ \underbrace{a_{1K}y_1}_{K_1} + \underbrace{a_{2K}y_2}_{K_2} &= K. \end{aligned} \tag{1.8}$$

Notice that (1.7) and (1.8) together are *four* equations in *four* unknowns, namely, (w, r) and (y_1, y_2) . The parameters of these equations, $p_1, p_2, L,$ and $K,$ are given exogenously. Because the unit-cost functions are nonlinear, however, it is not enough to just count equations and unknowns: we need to study these equations in detail to understand whether the solutions are unique and strictly positive, or not. Our task for the rest of this chapter will be to understand the properties of these equations and their solutions.

To guide us in this investigation, there are three key questions that we can ask: (1) what is the solution for factor prices? (2) if prices change, how do factor prices change? (3) if endowments change, how do outputs change? Each of these questions is taken up in the sections that follow. The methods we shall use follow the “dual” approach of Woodland (1977, 1982), Mussa (1979), and Dixit and Norman (1980).

Determination of Factor Prices

Notice that our four-equation system above can be decomposed into the zero-profit conditions as *two* equations in *two* unknowns—the wage and

rental—and then the full-employment conditions, which involve both the factor prices (which affect a_{iL} and a_{iK}) and the outputs. It would be especially convenient if we could uniquely solve for the factor prices from the zero-profit conditions, and then just substitute these into the full-employment conditions. This will be possible when the hypotheses of the following lemma are satisfied.

Lemma (Factor Price Insensitivity)

So long as both goods are produced, and factor intensity reversals (FIRs) do not occur, then each price vector (p_1, p_2) corresponds to unique factor prices (w, r) .

This is a remarkable result, because it says that the factor endowments (L, K) do not matter for the determination of (w, r) . We can contrast this result with a one-sector economy, with production of $y = f(L, K)$, wages of $w = pf_L$, and diminishing marginal product $f_{LL} < 0$. In this case, any increase in the labor endowments would certainly reduce wages, so that countries with higher labor/capital endowments (L/K) would have lower wages. This is the result we normally expect. In contrast, the above lemma says that in a two-by-two economy, with a fixed product price p , it is possible for the labor force or capital stock to grow *without* affecting their factor prices! Thus, Leamer (1995) refers to this result as “factor price insensitivity.” Our goal in this section is to prove the result and also develop the intuition for why it holds.

Two conditions must hold to obtain this result: first, that both goods are produced; and second, that factor intensity reversals (FIRs) do not occur. To understand FIRs, consider Figures 1.5 and 1.6. In the first case, presented in Figure 1.5, we have graphed the two zero-profit conditions, and the unit-cost lines intersect only *once*, at point A. This illustrates the lemma: given (p_1, p_2) , there is a *unique* solution for (w, r) . But another case is illustrated in Figure 1.6, where the unit-cost lines intersect *twice*, at points A and B. Then there are two possible solutions for (w, r) , and the result stated in the lemma no longer holds.

The case where the unit-cost lines intersect more than once corresponds to ‘*factor intensity reversals*.’ To see where this name comes from, let us compute the labor and capital requirements in the two industries. We have already shown that a_{iL} and a_{iK} are the derivatives of the unit-cost function with respect to factor prices, so it follows that the vectors (a_{iL}, a_{iK}) are the *gradient vectors* to the iso-cost curves for the two industries in Figure 1.5. Recall from calculus that gradient vectors point in the direction of the maximum increase of the function in question. This means that they are *orthogonal* to their respective iso-cost curves, as shown by (a_{1L}, a_{1K}) and

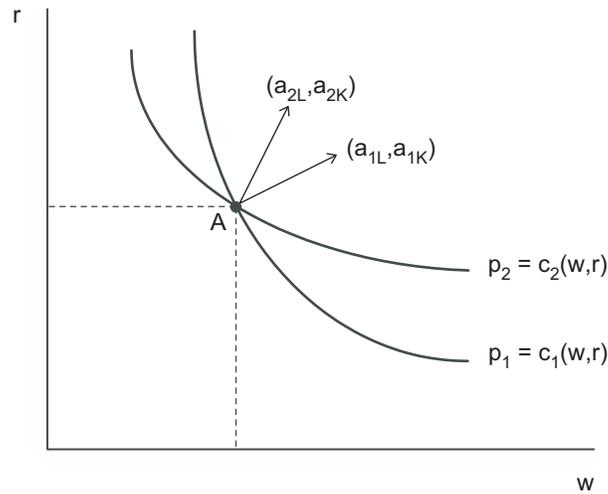


Figure 1.5

(a_{2L}, a_{2K}) at point A. Each of these vectors has slope (a_{iK}/a_{iL}) , or the capital-labor ratio. It is clear from Figure 1.5 that (a_{1L}, a_{1K}) has a smaller slope than (a_{2L}, a_{2K}) which means that *industry 2 is capital intensive*, or equivalently, *industry 1 is labor intensive*.⁷

In Figure 1.6, however, the situation is more complicated. Now there are two sets of gradient vectors, which we label by (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) at point A and by (b_{1L}, b_{1K}) and (b_{2L}, b_{2K}) at point B. A close inspection of the figure will reveal that industry 1 is *labor intensive* $(a_{1K}/a_{1L}) < a_{2K}/a_{2L}$ at point A but is *capital intensive* $(b_{1K}/b_{1L}) < b_{2K}/b_{2L}$ at point B. This illustrates a *factor intensity reversal*, whereby the comparison of factor intensities changes at different factor prices.

While FIRs might seem like a theoretical curiozum, they are actually quite realistic. Consider the footwear industry, for example. While much of the footwear in the world is produced in developing nations, the United States retains a small number of plants. In sneakers, New Balance has a plant in Norridgewock, Maine, where employees earn some \$14 per hour.⁸ Some operate computerized equipment with up to 20 sewing

⁷Alternatively, we can totally differentiate the zero-profit conditions, holding prices fixed, to obtain $0 = a_{iL}dw + a_{iK}dr$. It follows that the slope of the iso-cost curve equals $dr/dw = -a_{iL}/a_{iK} = -L_i/K_i$. Thus, the slope of each iso-cost curve equals the relative demand for the factor on the horizontal axis, whereas the slope of the gradient vector (which is orthogonal to the iso-cost curve) equals the relative demand for the factor on the vertical axis.

⁸The material that follows is drawn from Aaron Bernstein, "Low-Skilled Jobs: Do They Have to Move?" *Business Week*, February 26, 2001, 94–95.

