Chapter 1

Introduction

Flying has become so common that we tend to take many details of flight for granted. Nevertheless, flight is a complex process, involving the equilibrium, stability, and control of a machine that is both intricate and elegant in its design. All aircraft are governed by the same rules of physics, but the details of their motions can be quite different, depending not only on the shapes, weights, and propulsion of the craft but on their structures, control systems, speed, and atmospheric environment. This book presents the flight dynamics of aircraft, with particular attention given to mathematical models and techniques for analysis, simulation, evaluation of flying qualities, and control system design.

In this chapter, we introduce the basic components of configuration that are common to most aircraft (Section 1.1) and provide illustrative examples through descriptions of contemporary aircraft (Section 1.2). Notation that is used throughout the book is presented in Section 1.3, with an introductory example based on the flight of a paper airplane.

1.1 Elements of the Airplane

Aircraft configurations are designed to satisfy operational mission and functional requirements, with considerable consideration given to cost, manufacture, reliability, and safety. All aircraft have structures that generate aerodynamic lift, mechanisms for effecting control, and internal spaces for carrying payloads. Most aircraft also have propulsion systems and associated propellant tanks, undercarriage for takeoff and/or landing, and navigation systems. The layout and interactions of all these components have major influence on an aircraft’s motions [e.g., B-1, C-1, G-2, H-1, H-2, K-2, N-1, R-1, S-1, S-2, S-3, T-1, and W-1].

Airframe Components

Consider the Cirrus SR20 aircraft shown in Fig. 1.1-1. A single-propeller-driven aircraft, the SR20 has all the elements of a conventional aircraft:
wing, fuselage, horizontal tail, vertical tail, and control surfaces, as well as cockpit/cabin, engine, and landing gear. The \textit{wing} provides the aircraft’s largest aerodynamic force, used principally for supporting the vehicle’s weight and changing the direction of flight. The wing’s fore-aft location is near the vehicle’s center of mass for good balance, and the wing itself is shaped to provide a large amount of lift with as little energy-absorbing drag as possible. The \textit{fuselage} is the aircraft’s principal structure for containing payload and systems. Its slender shape provides a usable volume for the payload and a mounting base for engine and tail components, with minimum drag and weight penalty.

Just like a weathervane, the \textit{vertical tail} produces directional stability about an aircraft-relative vertical axis; it gives the aircraft a tendency to nose into the \textit{relative wind} that results from forward motion. Similarly, the \textit{horizontal tail} provides angular stability for rotations about an axis parallel to the wingspan. Aerodynamically, the vertical and horizontal tails are small wings whose lift acts through moment arms (between aerodynamic centers and the center of mass) to generate restoring torques that are proportional to angular perturbations. Collectively, the tail surfaces are called the \textit{empennage}. Stability about the third (rolling) axis is a complex effect, dominated by the wing’s vertical location on the fuselage and the upward inclination of the wing tips relative to the wing roots (the \textit{dihedral angle}).

The SR20 has conventional control effectors: ailerons, elevator, rudder, flaps, and thrust setting. The \textit{ailerons} are movable surfaces located near the wing tips that produce large rolling torques; the two surfaces are linked so that the trailing edge of one moves up when the trailing edge of
the other moves down. The elevator is a movable surface that extends across the trailing edge of the horizontal tail for angular control. In addition to regulating the aircraft’s angle of attack, which, in turn, governs the amount of lift generated by the wing, the elevator setting controls the pitch angle when the wings are level, an important function during take-off and landing. The rudder has a similar effect for yawing motions, controlling the sideslip angle for turn coordination and crosswind takeoffs and landings. It is a movable surface mounted on the trailing edge of the vertical tail.

The wing’s trailing-edge flaps are movable surfaces mounted inboard of the ailerons for direct control of lift and drag during takeoff and landing. The left and right surfaces act in unison, deflecting downward from their cruising-flight (stowed) positions. Whereas the pilot exerts continuous control of the ailerons, elevator, and rudder, the flaps normally are adjusted to discrete settings that depend on the flight phase. Engine thrust setting is regulated only occasionally to achieve takeoff acceleration, climb, desired cruising conditions, descent, and landing sink rate.

A number of other subsystems have dynamic and control effects, including landing gear, trim tabs, leading-edge flaps, spoilers, speed brakes, thrust reversers, engine inlet shape and bleed air, jet exhaust deflection, and external stores. Functions may be combined or distributed, as for tailless aircraft or those possessing redundant control surfaces. These are configuration-dependent effects that are explored in later sections.

If each of the aircraft’s elements performed only the functions described above and performed them perfectly, aircraft dynamics and control could be relatively simple topics; however, there are numerous complicating factors. Perhaps the greatest is that important physical phenomena are inherently nonlinear. The significance of nonlinearity is revealed in the remainder of the book, but the general notion is that doubling a cause does not always double the effect. For example, the wing’s lift is linearly proportional to the angle of attack up to a point; then, the wing stalls and no greater lift can be achieved, even if the angle continues to increase.

