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Chapter 1

INTRODUCTION

Astrophysics makes use of nearly every branch of physics: mechanics, electromagnetism, quantum mechanics, fluid mechanics, nuclear physics and general relativity. In addition, the subject of plasma physics has been playing an increasing role in astrophysics. For a large part of astrophysics, we can get along without plasma physics except in a passive role. For example, magnetic fields in a fluid exert forces that modify its behavior in a way that is easy to grasp. But many of the more extreme events in astrophysics involve plasma physics in an essential way, and to comprehend them a deeper understanding of the more subtle and surprising properties of plasma physics is necessary.

Plasmas in the laboratory have many striking properties, but the laboratory involves sizes and other parameters with rather mundane values. When we pass to astrophysics these sizes and parameters take on extreme values, which bring out these properties more forcefully.

A particular interesting example is the so-called Langmuir paradox. In the late 1920s when Tonks and Langmuir were first creating and examining plasmas, they shot a beam of energetic electrons into a plasma and found that the beam came to a screeching halt, in a very short distance, much shorter than the collisional length over which the beam is expected to decelerate (Tonks and Langmuir 1929). A similar phenomenon occurs in astrophysics for cosmic rays. They are injected into the interstellar medium by acceleration around a supernova and rather quickly lose their directed motion becoming isotropic in their velocity space. Both phenomena are caused by strong instabilities on small scales. The intense electric fields in these instabilities interact with the beams and decelerate them or scatter them. The instabilities arise from the free energy in the beams themselves. Such plasma phenomena occur on a small scale that is not visible to the observer, and thus appear hidden, so that the results of their action often appear mysterious.

The solar flare is another important consequence of deeper plasma phenomena. The sudden appearance far above the sun's surface of energy in the form of radiation, particle acceleration, and heating at first seemed very remarkable and incomprehensible, until it was appreciated that the source of energy was the sun's magnetic field. How this conversion occurs is still unclear, but it is believed to be associated with the reconnection of magnetic field lines.

The ideas behind this are rooted in the concept that magnetic field lines have a reality much deeper than the mere vector representation of the magnetic field. Once these lines are present in a plasma they can be considered as real lines bodily transported by macroscopic plasma motions. This

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bodily transport can be destroyed only by resistivity. However, because of the large scale of astrophysical plasmas this occurs only very slowly. As a consequence, magnetic field lines can be wound up, much as a spring can, and the tightly wound lines represent a great increase in the magnetic energy. In the solar flare, for example, this continues until the lines suddenly break, allowing the rapid conversion of the excess magnetic energy into thermal and radiated energy. How this happens is still not fully understood and is currently the subject of intense research.

The remarkable freezing of magnetic field lines into a plasma can be traced to the large induction $\mathcal{L}$ of a typical large-scale astrophysical plasma and a correspondingly very small resistance $\mathcal{R}$. In electrical circuits the timescale for decay of currents is $\mathcal{L}/\mathcal{R}$ and, correspondingly, the timescale during which the flux freezing holds is $\mathcal{L}/\mathcal{R}$ and in astrophysics is generally very large.

This freezing is one of the outstanding characteristics of plasmas on large scales. As an example, it easily explains why a galaxy can have a long-lasting magnetic field, a field whose decay time is longer than the age of the universe by many orders of magnitude. On the other hand, it makes it difficult to see how a magnetic field on this scale can arise in a time shorter than the life of the galaxy. In fact, such a start-up of a field can happen only while conserving the actual number of magnetic field lines. It is thought to be accomplished in the galaxy by a mechanism in which helical motions fold the lines in a complex topological manner so that a large amount of positive and negative flux results. Then the observed large-scale field of constant sign results by the removal of the negative flux from the galaxy. Such processes are called dynamos. Similar processes seem to be present in the sun, which could explain the mysterious reversal of its magnetic field every eleven years. Accretion disks and compact objects have dynamo mechanisms that generate and amplify magnetic fields. Here the field is amplified to a point where it can provide the viscous force needed for the matter to fall onto the compact body at the center of the disk.

Plasma processes play many other roles, which are not immediately obvious and are detected only through their consequences, such as in particle acceleration, generation of intense radiation, confinement of energetic particles such as cosmic rays, and stellar formation.