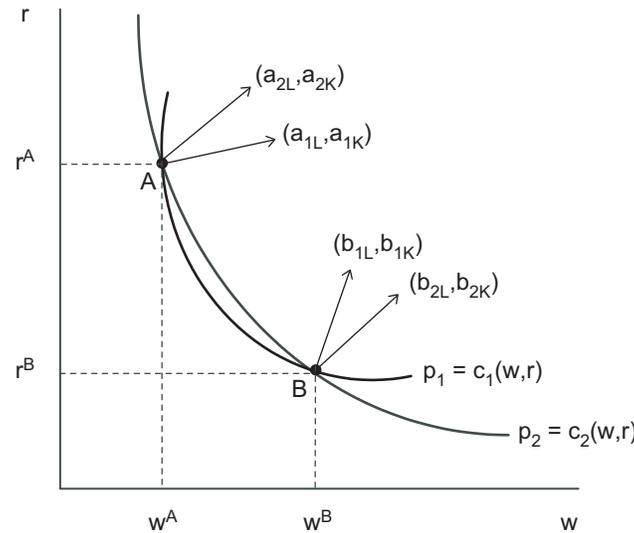


Figure 1.6

machine heads running at once, while others operate automated stitchers guided by cameras, which allow one person to do the work of six. This is a far cry from the plants in Asia that produce shoes for Nike, Reebok, and other U.S. producers, using century-old technology and paying less than \$1 per hour. The technology used to make sneakers in Asia is like industry 1 at point A in Figure 1.5, using labor-intensive technology and paying low wages w^A , while industry 1 in the United States is at point B, paying higher wages w^B and using a capital-intensive technology.

As suggested by this discussion, when there are two possible solutions for the factor prices such as points A and B in Figure 1.6, then some countries can be at one equilibrium and others countries at the other. How do we know which country is where? This is a question that we will answer at the end of the chapter, where we will argue that a *labor abundant* country will likely be at equilibrium A of Figure 1.6, with a low wage and high rental on capital, whereas a *capital abundant* country will be at equilibrium B, with a high wage and low rental. Generally, to determine the factor prices in each country we will need to examine its full-employment conditions in addition to the zero-profit conditions.

Let us conclude this section by returning to the simple case of no FIR, in which the lemma stated above applies. What are the implications of this result for the determination of factor prices under free trade? To answer this question, let us sketch out some of the assumptions of the Heckscher-Ohlin model, which we will study in more detail in the next chapter. We assume that there are two countries, with identical technologies but

different factor endowments. We continue to assume that labor and capital are the two factors of production, so that under free trade the equilibrium conditions (1.7) and (1.8) apply in *each* country with the *same* product prices (p_1, p_2) . We can draw Figure 1.5 for each country, and in the absence of FIR, this *uniquely* determines the factor prices in each country. In other words, the wage and rental determined by Figure 1.5 are *identical* across the two countries. We have therefore proved the factor price equalization (FPE) theorem, which is stated as follows.

Factor Price Equalization Theorem (Samuelson 1949)

Suppose that two countries are engaged in free trade, having identical technologies but different factor endowments. If both countries produce both goods and FIRs do not occur, then the factor prices (w, r) are equalized across the countries.

The FPE theorem is a remarkable result because it says that *trade in goods* has the ability to equalize factor prices: in this sense, trade in goods is a “perfect substitute” for trade in factors. We can again contrast this result with that obtained from a one-sector economy in both countries. In that case, equalization of the product price through trade would certainly not equalize factor prices: the labor-abundant country would be paying a lower wage. Why does this outcome *not occur* when there are two sectors? The answer is that the labor-abundant country can *produce more of, and export*, the labor-intensive good. In that way it can fully employ its labor while still paying the same wages as a capital-abundant country. In the two-by-two model, the opportunity to disproportionately produce more of one good than the other, while exporting the amounts not consumed at home, is what allows factor price equalization to occur. This intuition will become even clearer as we continue to study the Heckscher-Ohlin model in the next chapter.

Change in Product Prices

Let us move on now to the second of our key questions of the two-by-two model: if the product prices change, how will the factor prices change? To answer this, we perform comparative statics on the zero-profit conditions (1.7). Totally differentiating these conditions, we obtain

$$dp_i = a_{iL}dw + a_{iK}dr \Rightarrow \frac{dp_i}{p_i} = \frac{wa_{iL}}{c_i} \frac{dw}{w} + \frac{ra_{iK}}{c_i} \frac{dr}{r}, \quad i = 1, 2. \quad (1.9)$$

The second equation is obtained by multiplying and dividing like terms, and noting that $p_i = c_i(w, r)$. The advantage of this approach is that it

allows us to express the variables in terms of *percentage changes*, such as $d \ln w = dw/w$, as well as *cost shares*. Specifically, let $\theta_{iL} = wa_{iL}/c_i$ denote the cost share of labor in industry i , while $\theta_{iK} = ra_{iK}/c_i$ denotes the cost share of capital. The fact that costs equal $c_i = wa_{iL} + ra_{iK}$ ensures that the shares sum to unity, $\theta_{iL} + \theta_{iK} = 1$. In addition, denote the percentage changes by $dw/w = \hat{w}$ and $dr/r = \hat{r}$. Then (1.9) can be rewritten as

$$\hat{p}_i = \theta_{iL} \hat{w} + \theta_{iK} \hat{r}, \quad i = 1, 2. \quad (1.9')$$

Expressing the equation using these cost shares and percentage changes follows Jones (1965) and is referred to as the “Jones’ algebra.” This system of equation can be written in matrix form and solved as

$$\begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \end{pmatrix} = \begin{pmatrix} \theta_{1L} & \theta_{1K} \\ \theta_{2L} & \theta_{2K} \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix} \Rightarrow \begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix} = \frac{1}{|\theta|} \begin{pmatrix} \theta_{2K} & -\theta_{1K} \\ -\theta_{2L} & \theta_{1L} \end{pmatrix} \begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \end{pmatrix}, \quad (1.10)$$

where $|\theta|$ denotes the determinant of the two-by-two matrix on the left. This determinant can be expressed as

$$\begin{aligned} |\theta| &= \theta_{1L}\theta_{2K} - \theta_{1K}\theta_{2L} \\ &= \theta_{1L}(1 - \theta_{2L}) - (1 - \theta_{1L})\theta_{2L} \\ &= \theta_{1L} - \theta_{2L} = \theta_{2K} - \theta_{1K}, \end{aligned} \quad (1.11)$$

where we have repeatedly made use of the fact that $\theta_{iL} + \theta_{iK} = 1$.

