Chapter 1

Finding the Optimal Use of a Limited Income

Main Economic Concepts	1. More is better. 2. Free choice is a valuable commodity. 3. Freedom to trade can make everyone better off.
New Terms	1. Indifference curve 2. Diminishing marginal utility 3. Budget constraint 4. Optimal use of a limited income 5. Pareto efficiency 6. Pareto optimal allocation 7. Pareto efficient allocation 8. Edgeworth box diagram 9. Contract curve 10. Compensation principle 11. Substitution effect 12. Price effect 13. Income effect 14. Corner solution

The best place to start the study of economics is with a model of consumer decisions. Each of us has a limited income and must make choices about how best to allocate it among competing uses. Compared to a bundle of goods and services that are given to us with a market value of $20,000, most of us would prefer to have a $20,000 income to spend as we want. Why? Because each of us has different preferences for different goods and services, and thus, the "value" of a dollar is higher if we have the opportunity to spend it as we please. The value of free choice is a central tenet in economics and provides the basis to understanding the concept of a demand curve.

I. Indifference Curves

I am going to pursue this problem in a simplified way. There are two goods, clothing and housing. There are no other uses of income, no savings and no taxes.

A. The Main Question

A person has $100 to spend during some period. Using the assumptions below, how much does he spend on clothing and housing?

Assumptions about consumer preferences:

1. Each consumer knows his or her preferences and is able to articulate them so that we can portray them in the form of a chart.
2. Preferences are consistent. If a consumer tells us that some bundle of goods *A* is superior to bundle *B*, and bundle *B* is superior to bundle *C*, then it must follow that *A* is preferred to *C*.
3. More is better. Consumers prefer a bundle of goods that has more of both goods. Likewise, a bundle with fewer of both goods is inferior.

To answer this question, I need to introduce the concept of an **indifference curve**. An indifference curve merely tells us the various combinations of goods that make a particular consumer indifferent. Consider figure 1-1, panel (*a*). I assume that we can create a homogeneous unit of clothing, like yards of quality-adjusted material. This measure is shown on the horizontal axis. I also assume that we can create a homogeneous unit of housing, like number of quality-adjusted square feet, which I show along the vertical axis. Suppose we consider some combination of clothing and housing labeled *B*, which corresponds to 25 units of clothing and 50 units of housing. What other combinations of clothing and housing would make this consumer indifferent to this particular allocation?

B. Indifference Curves Slope Downward

We know that any bundle that has both more housing and more clothing must be superior to *B*, and thus, any such bundle cannot be on the same indifference curve. This inference follows from the axiom "More is better." The combinations of housing and clothing labeled II in the figure denote superior bundles as compared to *B*. Likewise, our consumer cannot be indifferent between the bundle labeled *B* and any combination of both less housing *and* less clothing, denoted by area IV in the figure. This means that the indifference curve passing though point *B* must pass through areas I and III. In other words, the indifference curve must be downsloping from left to right. Panel (*b*) in figure 1-1 shows one such indifference curve that satisfies this criterion.

This particular indifference curve is unique to some hypothetical person that we are considering. To be concrete, suppose that we are drawing an indifference curve for Jane Smith, who in fact possesses the bundle of goods labeled *A*. This bundle comprises 100 units of housing and 12.5 units of clothing. And suppose that we quiz her as follows: if we take away some units of housing, leaving her with only 50 instead of 100, how many additional units of clothing would she require in order to be indifferent to bundle *A*? We suppose that she answers, 12.5 units,