Why does a plasma have these remarkable properties? When we inspect a plasma closely, we appreciate that it can have a very large number of degrees of freedom, which manifest themselves as waves with a large range of wavelengths, extending from microscopic to macroscopic length scales, The amplitudes of these waves are generally lifted above thermal levels by specific plasma instabilities. The collective motions of particles in these waves and the resulting strong fields are the agents by which these properties are produced.

A plasma consists of enumerable particles of either sign and the particles carry large amounts of charge and electrical currents. In a normal situation

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the charges cancel to an extremely high degree of approximation. However, even the slightest imbalance in the numbers of particles of opposite sign, ions and electrons, can produce intense electric fields. Also, a slight anisotropy in their motions can produce extremely strong magnetic fields.

As an example, there are roughly $N = 10^{57}$ electrons and $N = 10^{57}$ ions in the sun. If these numbers were seriously out of balance, say only half the number of electrons as ions, there would be an electric field at the surface of the sun of $Ne/R^2 = 10^{57} \times 4.8 \times 10^{-10}/(6 \times 10^{10})^2 \approx 1.5 \times 10^{26}$ statvolts/cm $= 4.5 \times 10^{28}$ volts/centimeter (V/cm). Such a field would easily pull the sun apart. In fact, the surface electric field would actually be outward and of such a strength so as to balance one-half of the inward gravitational force on an ion ($\approx 10^{-8}$V/cm). (This is the field necessary to keep the electrons from evaporating from the solar surface.) Actually, there is such a surface field and it satisfies $eE = m_H g/2 \approx \nabla p_e/n$. What imbalance between N_i and N_e is necessary to produce this field? We have $(N_i - N_e)e = 4\pi R^2 E$, and if we use $4\pi R^2 g = G N_i m_H$ we easily get $(N_i - N_e)/N_i = G m_H^2/e^2 \approx 10^{-36}$ so the cancellation is extremely high. The same degree of cancellation holds at every point in the sun.

The cancellation is general and represents the extremely large amount of charge that a plasma can potentially supply if necessary. In fact, suppose we have a plasma slab of thickness L. If there is an imbalance of charge density Δn, then there is an electric field of magnitude $4\pi \Delta n e L$ and an electric potential $\Delta\phi = 4\pi \Delta n e L^2/2$. The potential energy of an ion $e\Delta\phi$ is comparable to its one-dimensional thermal energy $k_B T/2$ when $\Delta n \approx T/4\pi n e^2 L^2$ or when $\Delta n/n \approx \lambda_D^2/L^2$, where

$$\lambda_D = \sqrt{\frac{T}{4\pi n e^2}} \approx 7\sqrt{\frac{T}{n}} \text{ cm} \tag{1}$$

This expression has the dimensions of length and is called the Debye length. It is a fundamental length in plasma physics. For the central region of the sun of radius $R/10$, $T \approx 10^7$ kelvins (K) and $n \approx 10^{26}$, so that $\lambda_D \approx 2 \times 10^{-9}$ cm and $(\lambda_D/10R)^2 \approx 10^{-37}$, which is consistent with our first estimate. ($\Delta n/n \approx 1$, can hold only when $L \approx \lambda_D$. Since this is the case for electron plasma oscillations, this is also the scale of electron plasma oscillations.)

Similarly, we can find how much potential current the sun could have by taking all the N electrons to move in the same direction. It is $I = Nev/c$, where v is the thermal velocity $v \approx 10^9$ cm/sec at $T \approx 100$ eV. Thus, $I \approx 10^{57} \times 4.8 \times 10^{-10} \times 10^9/(3 \times 10^{10}) = 10^{46}$ abamps ($\approx 10^{47}$ amps). Such a current would produce a magnetic field strength of order $B \approx 2I/R \approx 3 \times 10^{35}$ gauss (G). This enormous field derived from the assumption of totally directed motion of the electrons indicates that even the smallest anisotropy in velocities will produce almost any conceivable field. For example, an anisotropy of $\delta \approx 10^{-29}$ would produce a field strength of about a megagauss.