In order to fix ideas, let us assume henceforth that *industry 1 is labor intensive*. This implies that its cost share in industry 1 exceeds that in industry 2, $\theta_{1L} - \theta_{2L} > 0$, so that $|\theta| > 0$ in (1.11).⁹ Furthermore, suppose that the relative price of good 1 *increases*, so that $\hat{p} = \hat{p}_1 - \hat{p}_2 > 0$. Then we can solve for the change in factor prices from (1.10) and (1.11) as

$$\hat{w} = \frac{\theta_{2K}\hat{p}_1 - \theta_{1K}\hat{p}_2}{|\theta|} = \frac{(\theta_{2K} - \theta_{1K})\hat{p}_1 + \theta_{1K}(\hat{p}_1 - \hat{p}_2)}{(\theta_{2K} - \theta_{1K})} > \hat{p}_1, \quad (1.12a)$$

since $\hat{p}_1 - \hat{p}_2 > 0$, and,

$$\hat{r} = \frac{\theta_{1L}\hat{p}_2 - \theta_{2L}\hat{p}_1}{|\theta|} = \frac{(\theta_{1L} - \theta_{2L})\hat{p}_2 - \theta_{2L}(\hat{p}_1 - \hat{p}_2)}{(\theta_{1L} - \theta_{2L})} < \hat{p}_2, \quad (1.12b)$$

since $\hat{p}_1 - \hat{p}_2 > 0$.

⁹As an exercise, show that $L_1/K_1 > L_2/K_2 \Leftrightarrow \theta_{1L} > \theta_{2L}$. This is done by multiplying the numerator and denominator on both sides of the first inequality by like terms, so as to convert it into cost shares.

From the result in (1.12a), we see that the wage increases *by more* than the price of good 1, $\hat{w} > \hat{p}_1 > \hat{p}_2$. This means that workers can afford to buy more of good 1 (w/p_1 has gone up), as well as more of good 2 (w/p_2 has gone up). When labor can buy more of *both goods* in this fashion, we say that the *real wage* has increased. Looking at the rental on capital in (1.12b), we see that the rental r changes by *less than* the price of good 2. It follows that the capital owner can afford less of good 2 (r/p_2 has gone down), and also less of good 1 (r/p_1 has gone down). Thus the *real return to capital* has fallen. We can summarize these results with the following theorem.

Stolper-Samuelson (1941) Theorem

An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor.

To develop the intuition for this result, let us go back to the differentiated zero-profit conditions in (1.9'). Since the cost shares add up to unity in each industry, we see from equation (1.9') that \hat{p}_i is a weighted average of the factor price changes \hat{w} and \hat{r} . This implies that \hat{p}_i necessarily lies in between \hat{w} and \hat{r} . Putting these together with our assumption that $\hat{p}_1 - \hat{p}_2 > 0$, it is therefore clear that

$$\hat{w} > \hat{p}_1 > \hat{p}_2 > \hat{r}. \quad (1.13)$$

Jones (1965) has called this set of inequalities the “magnification effect”: they show that any change in the product prices has a *magnified effect* on the factor prices. This is an extremely important result. Whether we think of the product price change as due to export opportunities for a country (the export price goes up), or due to lowering import tariffs (so the import price goes down), the magnification effect says that there will be both gainers and losers due to this change. Even though we will argue in chapter 6 that there are gains from trade in some overall sense, it is still the case that trade opportunities have strong *distributional* consequences, making some people worse off and some better off!

We conclude this section by illustrating the Stolper-Samuelson theorem in Figure 1.7. We begin with an initial factor price equilibrium given by point A, where industry 1 is labor intensive. An increase in the price of that industry will shift out the iso-cost curve, and as illustrated, move the equilibrium to point B. It is clear that the wage has gone up, from w_0 to w_1 , and the rental has declined, from r_0 to r_1 . Can we be sure that the wage has increased in percentage terms *by more* than the relative price of good 1? The answer is yes, as can be seen by drawing a ray from the origin

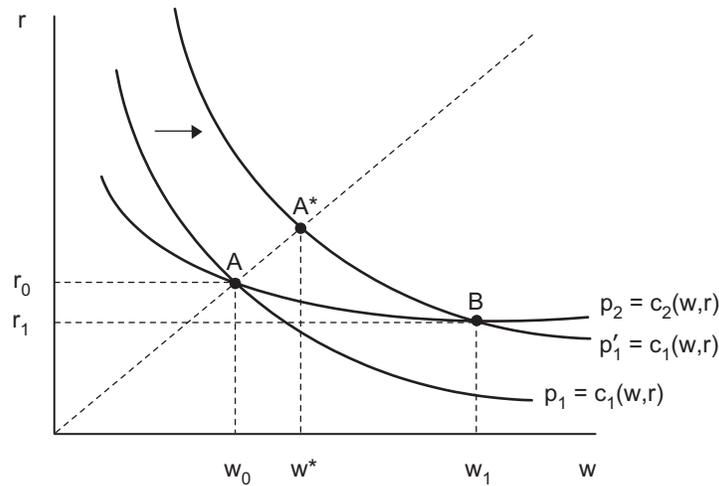


Figure 1.7

through the point A. Because the unit-cost functions are homogeneous of degree one in factor prices, moving along this ray increases p and (w, r) in the same proportion. Thus, at the point A^* , the increase in the wage exactly matched the percentage change in the price p_1 . But it is clear that the equilibrium wage increases by *more*, $w_1 > w^*$, so the percentage increase in the wage *exceeds* that of the product price, which is the Stolper-Samuelson result.

Changes in Endowments

We turn now to the third key question: if endowments change, how do the industry outputs change? To answer this, we hold the product prices *fixed* and totally differentiate the full-employment conditions (1.8) to obtain

$$\begin{aligned} a_{1L}dy_1 + a_{2L}dy_2 &= dL, \\ a_{1K}dy_1 + a_{2K}dy_2 &= dK. \end{aligned} \tag{1.14}$$

Notice that the a_{ij} coefficients *do not* change, despite the fact that they are functions of the factor prices (w, r) . These coefficients are fixed because p_1 and p_2 do not change, so from our earlier lemma, the factor prices are also fixed.