A particular element (e.g., the wing) produces a primary effect (lift), but it may also produce secondary effects (drag and side forces, as well as pitching, yawing, and rolling moments). Because the components are in close proximity, the aerodynamic forces and moments that they

1. The angle of attack is the aircraft-relative vertical angle between the centerline and the relative wind. The relative wind is the velocity of the aircraft relative to the air mass through which it flies.
2. The pitch angle is the angle of the aircraft’s centerline relative to the horizon.
3. The sideslip angle is the aircraft-relative horizontal angle between the centerline and the relative wind.
generate are interrelated. Hence, the airstream downwash and vorticity induced by the lifting wing have major effects on tail aerodynamics, thrust changes may upset pitch or yaw equilibrium, and so on. A good deal of useful analysis can be accomplished with single-input/single-output (scalar) models, but the actual dynamic system has multiple inputs and outputs, requiring a more comprehensive multi-input/multi-output (vector) approach.

There are challenging structural and inertial effects as well. The aircraft’s structure must be lightweight for good overall performance, but it is very flexible as a result; consequently, the wing, fuselage, and tail deflect under air loads, and the changes in shape have both static and dynamic effects on equilibrium and motion. In modern aircraft, natural frequencies of significant vibrational modes may be low enough to be excited by and to interfere with aircraft maneuvers, leading to major problems to be solved in control system design. They also contribute to poor ride quality for the passengers and crew and to reduced fatigue life of the airframe. Although most aircraft possess mirror symmetry about a body-fixed vertical plane, inertial and aerodynamic coupling of motions can occur in asymmetric maneuvers. The loading of asymmetric wing-mounted stores on combat aircraft clearly presents a dynamic coupling effect in otherwise symmetric flight, and even a steady, banked turn couples longitudinal and lateral-directional motions.

**Propulsion Systems**

All operational aircraft other than unpowered gliders and rocket-powered “aerospace planes” produce thrust by moving air backward with power from air-breathing engines. Power is generated either by the combustion of fuel and air in *reciprocating engines*, which use crankshafts to transform the linear motion of pistons moving in hollow cylinders to rotary motion, or in *turbine engines*, which produce rotary motion by the action of combustion gasses flowing through a finned wheel (i.e., a turbine). Power is most efficiently transformed to thrust by moving large masses of air at low speed rather than small masses of air at high speed; therefore, large-diameter *propellers* spinning at low rotational rate are indicated. Propellers are driven by either reciprocating engines or turbines; the latter arrangement is called a *turboprop* propulsion system if the propeller is of conventional design or a *propfan* if the propeller is designed for high-speed flow.

Propellers absorb unacceptable amounts of power as airspeed increases, inducing tip speeds that approach or exceed the speed of sound. The *turbofan* engine overcomes these problems by enclosing a smaller-diameter fan within a streamlined duct, allowing higher aircraft speeds but reducing
power conversion efficiency. The proportion of air that is merely accelerated by the fan compared to that which flows through the engine core is called the bypass ratio. The propfan mentioned earlier can be considered a turbofan with ultrahigh bypass ratio. With no bypass air accelerated by a fan, the turbine engine is called a turbojet—all the thrust-producing air mass passes through the combustion chambers and turbines. Turbojets were used in early supersonic aircraft, where the drag losses of the fan outweighed the fan’s conversion efficiency. Advances in fan design have led to supersonic aircraft with turbofan engines. Afterburning produces increased thrust by burning additional fuel in the oxygen-rich exhaust of a turbojet engine, at the expense of reduced combustion efficiency.

For flight at a Mach number $\mathcal{M}$ of two or more, the air can be satisfactorily compressed for combustion by a properly designed inlet, so the compressor blades and the turbine that turns them are no longer necessary. The ramjet is an engine consisting of an inlet (which compresses the air and slows it to subsonic speed), combustion chamber, and exhaust nozzle. It proves to be the most fuel-efficient powerplant for $\mathcal{M} = 3–6$. Compressing the air increases its temperature; at even higher speeds, this temperature rise is excessive, challenging the structural strength of engine materials. The supersonic-combustion ramjet (or scramjet) provides one possible solution to this problem, slowing the air enough for adequate compression, though yielding a supersonic flow through the combustion chamber.

The rocket engine is capable of producing thrust at any Mach number and in a complete vacuum. Moreover, its structural weight per unit of thrust is less than that of any of the previously mentioned devices. The principal penalty is that the oxidizer as well as the fuel must be carried onboard the aircraft, increasing the size and weight of the vehicle. Consequently, the rocket is useful for launch to orbit, but it has very restricted roles to play in atmospheric flight, such as in short-term thrust augmentation for takeoff, maneuvering, or braking.

Future aircraft that operate over a wide range of Mach numbers will almost certainly use variable- or multiple-cycle engines (referring to the thermal cycles of combustion). For example, a turboramjet would operate as a turbojet at low speeds and a ramjet at high speeds, with the percentage of inlet air ducted directly to the afterburner increasing as Mach number increases. A rocket could accelerate an aircraft to $\mathcal{M} = 3$, possibly driving a turbine to compress inlet air for combustion, with a ramjet taking over.