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The large quantity of potential charge and current available in an astrophysical body is quite general. For this reason, we generally calculate the electric and magnetic field first, and afterward check that the required charge and current is quite small compared to what is available. It almost always is.

Two other extreme quantities typical in an astrophysical body, which we have already mentioned, are very large electrical inductance and very small resistance. Inductance $\mathcal{L}$ is the number of lines produced by a unit amount of current, so since $B = 2I/R$ and since the number of lines is BR^2, the inductance is of order R. The resistivity is the reciprocal of the conductivity σ and the total resistance is $\mathcal{R} = \eta c/R$. Thus, the $\mathcal{L}/\mathcal{R}$ decay time of the currents and magnetic field is proportional to $R^2/\eta c$. Since η is about the same in the laboratory as in astrophysics, it is the very large size of R that makes the decay time so long. η is inversely proportional to the electron temperature to the three halves power and independent of density, so we find on establishing the correct constant that the decay time is

$$T_{\text{decay}} = \frac{\mathcal{L}}{\mathcal{R}} = \frac{R^2}{10^7} T_{\text{eV}}^{3/2} \quad \text{sec} \tag{2}$$

where the temperature is measured in electron volts. (One electron volt is the same as 10^4 K.) For the sun $R \approx 10^{11}$ cm, and $T_{\text{eV}} \approx 100$, so that $T_{\text{decay}} \approx 10^{17}$ sec. For a piece of the interstellar medium of size 1 parsec, $T_{\text{eV}} \approx 1$, we get $T_{\text{decay}} \approx 10^{30}$ sec. Thus, we see that resistivity cannot destroy a magnetic field in a static astrophysical plasma of a size greater than the size of a star in a Hubble time.

We can imagine what might happen. The electron motion need only be a very small mean drift v_D to produce the field ($v_D/v_{\text{th}} \approx 10^{-30}$ in the sun and $v_D/v_{\text{th}} \approx 10^{-15}$ in the interstellar medium). Our first impression is that a single collision can easily wipe out this incredibly small anisotropy. But if it did, the magnetic field would collapse to zero in the same short time. The resulting rapid change in the flux would create an enormous electric field, which would very quickly reaccelerate the electrons, recreating the necessary anisotropy to produce the magnetic field. Actually, there is always a very small electric field present that sustains the necessary anisotropy against collisions, and this electric field is produced by induction from the magnetic field decaying on the extremely long $\mathcal{L}/\mathcal{R}$ time mentioned above. In fact, this is an alternative way to arrive at the decay time of the magnetic field.

However, the above picture applies to the grossly simplified case of a static plasma and a magnetic field of scale R. If the plasma is moving, the same considerations apply locally in the moving frame. This brings us back to the flux-freezing idea and actually shows that flux-freezing holds for the $\mathcal{L}/\mathcal{R}$ timescale. It is useful to note that flux freezing holds for any astrophysical plasma in which the magnetic field varies on a scale larger than $10^{12}/T_{\text{eV}}^{3/2}$ cm, since at this scale the $\mathcal{L}/\mathcal{R}$ time is the Hubble time.

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The above highly qualitative discussion indicates some of the powerful effects that plasma physics can exert on astrophysical phenomena.

In general, we can most simply understand plasma physics as a double reaction between particles and electromagnetic fields. The plasma moves and generates currents, which produce and change the electromagnetic field. The electromagnetic field reacts back on the ionized plasma controlling its motion.

For example, in the corona the magnetic fields force the particles to move along their field lines. Electric fields accelerate the ions and electrons, causing currents to flow, which twist the field lines and change the paths along which the particles flow. Waves arising from below the photosphere propagate into the corona. These waves consist of perturbed magnetic fields accelerating the plasma particles. The accelerated plasma particles then oscillate the perturbed fields, resulting in the wave motion.

We will see that nearly all plasma physics can best be grasped in this manner: the electromagnetic field influences the plasma, and the plasma influences the electromagnetic field. This parallels what we learned in electromagnetism, where we were taught that charged particles generate fields, and fields exert forces on charged particles controlling their motions.