By rewriting the equations in (1.14) using the “Jones’ algebra,” we obtain

$$\begin{aligned} \frac{y_1 a_{1L}}{L} \frac{dy_1}{y_1} + \frac{y_2 a_{2L}}{L} \frac{dy_2}{y_2} &= \frac{dL}{L} \\ \frac{y_1 a_{1K}}{K} \frac{dy_1}{y_1} + \frac{y_2 a_{2K}}{K} \frac{dy_2}{y_2} &= \frac{dK}{K} \end{aligned} \Rightarrow \begin{aligned} \lambda_{1L} \hat{y}_1 + \lambda_{2L} \hat{y}_2 &= \hat{L} \\ \lambda_{1K} \hat{y}_1 + \lambda_{2K} \hat{y}_2 &= \hat{K} \end{aligned} \quad (1.14')$$

To move from the first set of equations to the second, we denote the percentage changes $dy_1/y_1 = \hat{y}_1$, and likewise for all the other variables. In addition, we define $\lambda_{iL} \equiv (y_i a_{iL}/L) = (L_i/L)$, which measures the *fraction of the labor force employed in industry i* , where $\lambda_{1L} + \lambda_{2L} = 1$. We define λ_{iK} analogously as the fraction of the capital stock employed in industry i .

This system of equations is written in matrix form and solved as

$$\begin{bmatrix} \lambda_{1L} & \lambda_{2L} \\ \lambda_{1K} & \lambda_{2K} \end{bmatrix} \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} = \begin{pmatrix} \hat{L} \\ \hat{K} \end{pmatrix} \Rightarrow \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} = \frac{1}{|\lambda|} \begin{bmatrix} \lambda_{2K} & -\lambda_{2L} \\ -\lambda_{1K} & \lambda_{1L} \end{bmatrix} \begin{pmatrix} \hat{L} \\ \hat{K} \end{pmatrix}, \quad (1.15)$$

where $|\lambda|$ denotes the determinant of the two-by-two matrix on the left, which is simplified as

$$\begin{aligned} |\lambda| &= \lambda_{1L} \lambda_{2K} - \lambda_{2L} \lambda_{1K} \\ &= \lambda_{1L} (1 - \lambda_{1K}) - (1 - \lambda_{1L}) \lambda_{1K} \\ &= \lambda_{1L} - \lambda_{1K} = \lambda_{2K} - \lambda_{2L}, \end{aligned} \quad (1.16)$$

where we have repeatedly made use of the fact that $\lambda_{1L} + \lambda_{2L} = 1$ and $\lambda_{1K} + \lambda_{2K} = 1$.

Recall that we assumed *industry 1 to be labor intensive*. This implies that the share of the labor force employed in industry 1 exceeds the share of the capital stock used there, $\lambda_{1L} - \lambda_{1K} > 0$, so that $|\lambda| > 0$ in (1.16).¹⁰ Suppose further that the endowment of labor is increasing, while the endowment of capital remains fixed such that $\hat{L} > 0$, and $\hat{K} = 0$. Then we can solve for the change in outputs from (1.15)–(1.16) as

$$\hat{y}_1 = \frac{\lambda_{2K}}{(\lambda_{2K} - \lambda_{2L})} \hat{L} > \hat{L} > 0 \quad \text{and} \quad \hat{y}_2 = \frac{-\lambda_{1K}}{|\lambda|} \hat{L} < 0. \quad (1.17)$$

From (1.17), we see that the output of the labor-intensive industry 1 expands, whereas the output of industry 2 contracts. We have therefore established the Rybczynski theorem.

¹⁰ As an exercise, show that $L_1/K_1 > L/K > L_2/K_2 \Leftrightarrow \lambda_{1L} > \lambda_{1K}$ and $\lambda_{2K} > \lambda_{2L}$.

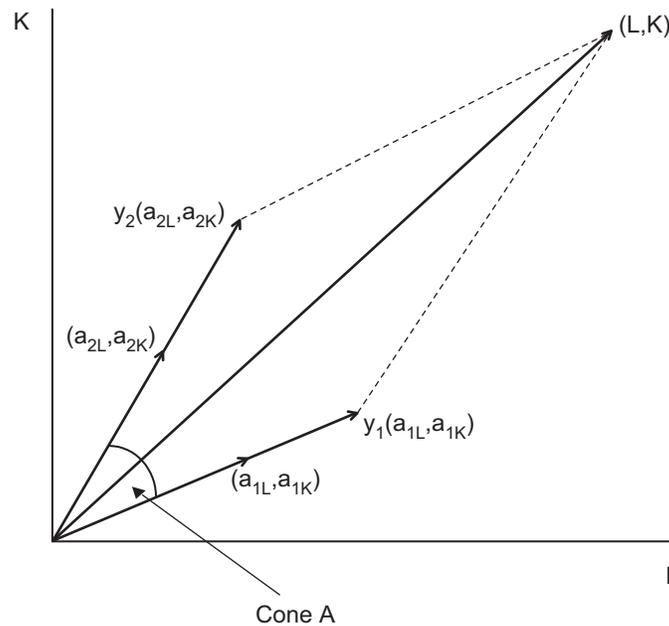


Figure 1.8

Rybczynski (1955) Theorem

An increase in a factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry.

To develop the intuition for this result, let us write the full-employment conditions in vector notation as

$$\begin{pmatrix} a_{1L} \\ a_{1K} \end{pmatrix} y_1 + \begin{pmatrix} a_{2L} \\ a_{2K} \end{pmatrix} y_2 = \begin{pmatrix} L \\ K \end{pmatrix}. \quad (1.8')$$

We have already illustrated the gradient vectors (a_{iL}, a_{iK}) to the iso-cost curves in Figures 1.5 (with no FIR). Now let us take these vectors and re-graph them, in Figure 1.8. Multiplying each of these by the output of their respective industries, we obtain the total labor and capital demands $y_1(a_{1L}, a_{1K})$ and $y_2(a_{2L}, a_{2K})$. Summing these as in (1.8') we obtain the labor and capital endowments (L, K) . But this exercise can also be performed in reverse: for any endowment vector (L, K) , there will be a *unique* value for the outputs (y_1, y_2) such that when (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) are multiplied by these amounts, they will sum to the endowments.