4. The Mach number $\mathcal{M}$ is the ratio of airspeed $V$ to the speed of sound $a$ in the surrounding air. The flow is supersonic when $\mathcal{M}$ is greater than 1 and subsonic when $\mathcal{M}$ is less than 1. Air moves faster or slower than the freestream airspeed over different parts of the aircraft. The transonic region begins at the freestream Mach number for which the flow reaches sonic speed over some part of the aircraft and ends when the entire flow is supersonic, typically for $0.4 < \mathcal{M} < 1.4$. 

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at $\mathcal{M} = 3$, a scramjet coming on line at $\mathcal{M} = 6$, and the rocket resuming for orbital insertion. There are numerous alternatives; which ones are practical is a matter of technological development.

1.2 Representative Aircraft

Attributes of several aircraft types are presented in preparation for the more general study of flight dynamics and control. The principal objectives are to indicate the effect that flight conditions and missions have on aircraft configurations and, conversely, to suggest the latitude for motions and control afforded by existing designs. The following descriptions are approximate.

**Light General Aviation Aircraft**

The *Cirrus SR20* (Fig. 1.1-1) is a four-place aircraft with a 149 kW (200 hp) reciprocating engine. The wingspan is 10.8 m, the length is 7.9 m, and the wing area is 12.6 m². The empty and maximum takeoff masses are 885 and 1,315 kg; hence, the maximum wing loading\(^5\) is 105 kg/m².

The cruising airspeed\(^6\) of this unpressurized aircraft is 296 km/hr (184 statute miles per hour [mph]) at 75% power and 1,980 m (6,500 ft) altitude, and its range is 1,480 km (920 mi). The maximum Mach number is 0.24. The SR20’s straight wing has a moderate aspect ratio\(^7\) (9.1) for efficient subsonic cruising, and it is mounted low on the fuselage with 5 deg dihedral angle. The horizontal tail is also mounted low, and the vertical tail is swept, with a full-length rudder whose throw is unrestricted by the horizontal tail location. The principal airframe material is a structural composite, and the structure uses semimonocoque construction (i.e., the skin carries a significant portion of the loads), with surfaces bonded to spars and ribs. A recovery parachute that can bring the entire aircraft safely to the ground is standard equipment.

**Variable-Stability Research Aircraft**

The *Princeton Variable-Response Research Aircraft* (VRA) is a highly modified single-engine North American Navion A airplane designed for research on flight dynamics, flying qualities, and control (Fig. 1.2-1). The

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5. Wing loading is defined in SI units as aircraft mass (kg) divided by wing area (m²). Using U.S. Customary Units, it is defined as weight (lb) divided by wing area (ft²).

6. The airspeed is the magnitude of the aircraft's velocity relative to the surrounding atmosphere, i.e., of the relative wind. If the wind is blowing, airspeed and ground speed are not the same.

7. The wing aspect ratio is defined as the span squared divided by the wing area.
most distinguishing feature of the VRA is the pair of vertical side-force-generating surfaces mounted midway between wing roots and tips, but equally important elements of its design are the digital fly-by-wire (DFBW) control system, first installed in 1978, which parallels the standard Navion’s mechanical control system, and the fast-acting wing flaps that produce negative as well as positive lift. In operation, a safety pilot/test conductor has direct control of the aircraft through the mechanical system, while the test subject controls the aircraft through the experimental electronic system.

Although limited to an airspeed of about 200 km/hr (125 mph), the VRA can simulate the perturbational motions of other aircraft types through independent, closed-loop control of all the forces and moments acting on the airplane. Feedback control of motion variables to the control surfaces allows the natural frequencies, damping ratios, and time constants of the Navion airframe to be shifted to values representative of other airplanes, while the direct-force surfaces (side force and lift) provide realistic accelerations in the cockpit.

Consider, for example, pure pitching and yawing motions of the 747 airliner. The aircraft rotates about its center of mass, which is considerably aft of the cockpit; consequently, substantial lateral and vertical motions and accelerations are felt in the cockpit when simple angular rotations occur. The VRA can simulate these lateral and vertical motions through
the actions of its direct-force control surfaces. Conversely, these surfaces make it possible to decouple or modify the VRA's natural modes of motion for experimentation. Thus, the VRA can execute a turn without banking or can bank without turning. Its control surfaces can also be used to simulate turbulence for test purposes.

Having been used for over twenty years of research at Princeton University, the VRA and its sister ship, the Avionics Research Aircraft (which is virtually identical to the VRA but does not have side-force panels) are currently owned and operated by the University of Tennessee Space Research Institute.

Sailplane

The Schweizer SGS 2-32 sailplane (Fig. 1.2-2) is a three-place aircraft with a wingspan of 17.4 m, an empty mass of 380 kg, and a typical loaded mass of 600 kg. With an aspect ratio of 18.1 and a wing area of 16.7 m²,
the aircraft’s minimum sink rate is 0.6 m/s. The maximum speed is 255 km/hr, while its speed for maximum lift-drag ratio is about 80 km/hr. The aircraft held numerous speed and distance soaring records in the 1970s, and it was used for aerodynamic studies at Princeton’s Flight Research Laboratory. Several 2-32s were purchased by the U.S. government and designated the X-26A. As the QT-2 or X-26B, Lockheed fitted a quiet engine to the aircraft for testing of stealthy reconnaissance concepts that led to the operational YO-3A. The mechanical flight control system is conventional, with all-moving horizontal tail, ailerons, rudder, flaps, and speed brakes (or “spoilers”).