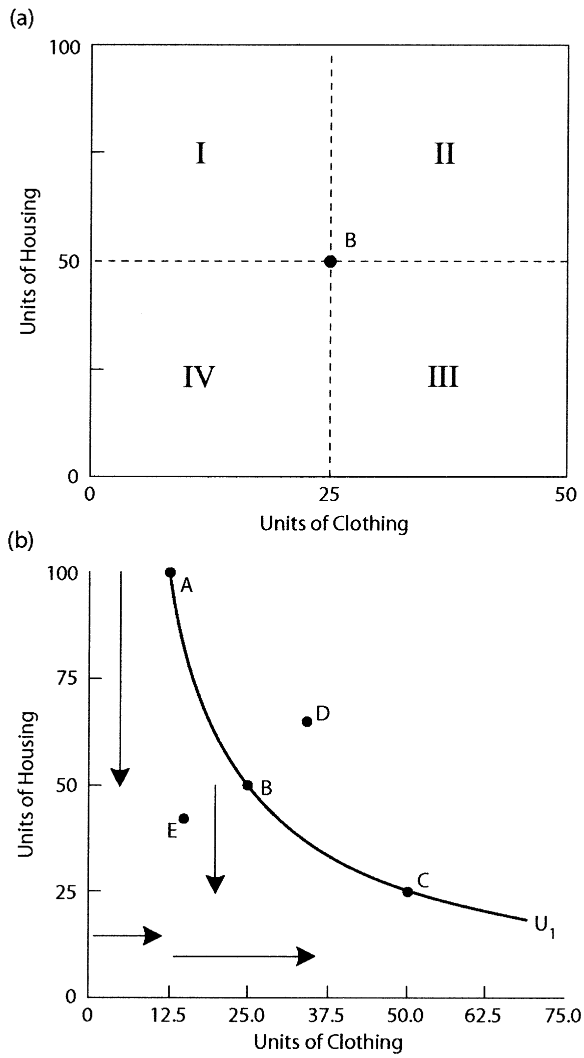

Figure 1-1. The Shape of an Indifference Curve

which I show in the diagram. This corresponds to bundle *B*. Thus, we know that bundles *A* and *B* must lie on the indifference curve. Note that over the relevant range, Jane is willing to give up an average of 4 units of housing for each unit of clothing she obtains.[1]

[1]This is the average trade-off of clothes for housing over the relevant range. The trade-off is different for each individual unit.

Assuming that we continue asking her questions like this, we could draw a line through all the points of her indifference curve, which I label as U_1 in panel (*b*). That is, U_1 describes all the combinations of clothing and housing that make Jane indifferent to bundle *A*; we can think of all these combinations as yielding the same *utility* to her, which is why I use the letter *U* to denote the indifference curve.

C. Other Things to Know about Indifference Curves

A few other features of indifference curves are important to know: they (*a*) are convex to the origin, (*b*) are infinite in number, (*c*) never cross each other, and (*d*) different consumers have different indifference curves.

Indifference curves are convex from the origin. This phenomenon is due to the concept of **diminishing marginal utility**, meaning that consumers attach a higher value to the first units of consumption of clothing or housing, and less value to marginal units of clothing or housing once they have an abundance of them. Thus, if Jane has lots of housing and little clothing, as for example at point *A* in the figure, she is willing to trade 50 units of housing for 12.5 units of clothing to form bundle *B*. But once she attains this bundle, she attaches less value to obtaining still more clothing and is more reluctant to give up more units of housing.

For example, starting at point *B*, suppose that we take 25 units of housing from Jane, say from 50 to 25 units in the figure. She requires 25 more units of clothing to make her indifferent to bundle *B*. Bundle *C* denotes the new allocation. Over the range *B* to *C*, she is willing to sacrifice only 1 unit of housing to receive 1 unit of clothing, on average. Compare this to the move from point *A* to point *B*, where she was willing to give up four times as much housing for each unit of additional clothing, on average. The difference is that at point *B*, she already has a fair amount of clothing and thus is not willing to give up as much housing to obtain even more clothing.

There are an infinite number of indifference curves. Panel (*b*) in figure 1-1 depicts a single indifference curve for Jane. That is, I started with bundle *A* and then drew an indifference curve through all the other bundles like *B* and *C* that yield the same utility to her. But suppose that Jane started with an allocation of goods labeled *D*. We know that this bundle of goods cannot be on indifference curve U_1 because in comparison to bundle *B*, for example, bundle *D* has both more clothing *and* more housing. Since more is better, then it follows that *D* must be on a higher indifference curve than *B*. Following the same reasoning, bundle *E* must be on a lower indifference curve.