How can we be sure that the outputs obtained from (1.8') are positive? It is clear from Figure 1.8 that the outputs in both industries will be positive

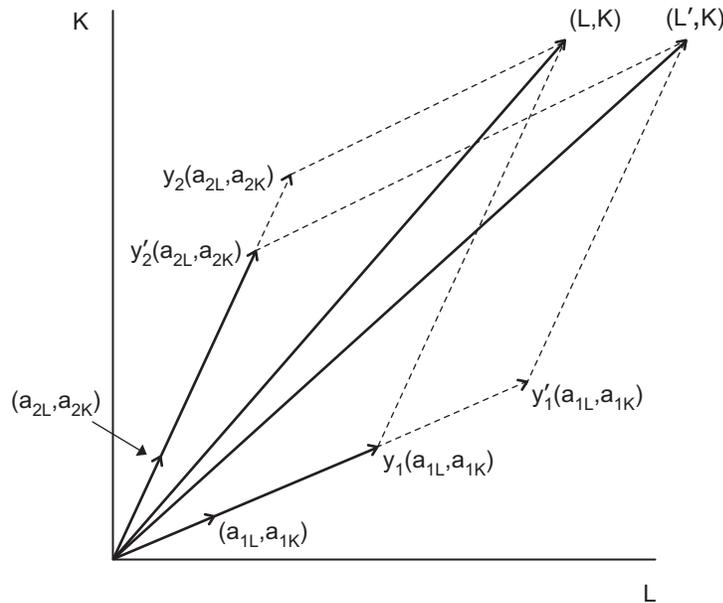


Figure 1.9

if and only if the endowment vector (L, K) lies *in between* the factor requirement vectors (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) . For this reason, the space spanned by these two vectors is called a ‘*cone of diversification*’, which we label by cone A in Figure 1.8. In contrast, if the endowment vector (L, K) lies *outside* of this cone, then it is *impossible* to add together any positive multiples of the vectors (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) and arrive at the endowment vector. So if (L, K) lies outside of the cone of diversification, then it must be that only *one good* is produced. At the end of the chapter, we will show how to determine which good it is.¹¹ For now, we should just recognize that when only one good is produced, then factor prices are determined by the marginal products of labor and capital as in the one-sector model, and will certainly depend on the factor endowments.

Now suppose that the labor endowment increases to $L' > L$, with no change in the capital endowment, as shown in Figure 1.9. Starting from the endowments (L', K) , the *only* way to add up multiples of (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) and obtain the endowments is to *reduce* the output of industry 2 to y'_2 , and *increase* the output of industry 1 to y'_1 . This means that not only does industry 1 absorb the entire amount of the extra labor endowment, it also absorbs further labor and capital from industry 2 so that its

¹¹ See problem 1.5.

ultimate labor/capital ratio is unchanged from before. The labor/capital ratio in industry 2 is also unchanged, and this is what permits both industries to pay exactly the same factor prices as they did before the change in endowments.

There are many examples of the Rybczynski theorem in practice, but perhaps the most commonly cited is what is called the “Dutch Disease.”¹² This refers to the discovery of oil off the coast of the Netherlands, which led to an increase in industries making use of this resource. (Shell Oil, one of the world’s largest producers of petroleum products, is a Dutch company.) At the same time, however, other “traditional” export industries of the Netherlands contracted. This occurred because resources were attracted away from these industries and into those that were intensive in oil, as the Rybczynski theorem would predict.

We have now answered the three questions raised earlier in the chapter: how are factor prices determined; how do changes in product prices affect factor prices; and how do changes in endowments affect outputs? But in answering all of these, we have relied on the assumptions that *both goods are produced*, and also that factor intensity reversals do not occur, as was stated explicitly in the FPE theorem. In the remainder of the chapter we need to investigate both of these assumptions, to understand either when they will hold or the consequences of their not holding.

We begin by tracing through the changes in the outputs induced by changes in endowments, along the equilibrium of the production possibility frontier. As the labor endowment grows in Figure 1.9, the PPF will shift out. This is shown in Figure 1.10, where the outputs will shift from point A to point A’ with an increase of good 1 and reduction of good 2, at the unchanged price p . As the endowment of labor rises, we can join up all points such as A and A’ where the slopes of the PPFs are equal. These form a downward-sloping line, which we will call the *Rybczynski line* for changes in labor (ΔL). The Rybczynski line for ΔL indicates how outputs change as labor endowment expands.

Of course, there is also a Rybczynski line for ΔK , which indicates how the outputs change as the capital endowment grows: this would lead to an increase in the output of good 2, and reduction in the output of good 1. As drawn, we have illustrated both of the Rybczynski lines as *straight* lines: can we be sure that this is the case? The answer is yes: the fact that the product prices are fixed along a Rybczynski line, implying that factor prices are also fixed, ensures that these are straight lines. To see this, we can easily calculate their slopes by differentiating the full-employment conditions (1.8). To compute the slope of the Rybczynski line for ΔL , it is convenient

¹² See, for example, Corden and Neary 1982 and Jones, Neary, and Ruane 1987.

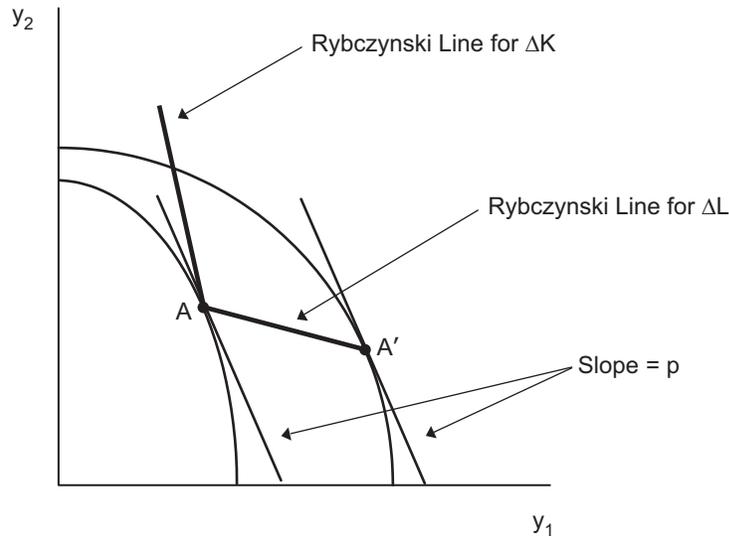


Figure 1.10

to work with the full-employment condition for *capital*, since that endowment does not change. Total differentiating (1.8) for capital gives

$$a_{1K}y_1 + a_{2K}y_2 = K \Rightarrow a_{1K}dy_1 + a_{2K}dy_2 = 0 \Rightarrow \frac{dy_2}{dy_1} = -\frac{a_{1K}}{a_{2K}}. \quad (1.18)$$

Thus, the slope of the Rybczynski line for ΔL is the negative of the ratio of capital/output in the two industries, which is constant for fixed prices. This proves that the Rybczynski lines are indeed straight.