**Business Jet Aircraft**

The *Cessna Citation CJ2* is representative of many small executive jet transports (Fig. 1.2-3); it has a “T” tail, two 10.7 kN (2,400 lb) thrust
turbofan engines in separate nacelles, and an unswept wing. Mounting the engines on the fuselage aft of the rear pressure bulkhead has several advantages in this aircraft class, including good ground clearance, protection of the engines against ingesting foreign objects while on the ground, reduced cabin noise, low contribution to drag, and structural efficiency. The Model CJ2 carries one or two crew members and up to six passengers in its 14.3-m-long fuselage. The wingspan, area, and aspect ratio are 15.1 m, 24.5 m², and 9.3, while the minimum and maximum masses are 3,460 and 5,580 kg. The maximum wing loading is 228 kg/m². The economical cruise speed at 12,500 m (47,300 ft) altitude is 704 km/hr (437 mph), and the maximum Mach number is 0.72. With maximum fuel, the maximum range of the Model CJ2 is 3,110 km (1,935 mi).

**Turboprop Commuter Aircraft**

The Bombardier Q200 (Fig. 1.2-4) exemplifies the many twin-turboprop designs used by regional airlines to move dozens of passengers on relatively short trips. The Q200 has a “T” tail and a high, straight wing with span, area, and aspect ratio of 25.9 m, 54.4 m², and 12.4, respectively. The craft is 22.3 m long, and it carries 37 passengers plus a crew of two; its empty and maximum masses are 10,380 and 16,465 kg, with a maximum wing loading of 305 kg/m². Typical cruise speed is 535 km/hr (335 mph), corresponding to $\frac{V}{V_0} = 0.44$. The maximum operating altitude is at 7,620 m (25,000 ft), and the maximum range is 1,715 km (1,065 mi). The Q200’s primary structure is aluminum, and its engines each generate 1,490 kW (2,000 hp). The aircraft has an active noise and vibration suppression system for quieting the passenger cabin.

**Small Commercial Transport Aircraft**

The Airbus A318 (Fig. 1.2-5), a shortened derivative of the A320, is on the boundary in flight performance and accommodations between typical regional jets and trunkline aircraft. The A320 was the first subsonic commercial jet to use a DFBW control system and sidemount hand controllers in the cockpit, and it was the first to use composite materials for primary structures. Like its predecessor, the A318 has an advanced technology wing, winglets at its wing tips, and a high degree of redundancy in flight control. Hydraulically powered flight control surfaces include spoilers for roll control and speed braking, leading-edge slats for increased lift at higher angles of attack, and conventional surfaces for roll.

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8. A nacelle is a streamlined fairing over an engine.
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Fig. 1.2-4. Bombardier Q200 regional airliner. (courtesy of Bombardier Aerospace DeHavilland)
pitch, and yaw control. There are mechanical backups for rudder control and pitch trim. The single-aisle A318 carries 107–129 passengers, depending on configuration, a flight crew of two, and a cabin crew of three. The empty mass is 85,900 kg, and normal maximum takeoff mass is 59,000 kg. The wingspan and fuselage length are 34.1 and 31.4 m, while the wing area, sweep angle, and aspect ratio are 122 m², 25 deg, and 9.5. Each of the A318’s engines produces 98–107 kN (22,000–24,000 lb) of maximum thrust. The optimal cruising Mach number is 0.78. The maximum range with 107 passengers is over 2,750 km.

Large Commercial Transport Aircraft

At 73.9 m, the Boeing 777-300 was the longest twin-engine aircraft built at its introduction. It has a maximum takeoff mass of over 299,000 kg and an empty mass of 160,530 kg (Fig. 1.2-6). It is powered by two 435 kN (98,000 lb) thrust turbofan engines mounted in nacelles under the wing. The aircraft has a maximum range of 6,715–15,370 km (3,625–8,300 nautical mi), assuming flight at the economic cruising Mach number of
0.84, with step climbs in altitude as fuel is expended. The maximum certified altitude is 13,135 m (43,100 ft). The 777-300 has a wingspan of 60.9 m and its wing area is 427.8 m², giving an aspect ratio of 8.7 and a maximum wing loading of 700 kg/m². The span of the horizontal tail is about double the wingspan of the Cirrus SR20. The double-aisle 777-300 can be configured to carry 450 passengers in two-class seating or 550 passengers in single-class seating.

In spite of its size, the 777 is certified to be flown by a two-person crew. The wing, mounted low on the fuselage, is swept 31.5 deg. Like other large aircraft with long, flexible wings, the 777 has two sets of roll

9. Current air traffic regulations typically require aircraft to cruise at constant altitude, allowing discrete altitude changes between assigned flight levels. With improving navigation and air traffic control systems, continuous cruise-climb will become more common, as it can provide additional fuel savings.
controls: ailerons near the wing tips for low-speed roll control and mid-wing flaperons plus spoilers for high-speed roll control with reduced wing bending moments. The 777 is the first U.S. commercial transport to use DFBW controls, with a triply redundant system; each of these is internally triply redundant, with dissimilar microprocessors to minimize the impact of hardware or software failures. While control signaling is electronic, the control surface actuators are powered hydraulically.