As is well known, a plasma consists of electrons and of ions of various species and charge states. However, it is not so universally appreciated that a plasma also consists of waves or collective motions. Of course, the waves are nothing more than electrons and ions coherently moving. But the wave motion is so coherent that waves can be thought of as independent entities. When we give the number density of ions and electrons in various states of motion, we should also specify the intensities of the waves, i.e., their energies at different wave numbers. Plasma physics also concerns itself with electric and magnetic fields some of whose structure is bound up in waves.

The subject of plasma physics is, on the face of it, a very complex subject. Yet a lot of progress has been made in systematizing it and laying down a number of fundamental principles and equations. We can divide the subject of plasma physics into one part that is well understood and in which reliable results can be obtained theoretically and demonstrated experimentally, and a second part that is still puzzling, in which results are still very much in doubt and over which there is much debate. The second part consists of tough problems important to astrophysics, such as dynamo theory, magnetic reconnection, and particle acceleration. These problems are slowly beginning to yield to intensive attack by theorists, computationalists, experimentalists, and observers.

Magnetic reconnection bears directly on the mechanism of solar flares, the magnetospheric interaction with the solar wind, and magnetic storms and aurora. Dynamo theory is important for the origin and evolution of cosmic magnetic fields, the understanding of the solar cycle, of accretion disks, and the sustainment and behavior of the earth's magnetic field.

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These problems are hard because they are primarily three dimensional and they contain no small dimensionless parameters that provide useful limiting cases that are simple to handle, as is the case with the better understood part of plasma physics. The small parameter for both magnetic reconnection and dynamo problems is the resistivity. In proper dimensionless units, it is a very small parameter. But as it becomes small the difficulties of solution become harder and more intractable, rather than easier to handle.

Plasma physics abounds in quantities, such as the Debye length, the gyroradius, and the collision time, which when expressed in the correct units of the problem are very small. These small parameters allow us to approach, through limiting cases, the simplification of a large number of problems. For example, the mean free path is often much smaller than the scale of variation of physical quantities. This allows the plasma to be treated as a fluid for most macroscopic problems. Where this is not a valid approximation, use can then be made of the smallness of the gyroradius, which leads to almost fluid-like equations.

1.1 How Do We Describe a Plasma and Its Electromagnetic Fields?

To completely describe a plasma at any time requires knowing the position and velocity of each particle of each type, for example, the position and velocity of each ion and electron. Each particle position and velocity changes under the influence of the electromagnetic fields, and we must know these fields at every point. Further, these fields change according to Maxwell's equations in which the current is obtained from the particle positions and velocities.

As a practical matter, we must give up such infinite precision and deal with the statistical distribution of the particle positions and velocities. Then from these distributions we can obtain a reasonable value for the charge and current densities that produce the **E** and **B** fields that move the particles. Similarly, the statistical distribution of the particle densities and currents is influenced by the electric and magnetic fields. To carry out this approach requires two steps in level of description.

First, we consider the distribution function f_j, where

$$f_j(r, v, t)d^3r d^3v \tag{3}$$

gives the mean number of particles of type j in a small volume in phase space $d^3\mathbf{r}d^3\mathbf{v}$:

$$d^6N_j = f_j(\mathbf{r_j}, \mathbf{v}, \mathbf{t})\mathbf{d^3rd^3v} \tag{4}$$

Then from the f_j we obtain the mean current density

$$\mathbf{j}(r, t) = \sum_k \frac{e_k}{c} \int f_k(\mathbf{r}, \mathbf{v}, t)\mathbf{v}d^3\mathbf{r} \tag{5}$$

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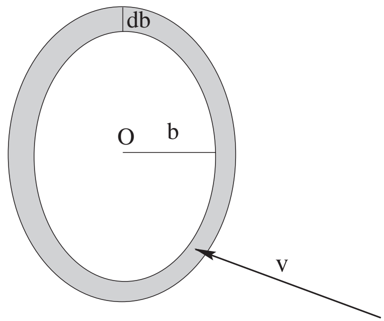

Figure 1.1. Collisions and the stosszahl ansatz

and the mean charge density

$$\rho(r,t) = \sum_k e_k \int f_K(\mathbf{v}, \mathbf{r}, t) d^3\mathbf{r} \tag{6}$$

These can be combined with Maxwell's equations to find **E** and **B**.