If we continue to increase the labor endowment, outputs will move downwards on the Rybczynski line for ΔL in Figure 1.10, until this line hits the y_1 axis. At this point the economy is fully specialized in good 1. In terms of Figure 1.9, the vector of endowments (L, K) is coincident with the vector of factor requirements (a_{1L}, a_{1K}) in industry 1. For further increases in the labor endowment, the Rybczynski line for ΔL then *moves right along the y_1 axis* in Figure 1.10, indicating that the economy remains specialized in good 1. This corresponds to the vector of endowments (L, K) lying outside and below the cone of diversification in Figure 1.9. With the economy fully specialized in good 1, factor prices are determined by the marginal products of labor and capital in that good, and the earlier “factor price insensitivity” lemma no longer applies.

country are measured relative to the origin 0^* —suppose that this allocation is at point B. Given the world endowments, we establish equilibrium prices for goods and factors in this “integrated world equilibrium.” The factor prices determine the demand for labor and capital in each industry (assuming no FIR), and using these, we can construct the diversification cone (since factor prices are the same across countries, then the diversification cone is also the same). Let us plot the diversification cone relative to the home origin 0 , and again relative to the foreign origin 0^* . These cones form the parallelogram $0A_10^*A_2$.

For later purposes, it is useful to identify precisely the points A_1 and A_2 on the vertices of this parallelogram. The vectors $0A_i$ and 0^*A_i are proportional to (a_{iL}, a_{iK}) , the amount of labor and capital used to produce one unit of good i in each country. Multiplying (a_{iL}, a_{iK}) by world demand for good i , D_i^w , we then obtain the *total* labor and capital used to produce that good, so that $A_i = (a_{iL}, a_{iK})D_i^w$. Summing these gives the total labor and capital used in world demand, which equals the labor and capital used in world production, or world endowments.

Now we ask whether we can achieve exactly the same world production and equilibrium prices as in this “integrated world equilibrium,” but *without* labor and capital mobility. Suppose there is some allocation of labor and capital endowments across the countries, such as point B. Then can we produce the same amount of each good as in the “integrated world equilibrium”? The answer is clearly yes: with labor and capital in each country at point B, we could devote $0B_1$ of resources to good 1 and $0B_2$ to good 2 at home, while devoting $0^*B_1^*$ to good 1 and $0^*B_2^*$ towards good 2 abroad. This will ensure that the same amount of labor and capital worldwide is devoted to each good as in the “integrated world equilibrium,” so that production and equilibrium prices must be the same as before. Thus, we have achieved the same equilibrium but without factor mobility. It will become clear in the next chapter that there is still *trade in goods* going on to satisfy the demands in each country.

More generally, for *any allocation* of labor and capital within the parallelogram $0A_10^*A_2$ both countries remain diversified (producing both goods), and we can achieve the same equilibrium prices as in the “integrated world economy.” It follows that factor prices *remain equalized across countries* for allocations of labor and capital within the parallelogram $0A_10^*A_2$, which is referred to as the *factor price equalization (FPE) set*. The FPE set illustrates the range of labor and capital endowments between countries over which both goods are produced in both countries, so that factor price equalization is obtained. In contrast, for endowments *outside* of the FPE set such as point B' , then at least one country would have to be fully specialized in one good and FPE no longer holds.

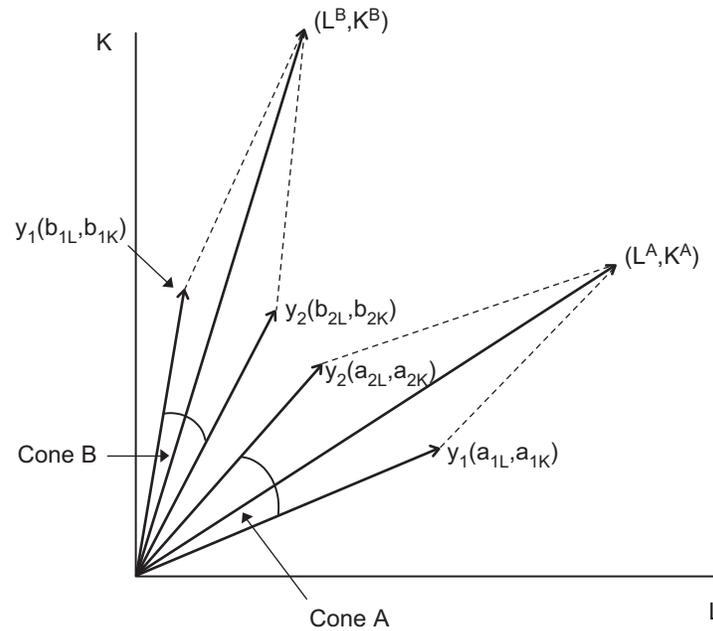


Figure 1.12

Factor Intensity Reversals

We conclude this chapter by returning to a question raised earlier: when there are “factor intensity reversals” giving multiple solutions to the zero-profit conditions, then how do we know which solution will prevail in each country? To answer this, it is necessary to combine the zero-profit with the full-employment conditions, as follows.

Consider the case in Figure 1.6, where the zero-profit conditions allow for two solutions to the factor prices. Each of these determine the labor and capital demands shown orthogonal to the iso-cost curves, labeled as (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) and (b_{1L}, b_{1K}) and (b_{2L}, b_{2K}) . We have redrawn these in Figure 1.12, after multiplying each of them by the outputs of their respective industries. These vectors create *two* cones of diversification, labeled as cone A and cone B. Initially, suppose that the factor endowments for each country lie within one cone or the other (then we will consider the case where the endowments are outside both cones).

Now we can answer the question of which factor prices will apply in each country: a *labor abundant* economy, with a high ratio of labor/capital endowments such as (L^A, K^A) in cone A of Figure 1.12, will have factor prices given by (w^A, r^A) in Figure 1.6, with low wages; whereas a *capital*

abundant economy, with a high ratio of capital/labor endowments, such as shown by (L^B, K^B) in cone B of Figure 1.12, will have factor prices given by (w^B, r^B) in Figure 1.6, with high wages. Thus, factor prices depend on the endowments of the economy. A labor-abundant country such as China will pay low wages and a high rental (as in cone A), while a capital-abundant country such as the United States will have high wages and a low rental (as in cone B). Notice that we have now reintroduced a link between factor endowments and factor prices, as we argued earlier in the one-sector model: when there are FIR in the two-by-two model, factor prices vary systematically with endowments *across* the cones of diversification, even though factor prices are independent of endowments *within* each cone.