Supersonic Transport Aircraft

The *Aérospatiale/British Aerospace Concorde* was the world’s only operational supersonic transport aircraft (Fig. 1.2-7), flying from 1976 until 2003. It carried 128 passengers and a crew of three at a maximum Mach number of 2.2 and a typical cruising speed of 2,180 km/hr (1,350 mph). The cruising altitude was over 15,000 m (50,000 ft). Concorde also had an economical subsonic cruising point at $M = 0.93$. The supersonic cruising range with a 10,100 kg payload and reserve fuel was 6,300 km (3,900 mi). Concorde was powered by four turbojet engines, each generating 169 kN (38,050 lb) of thrust with 17 percent afterburning for cruise; full afterburning was available for acceleration through the transonic region, where drag is especially high. The aircraft was 61.7 m long and had a wingspan of 25.6 m; its wing area and aspect ratio were 358.3 m$^2$ and 1.82, respectively. With empty and maximum-takeoff masses of 79,265 and 181,435 kg, its average wing loading was 365 kg/m$^2$. 

Fig. 1.2-7. *Aérospatiale/British Aerospace Concorde* commercial airliner. (courtesy of NewsCast Limited)
Concorde’s delta wing had a sweep angle that varied from root to tip in an “ogee” curve. This shape is important both for supersonic cruising efficiency and for satisfactory placement of the strong vortices that form over the wing at high angle of attack during takeoff and landing. The need for good pilot visibility during landing and sharp-nosed streamlining in supersonic flight conflict; hence, the aircraft had a two-position fairing (“droop snoot”) in front of the cockpit that accommodated both demands. As there was no separate fore or aft control surface, pitch and roll control were both supplied by the same set of “elevons” (deflected in sum for pitch and in difference for roll) at the wing’s trailing edge. The control moment arms were relatively short, and without a separate surface for pitch control, there could be no flaps for landing. Fuel was transferred fore and aft during flight to maintain proper balance as the aerodynamic center shifted with Mach number. Concorde was the first commercial airliner to incorporate a dual-redundant, analog fly-by-wire (FBW) control system in parallel with a mechanical control system.

**Fighter/Attack Aircraft**

The *Lockheed-Martin Joint Strike Fighter* is a single-engine, single-seat, high-performance aircraft intended for ground attack and air-to-air combat (Fig. 1.2-8). It is designed for low observability (“stealth”) as well as high performance. Three variants with 80–90 percent commonality are planned: a multirole conventional aircraft (F-35A), a carrier-based version (F-35B), and a short-takeoff and vertical-landing (STOVL) version (F-35C). The main turbofan engine for all variants produces an unaugmented...
maximum thrust of about 155 kN (35,000 lb), while the lift fan for the F-35C, driven by a shaft from the main engine, produces 80 kN (18,000 lb) of thrust for STOVL operations. The F-35A is 15.5 m long and it has a 10.7 m wingspan. The empty mass is 10,000 kg, while the maximum-takeoff mass is 22,700 kg. Given its wing area of 42.7 m², the aspect ratio is 2.7, and the maximum wing loading is 530 kg/m². The F-35’s maximum Mach number is about 1.5, although its typical maneuvering speed would be in the transonic range below Mach 1. The F-35B has a larger wing that folds for storage on an aircraft carrier, as well as larger stabilizing and control surfaces, strengthened landing gear, and a tail hook for arrested landing. The dimensions of the F-35C are similar to those of the F-35A, although the lift-fan engine and attendant structure increase the empty weight. Distinguishing features include twin vertical tails, side-mounted air inlets, and trapezoidal wing and horizontal tail.

Bomber Aircraft

The Northrop Grumman B-2 Spirit (Fig. 1.2-9) is a high-subsonic flying wing configuration with an unrefueled range of about 11,500 km and a payload of 18,000 kg. The angular planform, with straight edges and sharp corners, and the lack of a vertical tail are beneficial characteristics for low observability, although the latter provides a unique flight control challenge: providing adequate yaw stability and control. An active system using deflection of outboard “drag rudders” achieves this goal. There are four control surfaces on each wing, with elevator, aileron, and rudder functions achieved by blending commands to these surfaces. The aircraft’s wingspan, wing area, and length are 52.4 m, 465 m², and 21 m. The B-2 is powered by four 77 kN (17,300 lb) turbofan engines.