However, there is an a important correction to this picture associated with microscopic collisions. If a particle is at **r**, the probability that there is another particle nearby is not uniform, i.e., not exactly equal to $\int f d^3\mathbf{r}$ but on small scales varies erratically. Usually for microscopically stable plasmas this two-particle distribution function is most conveniently treated by Boltzmann's stosszahl ansatz principle. This principle can be roughly expressed as follows.

First, imagine a test particle at rest at O in Figure 1.1. Then imagine an annular zone about O of radius b and thickness db, perpendicular to some velocity **v**. The number of particles with velocity **v** that strike this zone in time dt is taken to be just the average number of particles in a cylinder with this zonal cross section and with height $v dt$. This number is equal to f times the volume $2\pi b db\, v dt$.

In general, the effect of the collisions between the "test" particle and the incoming particles in this zone is usually small enough so that the cumulative effect of these collisions can be estimated statistically by adding them together independently. The gradual evolution of the velocity of the test particle can thus be calculated. Since this collisional effect is random, the probability distribution for the particle's velocity will diffuse in velocity space. For the case of a test particle initially at rest, the particle will develop a velocity in a somewhat random direction. (If the test particle is moving with velocity **v**, we can apply the above picture by transferring to a frame moving with velocity **v**.) It turns out that there can also be a systematic change in the test particle velocity due to the effect of the many collisional encounters (dynamical friction).

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For electric fields on this microscopic scale we must realize that the rapidly varying electric field at a particle is due to the Coulomb fields of these incoming particles passing close by it.

Thus, the electric field (and magnetic field) should be considered as a sum of at least two parts: the large-scale smooth field produced by the large-scale distribution of particles, and the small-scale fields produced by the erratic encounters discussed above. In general, only the first field is discussed explicitly, the small-scale fields and their effect on the particles are treated statistically by collision theory. The large-scale field is predictive, while the small-scale fields are treated statistically.

There are also intermediate-scale fields and particle densities in phase space that are also treated statistically. These are the fields associated with various plasma waves. In a quiescent plasma these waves have very small amplitudes and they have little effect on the particles, However, in certain cases these waves can be excited to larger amplitudes, and their effect on the particles can dominate over normal collisional effects and lead to striking and important results.

To illustrate the behavior of these waves in an important example, consider the propagation of cosmic rays through interstellar medium. If a burst of cosmic rays is suddenly produced by a supernova, the cosmic rays will move away from the supernova at high speed along the interstellar magnetic field. This bulk motion of the cosmic rays will excite Alfven waves, which will grow rapidly. The current in these waves is intense and produces small-scale magnetic fields that interact with the cosmic rays and change the direction of their velocities, forcing their bulk velocity to slow down enough that they no longer excite the Alfven waves. As a consequence, a balance is reached between the waves and the bulk velocity of the cosmic rays, which in a steady state forces the bulk velocity to be approximately equal to the Alfven speed.

The waves themselves consist of coherent electromagnetic fields and the charge and current concentration that produce these fields. Their study is still at the forefront of plasma research and they are generally responsible for nonintuitive phenomena such as this reduction of the motion of cosmic rays to the Alfven speed. They are also responsible for most particle acceleration methods, for the entropy production in collisionless shocks, for the increase of plasma resistivity, and for the rapid conversion of the magnetic energy to heating and bulk particle motion in solar flares.

Although in each wave or wave packet the fields and perturbed particle densities and currents are quite coherent, the actual position and amplitudes of the waves themselves are usually quite random so that they can also be treated statistically.

Thus, the most convenient description of a plasma breaks down first into separate descriptions of the various particle distribution functions in phase space, $f_j(\mathbf{r}, \mathbf{v}, t)$ (six dimensional), and the electromagnetic field, $E(\mathbf{r}, t)$, $B(\mathbf{r}, t)$ (three dimensional). As usual, the fields affect the particles through the equation of motion and the particles affect the fields through Maxwell's equations.