What if the endowment vector of a country does not lie in either cone? Then the country will be fully specialized in one good or the other. Generally, we can determine which good it is by tracing through how the outputs change as we move through the cones of diversification, and it turns out that outputs depend *nonmonotonically* on the factor endowments.¹³ For example, textiles in South Korea or Taiwan expanded during the 1960s and 1970s, but contracted later as capital continued to grow. Despite the complexity involved, many trade economists feel that countries do in fact produce in different cones of diversification, and taking this possibility into account is a topic of current research.¹⁴

Conclusions

In this chapter we have reviewed several two-sector models: the Ricardian model, with just one factor, and the two-by-two model, with two factors both of which are fully mobile between industries. There are other two-sector models, of course: if we add a third factor, treating capital as specific to each sector but labor as mobile, then we obtain the Ricardo-Viner or “specific factors” model, as will be discussed in chapter 3. We will have an opportunity to make use of the two-by-two model throughout this book, and a thorough understanding of its properties—both the equations and the diagrams—is essential for all the material that follows.

One special feature of this chapter is the dual determination of factor prices, using the unit-cost function in the two industries. This follows the

¹³ See problem 1.5.

¹⁴ Empirical evidence on whether developed countries fit into the same cone is presented by Debaere and Demiroglu (2003), and the presence of multiple cones is explored by Leamer (1987); Harrigan and Zakrajšek (2000); Schott (2003); and Xu (2002). The latter papers draw on empirical methods that we introduce in chapter 3.

dual approach of Woodland (1977, 1982), Mussa (1979), and Dixit and Norman (1980). Samuelson (1949) used a quite different diagrammatic approach to prove the FPE theorem. Another method that is quite commonly used is the so-called Lerner (1952) diagram, which relies on the production rather than cost functions.¹⁵ We will not use the Lerner diagram in this book, but it will be useful to understand some articles, for example, Findlay and Grubert (1959) and Deardorff (1979), so we include a discussion of it in the appendix to this chapter.

This is the only chapter where we do not present any accompanying empirical evidence. The reader should not infer from this that the two-by-two model is unrealistic: while it is usually necessary to add more goods or factors to this model before confronting it with data, the relationships between prices, outputs, and endowments that we have identified in this chapter will carry over in some form to more general settings. Evidence on the pattern of trade is presented in the next chapter, where we extend the two-by-two model by adding another country, and then many countries, trading with each other. We also allow for many goods and factors, but for the most part restrict attention to situations where factor price equalization holds. In chapter 3, we examine the case of many goods and factors in greater detail, to determine whether the Stolper-Samuelson and Rybczynski theorems generalize and also how to estimate these effects. In chapter 4, evidence on the relationship between product prices and wages is examined in detail, using a model that allows for trade in intermediate inputs. The reader is already well prepared for these chapters that follow, based on the tools and intuition we have developed from the two-by-two model. Before moving on, you are encouraged to complete the problems at the end of this chapter.

Appendix: The Lerner Diagram and Factor Prices

The Lerner (1952) diagram for the two-by-two model can be explained as follows. With perfect competition and constant returns to scale, we have that revenue = costs in both industries. So let us choose a special isoquant in each industry such that revenue = 1. In each industry, we therefore choose the isoquant $p_i y_i = 1$, or

$$y = f_i(L_i, K_i) = 1/p_i \Rightarrow wL_i + rK_i = 1.$$

¹⁵This diagram was used in a seminar presented by Abba Lerner at the London School of Economics in 1933, but not published until 1952. The history of this diagram is described at the “Origins of Terms in International Economics,” maintained by Alan Deardorff at <http://www.personal.umich.edu/~alandear/glossary/orig.html>. See also Samuelson 1949, 181 n. 1.

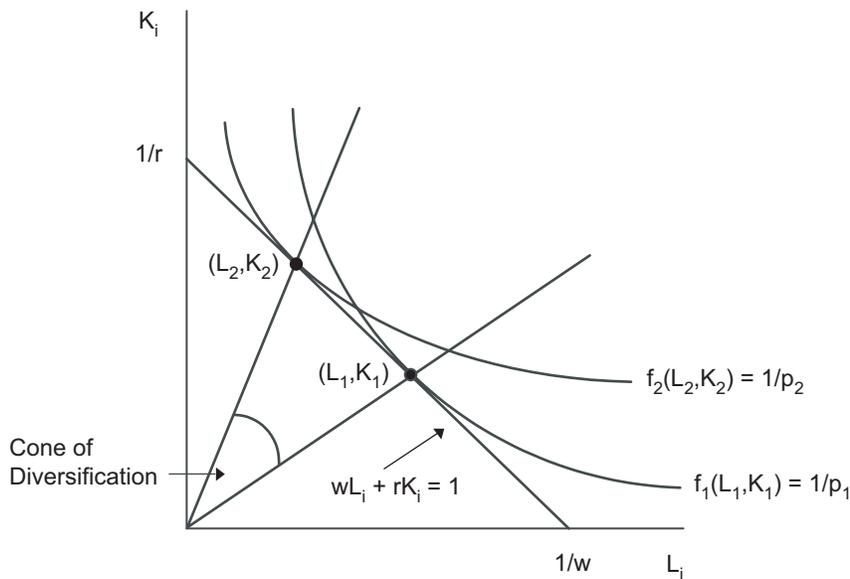


Figure 1.13

Therefore, from cost minimization, the $1/p_i$ isoquant in each industry will be *tangent* to the line $wL_i + rK_i = 1$. This is the same line for both industries, as shown in Figure 1.13.

Drawing the rays from the origin through the points of tangency, we obtain the cone of diversification, as labeled in Figure 1.13. Furthermore, we can determine the factor prices by computing where $wL_i + rK_i = 1$ intersects the two axes: $L_i = 0 \Rightarrow K_i = 1/r$, and $K_i = 0 \Rightarrow L_i = 1/w$. Therefore, given the prices p_i we determine the two isoquants in Figure 1.13, and drawing the (unique) line tangent to both of these, we determine the factor prices as the intercepts of this line. Notice that these equilibrium factor prices do not depend on the factor endowments, provided that the endowment vector lies within the cone of diversification (so that both goods are produced). We have thus obtained an alternative proof of the “factor price insensitivity” lemma, using a primal rather than dual approach. Furthermore, with two countries having the same prices (through free trade) and technologies, Figure 1.13 holds in both of them. Therefore, their factor prices will be equalized.