Space Shuttle

The Rockwell Space Shuttle (Fig. 1.2-10) is the second glider on our list, although it is powered into orbit by dual solid-fuel rockets and its three liquid-oxygen/hydrogen engines. The shuttle also has two orbital maneuvering rockets, as well as small thrusters for attitude control in space. During its return from orbit, the shuttle transitions from spacecraft to aircraft, reentering the atmosphere at $M = 25$ and a high angle of attack to dissipate heat efficiently without damaging the structure, and performing a “deadstick” horizontal landing at 335 km/hr (210 mph). The shuttle’s planform and aerodynamic control surfaces are functionally similar to the Concorde’s;
however, the shuttle has a good deal more drag and thus glides at a much steeper flight path angle. The craft is controlled by a quintuply redundant DFBW system with no mechanical backup. The shuttle carries a flight crew of two plus up to five mission specialists. Its empty mass is 70,600 kg, and it can return to earth with a 14,500 kg payload. With a wing area
of 250 m², the average wing loading during entry is 310 kg/m². The wing-span is 23.8 m, for an aspect ratio of 2.3, and the vehicle length is 37.2 m.

**Uninhabited Air Vehicle**

Many missions, including meteorological sampling, communications relay, surveillance, reconnaissance, target acquisition, and ground attack, may not require the presence of a human pilot on board the aircraft, may subject the pilot to high risk, or may require endurance or stress that exceeds normal human capabilities. Uninhabited air vehicles (UAV) are low-cost alternatives for performing these missions, and a number of designs have proved their worth to date. The **General Atomics RQ-1 Predator A** (Fig. 1.2-11) has physical and flight characteristics that are similar to those of a light general aviation aircraft, with a wingspan of 14.8 m, a length of 8.2 m, and maximum mass of 1,020 kg. Powered by a 75 kW (100 hp) reciprocating engine, the RQ-1 has a cruising speed of 130 km/hr, a top speed of 210 km/hr, and a maximum endurance of 40 hr. The Predator A carries a payload of 205 kg, and it has been outfitted to launch
small missiles as well as conduct surveillance. The aircraft has a straight wing, inverted-V tail, and retractable tricycle landing gear. Its ceiling is 7,600 m.

1.3 The Mechanics of Flight

Aircraft flight is described by the principles of classical mechanics. Mechanics deals with the motions of objects that possess a substantive scalar inertial property called mass. Objects may be modeled as individual particles (also called point masses) or assemblages of particles called bodies. The translational motions of both point masses and bodies are of interest, as such objects occupy positions in space and may possess three linear velocity components relative to some frame of reference. The three scalar velocity components can be combined in a single three-dimensional column vector \( \mathbf{v} \), where

\[
\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}
\]

(1.3-1)
The product of an object’s mass and velocity is called **translational momentum**, also a three-dimensional vector. **Forces** may act on an object to change its translational momentum, which otherwise remains constant relative to an inertial (absolute) frame of reference.

Unlike point masses, bodies have three-dimensional shape and volume; the **position** of a body is defined by the coordinates of a particular reference point on or in the body, such as its **center of mass** (the “balance point”). The velocity of the body refers to the velocity of that reference point. The angular orientation and rotational motion of a body are also important descriptors of its physical state. **Angular momentum**, the rotational equivalent of translational momentum, remains unchanged unless a **torque** (a force that is displaced from the center of mass by a moment arm and that is perpendicular to the moment arm) acts on the body. A body can be characterized by six rotational inertial properties called **moments and products of inertia**; the former portray the direct relationship between angular rate and momentum about a rotational axis, while the latter establish coupling effects between axes.

Mechanics is further broken down into kinematics, statics, dynamics, and control. **Kinematics** is the general description of an object’s motions without regard to the forces or torques that may induce change; thus, the geometry and coupling of position and velocity (both translational and rotational) are considered, while the means of effecting change are not. **Statics** addresses the balance of forces and torques with inertial effects to produce equilibrium. An aircraft can achieve static equilibrium when it is moving, as long as neither translational nor angular momentum is changing; for constant mass and rotational inertial characteristics, this implies unaccelerated flight. **Dynamics** deals with accelerated flight, when momentum is changing with time. While it is possible to achieve a steady, dynamic equilibrium (as in constant-speed turning flight), the more usual dynamics problem concerns continually varying motions in response to a variety of conditions, such as nonequilibrium initial conditions, disturbance inputs, or commanded forces and torques.

We refer to linear and angular positions and rates of the aircraft as its dynamic **state**; the corresponding twelve quantities are arrayed in the **state vector** described below. Motions occurring in the vertical plane are called **longitudinal motions**, while those occurring out of the plane are called **lateral-directional motions**. The variables of longitudinal motion, related to body axes, are axial velocity, normal velocity, pitching rate, and their inertial-axis integrals: range, altitude, and pitch angle. The lateral-directional variables are side velocity, roll rate, yaw rate, and their inertial-axis integrals, crossrange, roll angle, and yaw angle. **Stability** is an important dynamic characteristic that describes the tendency for the aircraft’s state to return to an equilibrium condition or to diverge in response to inputs or initial conditions. **Control** is the critical area of...
mechanics that develops strategies and systems for achieving goals and assuring stability, given an aircraft’s mission, likely disturbances, parametric uncertainties, and piloting tasks. The aircraft’s natural stability can be augmented by feedback control, in which measured flight motions drive control effectors on a continuing basis.