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This general description is simplified by breaking each of these descriptions into three parts. The distribution of particles is divided into three parts, a large-scale smooth part, $f(\mathbf{r}, \mathbf{v}, t)$, a smaller scale part on an intermediate scale associated with waves, and a very small-scale part associated with collisions. Similarly, the electric and magnetic fields are divided into the same three parts, a large-scale smooth part, a smaller intermediate-scale part associated with waves, and a very small-scale part associated with collisions.

The smallest scale part, the part that is important for collisional evolution, can be expressed in terms of $f(\mathbf{r}, \mathbf{v}, t)$ by means of Boltzmann's stosszahl ansatz and leads to Boltzmann's collision integral, or for plasmas, essentially the Fokker–Planck equation.

The intermediate part associated with waves can also be treated randomly since the positions of the waves are of no real consequence. However, their amplitudes are of importance. For each possible plasma wave of type j, the intensity $I_j(\mathbf{r}, \mathbf{k})$ as a function of its position $\mathbf{r}$, and its wave number $\mathbf{k}$ must be given.

Thus, a full description of a plasma requires the distribution function for each species j, $f_j(\mathbf{r}, \mathbf{v}, t)$, specified on the large scale, the large-scale electric and magnetic field $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$, and the mode intensity $I_j(\mathbf{r}, \mathbf{k}, t)$ for the waves. The evolution of each of these quantities is given by its own equations, which involves the other quantities as well. It will be found that this mode of describing the plasma is general enough to encompass most astrophysical plasma phenomena of interest.

It should be noted that the waves themselves can occur on large scales, but these waves are included in the description of the large-scale distributions and fields.

The intermediate-scale waves are described by $I_j(\mathbf{r}, \mathbf{k}, t)$, which depends on the wave number $\mathbf{k}$ as well as position. The wave numbers are large enough that the waves propagate in regions of local homogeneity of the large-scale quantities. The evolution of I will be described by the wave kinetic equation.

Although these matters will be discussed in detail in later chapters we here list the accepted evolution equations for the quantities f_j, E, B, and I. First, $f(\mathbf{r}, \mathbf{v}, t)$ is given by the Fokker–Planck equation

$$\begin{aligned} &\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \nabla f_j + \frac{e_j}{m_j}\left(\mathbf{E}(\mathbf{r}, t) + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \cdot \frac{\partial f_j}{\partial \mathbf{v}} + \mathbf{g} \cdot \frac{\partial f_j}{\partial \mathbf{v}} \\ &\qquad = \left(\frac{\partial f_j}{\partial t}\right)_c + \left(\frac{\partial f_j}{\partial t}\right)_w \end{aligned} \tag{7}$$

where e_j and m_j are the charge and mass of the jth species. The first term on the right represents the time rate of change of f_j due to collisions. For plasma particles it can be expressed in terms of the Fokker–Planck operator which in the Landau form reads

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$$\left(\frac{\partial f_j}{\partial t}\right)_c =$$
$$\sum_\ell \left[\frac{2\pi(e_j e_\ell)^2 \ln\Lambda}{m_j}\right] \frac{\partial}{\partial \mathbf{v}_j}\cdot \int \left(\frac{f_\ell}{m_j}\frac{\partial f_j}{\partial \mathbf{v}_j} - \frac{f_j}{m_\ell}\frac{\partial f_\ell}{\partial \mathbf{v}_\ell}\right)\cdot\left(\frac{\mathbf{I}}{g} - \frac{\mathbf{g}_{\ell,j}\mathbf{g}_{\ell,j}}{g^3}\right) d^3\mathbf{v}_\ell \tag{8}$$

where $\mathbf{I}$ is the unit dyadic and the collisional evolution of f_j is the sum of encounters with the ℓth species. $f_\ell \equiv f_\ell(\mathbf{r}, \mathbf{v}_\ell, t)$, $f_j = f_j(\mathbf{r}, \mathbf{v}_j, t)$, and $g_{\ell,j} = v_\ell - v_j$. This form of the collision operator was first given by Landau in 1937. It has the pleasing properties that it conserves number, momentum, and energy between the ℓth and jth species.