Lerner (1952) also showed how Figure 1.13 can be extended to the case of factor intensity reversals, in which case the isoquants intersect twice. In that case there will be *two* lines $wL_i + rK_i = 1$ that are tangent to both isoquants, and there are two cones of diversification. This is

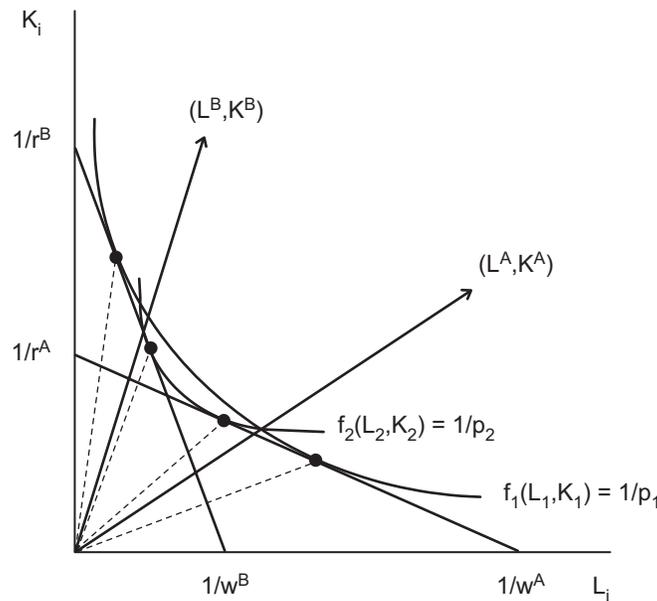


Figure 1.14

shown in Figure 1.14. To determine which factor prices apply in a particular country, we plot its endowments vector and note which cone of diversification it lies in: the factor prices in this country are those applying to that cone. For example, the endowments (L^A, K^A) will have the factor prices (w^A, r^A) , and the endowments (L^B, K^B) will have the factor prices (w^B, r^B) . Notice that the labor-abundant country with endowments (L^A, K^A) has the low wage and high rental, whereas the capital-abundant country with endowments (L^B, K^B) has the high wage and low rental.

How likely is it that the isoquants of industries 1 and 2 intersect twice, as in Figure 1.14? Lerner (1952, 11) correctly suggested that it depends on the elasticity of substitution between labor and capital in each industry. For simplicity, suppose that each industry has a constant elasticity of substitution production function. If the elasticities are the same across industries, then it is impossible for the isoquants to intersect twice. If the elasticities of substitution differ across industries, however, and we choose prices p_i , $i = 1, 2$, such that the $1/p_i$ isoquants intersect at least once, then it is *guaranteed* that they intersect twice. Under exactly the same conditions, the iso-cost lines in figure 1.6 intersect twice. Thus, the occurrence of FIR is very likely once we allow elasticities of substitution to differ across industries. Minhas (1962) confirmed that this was the case empirically, and dis-

cussed the implications of FIR for factor prices and trade patterns. This line of empirical research was dropped thereafter, perhaps because FIR seemed too complex to deal with, and has been picked up again only recently (see note 14).

Problems

1.1 Rewrite the production function $y_1 = f_1(L_1, K_1)$ as $y_1 = f_1(v_1)$, and similarly, $y_2 = f_2(v_2)$. Concavity means that given two points $y_1^a = f_1(v_1^a)$ and $y_1^b = f_1(v_1^b)$, and $0 \leq \lambda \leq 1$, then $f_1(\lambda v_1^a + (1 - \lambda)v_1^b) \geq \lambda y_1^a + (1 - \lambda)y_1^b$. Similarly for the production function $y_2 = f_2(v_2)$. Consider two points $y^a = (y_1^a, y_2^a)$ and $y^b = (y_1^b, y_2^b)$, both of which can be produced while satisfying the full-employment conditions $v_1^a + v_2^a \leq V$ and $v_1^b + v_2^b \leq V$, where V are the endowments. Consider a production point midway between these, $\lambda y^a + (1 - \lambda)y^b$. Then use the concavity of the production functions to show that this point can *also* be produced while satisfying the full-employment conditions. This proves that the production possibilities set is *convex*. (Hint: Rather than showing that $\lambda y^a + (1 - \lambda)y^b$ can be produced while satisfying the full-employment conditions, consider instead allocating $\lambda v_1^a + (1 - \lambda)v_1^b$ of the resources to industry 1, and $\lambda v_2^a + (1 - \lambda)v_2^b$ of the resources to industry 2.)

1.2 Any function $y = f(v)$ is homogeneous of degree α if for all $\lambda > 0$, $f(\lambda v) = \lambda^\alpha f(v)$. Consider the production function $y = f(L, K)$, which we assume is homogeneous of degree one, so that $f(\lambda L, \lambda K) = \lambda f(L, K)$. Now differentiate this expression with respect to L , and answer the following: Is the marginal product $f_L(L, K)$ homogeneous, and of what degree? Use the expression you have obtained to show that $f_L(L/K, 1) = f_L(L, K)$.

1.3 Consider the problem of maximizing $y_1 = f_1(L_1, K_1)$, subject to the full-employment conditions $L_1 + L_2 \leq L$ and $K_1 + K_2 \leq K$, and the constraint $y_2 = f_2(L_2, K_2)$. Set this up as a Lagrangian, and obtain the first-order conditions. Then use the Lagrangian to solve for dy_1/dy_2 , which is the slope of the production possibility frontier. How is this slope related to the marginal product of labor and capital?

1.4 Consider the problem of maximizing $p_1 f_1(L_1, K_1) + p_2 f_2(L_2, K_2)$, subject to the full-employment constraints $L_1 + L_2 \leq L$ and $K_1 + K_2 \leq K$. Call the result the GDP function $G(p, L, K)$, where $p = (p_1, p_2)$ is the price vector. Then answer the following:

- (a) What is $\partial G / \partial p_i$? (Hint: we solved for this in the chapter.)
- (b) Give an economic interpretation to $\partial G / \partial L$ and $\partial G / \partial K$.

(c) Give an economic interpretation to $\partial^2 G/\partial p_i \partial L = \partial^2 G/\partial L \partial p_i$, and $\partial^2 G/\partial p_i \partial K = \partial^2 G/\partial K \partial p_i$.

1.5 Trace through changes in outputs when there are factor intensity reversals. That is, construct a graph with the capital endowment on the horizontal axis, and the output of goods 1 and 2 on the vertical axis. Starting at a point of diversification (where both goods are produced) in cone A of Figure 1.12, draw the changes in output of goods 1 and 2 as the capital endowment grows outside of cone A, into cone B, and beyond this.