For the most part, kinematics is well described by algebraic and trigonometric equations. Because many scalar variables often must be addressed simultaneously, matrix-vector notation is helpful for kinematic and other problems. For example, suppose \( x \) is a vector with three components \( (x_1, x_2, x_3) \) and \( y \) is a vector with two components \( (y_1, y_2) \) such that

\[
\begin{align*}
y_1 &= x_1 + \sin x_2 + \cos x_3 \quad (1.3-2) \\
y_2 &= \cos x_1 + \sin x_3 \quad (1.3-3)
\end{align*}
\]

The two equations can be replaced symbolically by the single vector equation

\[ y = f(x) \quad (1.3-4) \]

where \( f(x) \) is a two-component vector composed of the right sides of eqs. 1.3-2 and 1.3-3. As a second example, let

\[
\begin{align*}
y_1 &= ax_1 + bx_2 + cx_3 \quad (1.3-5) \\
y_2 &= dx_1 + ex_2 + fx_3 \quad (1.3-6)
\end{align*}
\]

The two scalar equations are represented by the matrix-vector equation

\[
\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (1.3-7)
\]

where \( \mathbf{H} \) is the \((2 \times 3)\) matrix

\[
\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \quad (1.3-8)
\]

and the matrix-vector product is defined accordingly.\(^{10}\)

The scalar velocity \( v \) is the time rate of change (or derivative with respect to time) of a scalar position \( r \),

\[ v = \frac{dr}{dt} \quad (1.3-9) \]

10. Generic scalar quantities are denoted by lower-case italic letters \( (a, b, c) \), while vectors are represented by lower-case bold letters \( (\mathbf{a}, \mathbf{b}, \mathbf{c}) \) and matrices by upper-case bold letters \( (\mathbf{A}, \mathbf{B}, \mathbf{C}) \). Italic letters (upper or lower case) and other fonts are used throughout to express commonly accepted symbols without rigid adherence to this convention.
while the position is the time integral of the velocity plus the appropriate constant of integration:

\[ r(t) = r_0 + \int_0^t v(t) \, dt \quad (1.3-10) \]

The equations of *statics* can be combined in a single vector equation

\[ 0 = f(x, u, w) \quad (1.3-11) \]

which indicates that some combination of forces and inertial effects is simultaneously balanced along many coordinate directions. In eq. 1.3-11, \( f(\cdot) \) represents the vector of scalar momentum equations to be balanced, \( x \) is the solution variable (e.g., a six-component reduced state vector of translational and rotational velocities), \( u \) is a vector of constant control displacements, and \( w \) is a vector of constant disturbances.

In the *dynamic* case, the physical equations may vary in time \( t \), and rather than equaling zero, they equal the rates of change of the motion variables, term by term:

\[ \frac{dx(t)}{dt} = \dot{x}(t) = f[x(t), u(t), w(t), t] \quad (1.3-12) \]

This is an *ordinary differential equation* for the *state vector* \( x(t) \) as a function of the independent variable \( t \). The definitions of \( x \) and \( f[\cdot] \) can be extended to include position variables and their kinematics. A first-order differential equation of this form is called a *state-space model* of system dynamics.

The object of control is to apply forces and moments that produce desired values of the state during the time interval of interest \( (t_0, t_f) \). The control vector \( u(t) \) may be a function of any measurable variables, of time, or of external inputs such as pilot commands \( u_c(t) \); hence, the dynamic equation becomes

\[ \dot{x}(t) = f[x(t), u[x(t), w(t), u_c(t), t], w(t), t] \quad (1.3-13) \]

in the controlled case.

It is rarely adequate to know just the form of the aircraft’s differential equation: we want to know the state as a function of time or some other variable. Therefore, equation 1.3-13 must be integrated subject to the initial condition of the state \( x(t_0) \):

\[ x(t) = x(t_0) + \int_{t_0}^t f[x(t), u[x(t), w(t), u_c(t), t], w(t), t] \, dt \quad (1.3-14) \]
It is always possible to integrate eq. 1.3-13 numerically; however, simpler alternatives may provide greater insight into aircraft behavior. If, for example, \( f(\cdot) \) is a linear function of its arguments, the initial condition response can be written as the matrix-vector product

\[
x(t) = \Phi(t, t_0) x(t_0)
\]  

(1.3-15)

where \( \Phi(t, t_0) \) is a state transition matrix relating the state at time \( t_0 \) to the state at time \( t \). It is also possible to write the steady sinusoidal response of a linear system to a steady sinusoidal control input \( u(t) = a \sin \omega t \) with vector amplitude \( a \) at frequency \( \omega \) as

\[
x(\omega) = A(j\omega) u(\omega)
\]  

(1.3-16)

where \( A(j\omega) \) is the system's complex-valued frequency-response matrix and \( j = \sqrt{-1} \). Replacing \( j\omega \) by the complex operator, \( s = \sigma + j\omega \), \( A(s) \) is found to be the transfer function matrix, an important element of stability and control analysis.