The last term, $(\partial f/\partial t)_{\text{waves}}$, represents the effect of the waves on the evolution of the distribution function. This effect occurs mainly through the interaction of particles resonant with the waves. The resonance can be either a Cerenkov resonance, in which case the particles are traveling close to phase velocity of the wave $\omega - \mathbf{k}\cdot\mathbf{v} \approx 0$, or, in the presence of a magnetic field, a cyclotron resonance, where the Doppler-shifted frequency of the wave seen by the particle is close to a harmonic of the cyclotron frequency $\omega - \mathbf{k}\cdot\mathbf{v} \approx n\Omega$, where n is an integer (possibly negative). Higher order resonances in an expansion in the amplitude of the waves can also occur between the particle and two waves $\omega_1 - \omega_2 = (\mathbf{k}_1 - \mathbf{k}_2)\cdot\mathbf{v}$. In addition, there is spontaneous emission of the waves. Because of the multiplicity of the particle interactions we will not attempt to write a general formula for the wave–particle interactions, but will treat these interactions in detail in specific examples.

For illustrative purposes, we give the quasilinear form of the wave–particle collision term when the waves are electrostatic and the magnetic field is absent:

$$\left(\frac{\partial f}{\partial t}\right)_{\text{waves}} = \frac{\partial}{\partial \mathbf{v}}\cdot\left[\pi\frac{e^2}{m}\int d^3\mathbf{k}\hat{\mathbf{k}}\hat{\mathbf{k}} I(\mathbf{k},\mathbf{r},t)\delta(\omega - \mathbf{k}\cdot\mathbf{v})\cdot\frac{\partial f}{\partial \mathbf{v}}\right] \tag{9}$$

where I is the spectrum of the electric field normalized so that

$$\left\langle\frac{E^2}{\delta\pi}\right\rangle = \int I d^3\mathbf{k} \tag{10}$$

and $\hat{\mathbf{k}} = \mathbf{k}/k$ is the unit vector in the $\mathbf{k}$ direction.

Let us turn to the smooth large-scale parts of the electromagnetic fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, \mathbf{t})$. Their evolution is governed by Maxwell's equations:

$$\nabla\cdot\mathbf{B} = 0 \tag{11}$$

$$-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} = \nabla\times\mathbf{E} \tag{12}$$

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$$\nabla \cdot \mathbf{E} = 4\pi q \tag{13}$$

$$\nabla \times \mathbf{B} = 4\pi \mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \tag{14}$$

where q, the net charge density, and $\mathbf{j}$ are given by equations 5 and 6. These Maxwell equations, (11)–(14), represent the total effect of the smooth plasma on the smooth fields. The remainder of the fields, on smaller scales, is bound up with the Landau collisional term in equation 8, and with the waves described by the wave kinetic equation below.

Next, let us consider the intermediate scale and the waves. There are different types of waves in different wave number regions, but in any given wave number region $\mathbf{k}$ there are generally only one or two types. The behavior of these waves, their dispersion relation $\omega(\mathbf{r}, \mathbf{k}, t)$, and their mode structure or polarization depend on the large-scale fields and distributions of $\mathbf{E}, \mathbf{B}$, and f. Further, the change in their amplitude, growth, or damping rate γ also depends on the distribution of resonant particles given by f. If the waves are unstable, their amplitude will be limited by various nonlinear interactions among themselves.

The evolution of the waves is governed by the wave kinetic equation,

$$\frac{\partial I}{\partial t} - \frac{\partial \omega}{\partial \mathbf{r}}\frac{\partial I}{\partial \mathbf{k}} + \frac{\partial \omega}{\partial \mathbf{k}}\frac{\partial I}{\partial \mathbf{r}} = 2\gamma I + \left(\frac{\partial I}{\partial t}\right)_{\text{coll}} + S \tag{15}$$

The second term on the left-hand side refers to the refraction of the wave and the third to its group velocity. The first term on the right refers to the damping or growth of the waves. The second term on the right represents the nonlinear wave–wave interaction, while the last term S refers to the various ways in which the waves can get started, such as bremsstrahlung or Cerenkov emission.