**Example 1.3-1. Longitudinal Motions of a Paper Airplane**

The classic dart-shaped paper airplane (figure 1.3-1) provides an excellent introduction to flight mechanics, for both numerical analysis and experiment. Starting with a sheet of plain paper, which weighs11 about 3 g,

11. Strictly speaking, its mass (not weight) is 3 g; “weighs” is used here and throughout in a colloquial sense.
the experimental vehicle has a wingspan of 12 cm and a length of 28 cm, yielding an aspect ratio $AR$ and wing area $S$ of 0.86 and 0.017 m$^2$, respectively. As shown in Chapter 3, the point-mass longitudinal motions of the paper airplane can be predicted by integrating four differential equations, where $V$, $\gamma$, $h$, and $r$ are the airspeed, flight path angle, height, and range, respectively. The parameters of this model are contained in the drag coefficient $C_D$, lift coefficient $C_L$, air density $\rho$ (1.225 kg/m$^3$ at sea level), reference area $S$, mass $m$, and acceleration due to gravity, $g = 9.807$ m/s$^2$.

The lift coefficient is modeled as a linear function of the angle of attack $\alpha$,

$$C_L = C_{L_{\alpha}} \alpha$$

The lift-slope derivative is estimated as

$$C_{L_{\alpha}} = \frac{\pi AR}{1 + \sqrt{1 + (AR/2)^2}} \text{ (per radian)}$$

The drag coefficient is modeled as

$$C_D = C_{D_0} + \varepsilon C_L^2$$

with $C_{D_0} = 0.02$ and the induced-drag factor $\varepsilon = 1/\pi e AR$; $e$ is known as the Oswald efficiency factor, estimated to be 0.9 here. In this example, attitude dynamics are neglected, so the angle of attack is taken as the control variable; its trimmed value is a function of the center-of-mass location and the amount of upward deflection of the wing’s trailing edge.

This mathematical model can be put in the form of eq. 1.3-12 by making the following definitions:

$$x = \begin{bmatrix} V \\ \gamma \\ h \\ r \end{bmatrix}, \quad u = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \Delta = \alpha = u$$
\[ f(x, u) = \begin{pmatrix} -C_D(u) \left( \frac{1}{2} \rho x_1^2 \right) S/m - g \sin x_2 \\ C_L(u) \left( \frac{1}{2} \rho x_1^2 \right) S/m - g \cos x_2 \\ x_1 \sin x_2 \\ x_1 \cos x_2 \end{pmatrix} \]

\( f(x, u) \) is a nonlinear function of its arguments for several reasons: \( C_D(u) \) is a quadratic function of \( u \), and several nonlinear operations involve elements of the state, including the square of \( x_1 \), division by \( x_1 \), sines and cosines of \( x_2 \), and the product of \( x_1 \) with functions of \( x_2 \).

Three longitudinal paths are typical of paper airplane flight: constant-angle descent, vertical oscillation, and loop. Numerically integrating the equations of our mathematical model produces these paths (eq. 1.3-14), as shown in the accompanying figures. Four cases are simulated: (a) equilibrium glide at maximum lift-drag ratio (\( L/D \)), (b) oscillating glide due to zero initial flight path angle, (c) increased oscillation amplitude due to increased initial speed, and (d) loop due to a further increase in launch speed. In all cases, \( \alpha \) is fixed at 9.3 deg, yielding the best \( L/D \) (5.2) for the configuration. This angle of attack produces the shallowest equilibrium glide slope (\( -11 \) deg) and a constant speed of 3.7 m/s. The equilibrium trajectory is represented by the straight descending path in Fig. 1.3-2a and the constant speed and flight path angle in Fig. 1.3-2b (case a).

Obtaining this smooth, linear path is dependent on starting out with exactly the right speed and angle; case b illustrates that having the right speed but the wrong initial angle (0 deg) perturbs the dynamic system, introducing a lightly damped oscillation about the equilibrium path. The period of the oscillation is 1.7 s, and the wave shape is very nearly sinusoidal.

The initial-condition perturbation is increased in case c, where the zero initial angle is retained, and the speed is increased. The vertical oscillation has greater height, speed, and angle amplitude, and the wave shape is perceptibly scalloped in the first two variables. Nevertheless, the rate of amplitude decay and the period are about the same as in the previous case. Further increasing the launch speed results in a loop, followed by sharply scalloped, decaying undulations with a 1.7 s period (case d). The loop provides an upper limit on the vertical oscillations that can be experienced by an airplane with fixed control settings.
Fig. 1.3-2a. Flight of a paper airplane: Height versus range.

The MATLAB program used to generate these results can be found in Appendix D. You can replicate these flight paths with your own paper airplane, although you are not likely to get identical flight test results. The aerodynamic model used here is approximate, and the weight and dimensions of your airplane will be slightly different. It is difficult to trim the paper plane’s $\alpha$ precisely, but it is relatively easy to sweep a range of values by bending the wing’s trailing edge up or down. Similarly, launching by hand produces variable starting conditions, and it probably is not possible to launch a paper airplane at speeds much above 5 m/s without inducing significant aeroelastic deformations to the structure, changing its aerodynamics. Perhaps most important, attitude dynamics (not modeled above) play a role, particularly in large-amplitude maneuvers such as the loop. It is tricky to loop the classic dart-shaped paper airplane in practice, but other designs can be looped readily.
Fig. 1.3-2b. Flight of a paper airplane: Velocity, flight path angle, altitude, and range versus time.

References