These processes, represented by γ and S, are complementary to the wave–particle interaction terms in the kinetic equation (7) for the particle distribution function. Whenever waves gain energy due to an instability ($\gamma > 0$), the $(\partial f/\partial t)_{\text{wave}}$ term in the Fokker–Planck equation reduces the energy of the smoothed parts of the particle distribution function. There is, in addition, a more subtle effect related to the refraction of the waves. If the medium is being compressed, the waves gain coherent energy, which exerts a backpressure force on the smoothed plasma to reduce its momentum and energy. This is included as a nonresonant part $(\partial f/\partial t)_{\text{waves}}$ in (7). In a fluid description it is generally expressed in terms of a wave pressure.

In summary, a plasma and its electric and magnetic field interactions break down into six components. The smallest scale chaotic components are taken care of by collision theory in terms of the other four components. The more coherent intermediate-scale parts of $\mathbf{E}, \mathbf{B}$, and f are described by the distribution of energies over the wave spectrum of the various wave modes, I. Thus,

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we need only keep track of three components: $f(\mathbf{r}, \mathbf{v}, t)$, the one-particle distribution functions on large scales; $\mathbf{E}$ and $\mathbf{B}$, the electromagnetic fields on large scales; and $I(k)$, the intemediate-scale wave component. These quantities are governed by the kinetic or Fokker–Planck equations, Maxwell's equations, and the wave kinetic equation, respectively.

The above method of describing a plasma is quite general. It covers most of the phenomena encountered in astrophysics. However, because of its generality it is difficult to handle except in limiting cases. In specific examples these limiting cases are provided by some parameter being very small, which allows a limiting case to be taken. In this book we will frequently make use of these limiting cases when dealing with plasma phenomena.

For example, because the Debye length is very small, the positive and negative charge densities of the plasma very nearly cancel. Because each separate charge density is enormous, this cancellation is essential, otherwise the electric fields generated would be enormous and lead to rapid motions of the electrons, which would immediately short out these fields and restore this quasi-neutral balance. However, because charge densities of the individual species are so large, even after substantial cancellation there is more than enough net charge to produce the electric fields to enforce this near cancellation. This cancellation is termed "charge neutrality" and means no more than that the positive and negative charge densities are nearly equal.

Another equally important plasma limit is the fluid limit that is valid when the collision rate of each species with itself is large compared to the macroscopic rates of change. At the same time, the mean free paths are short compared to macroscopic dimensions. As a consequence, before plasma conditions change significantly each species collisionally relaxes to a local Maxwellian with its own mean density ρ, temperature T, and mean velocity V. These parameters, then, are sufficient to characterize the plasma state and they are governed by the fluid equations. These equations are the magnetohydrodynamic or MHD equations. They consist of

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

the continuity equation for the total density ρ,

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g}$$

the equation of motion for the velocity $\mathbf{V}$, and some sort of entropy equation for the pressure p. If collisions are very strong, then the entropy per unit mass is conserved following the fluid:

$$\frac{\partial}{\partial t} \left(\frac{p}{\rho^{\gamma}} \right) + \mathbf{V} \cdot \nabla \left(\frac{p}{\rho^{\gamma}} \right) = 0$$

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In addition to these equations, the equation of the motion of the electron fluid

$$\rho_e \frac{d\mathbf{V}_e}{dt} = -n_e e \left(\mathbf{E} + \frac{\mathbf{V_e x B}}{c} \right) - \nabla p_e + \mathbf{F}_{ei} + \rho_e \mathbf{g}$$

is also important. $\mathbf{F}_{ei}$ is the force exerted on the electrons by all the other species ions and neutrals. Except at very high frequencies the inertial and gravitational terms are negligible. The resulting equation is called the generalized Ohm's law. It couples the fluid properties to the electromagnetic fields. This fluid description is not as universally valid as the "charge neutrality" condition, but when applicable it leads to enormous simplification of plasma problems.

It is almost always the case in astrophysics that most of the plasma species are properly described by fluid equations. However, a single component, such as the cosmic rays, may be present, which is collisionless and must be described by the kinetic equation (7). For this species, binary collisions are negligible and the term $(\partial f/\partial t)_{\mathrm{waves}}$ is very important. This species is coupled through waves to the rest of the plasma by its pressure, which must be included in the main fluid equation of motion.

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