If we pursued the same experiment with Jane starting from bundle *D* as we did when she had bundle *A*, we could draw a second indifference curve running through bundle *D* in the figure. If we do, then we have an indifference curve labeled U_2. Similarly, we could draw an indifference curve passing through point *E*, labeled U_0. I show these indifference curves in figure 1-2, panel (*a*). U_2 is a higher indifference curve than U_1, and therefore any combination of housing and clothing on this curve is preferred to U_1. Similarly, U_0 is a lower indifference curve than U_1, and

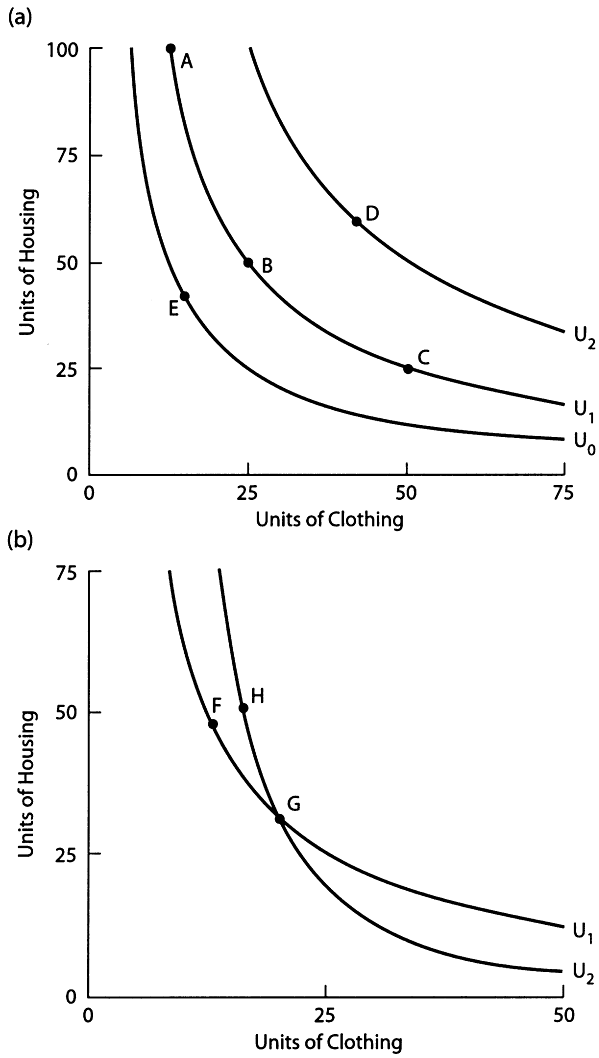

Figure 1-2. An Indifference Curve Map

therefore any combination of clothing and housing on this curve is inferior to U_1. In reality, there are an infinite number of indifference curves. To keep the figures simple, we normally portray only two or three in the relevant range to illustrate a problem.

Indifference curves do not cross. Each indifference curve is uniformly higher than the one below. Why? If they were not depicted this way, they would violate the rule of consistency. Consider panel (*b*) in figure 1-2. In this figure, I have drawn indifference curve U_1 and show points labeled *F* and *G*. I also portray indifference curve U_2 passing through point *G*. In drawing it this way, I am saying that bundle *G* yields the same amount of utility as bundle *F*. I also am saying that bundle *G* is the same as bundle *H*. But how can this be true, since bundle *H* has more housing and clothing than bundle *F*? This conundrum violates the consistency rule. We avoid this problem as long as we ensure that indifference curves never cross.

Note that we can use this same idea to remind ourselves that any bundle on a higher indifference curve is superior to any bundle on a lower indifference curve. Consider panel (*a*) in figure 1-2. How can we be sure that bundle *C* is inferior to bundle *D*? We know this because bundle *C* offers the same utility as bundle *B* because they are on the same indifference curve. But bundle *B* clearly is inferior to bundle *D* because there are fewer units of housing and clothing in bundle *B* compared to *D*. Since *C* is the same as *B*, it follows that *C* also must be inferior to *D*. This is another application of the principle that more is better.

Different consumers have different indifference curves. The indifference curves drawn for Jane are specific to her tastes. Ken Jones would have a different set of indifference curves depending on his tastes for clothing and housing. The basic look of his indifference curves would be similar to Jane's (downsloping, convex, etc.), but his trade-off of clothing and housing very likely would be somewhat different.

II. Gains from Trade Using the Edgeworth Box Diagram

With this small amount of modeling, we already can illustrate an important principle of economics—namely, the gains that result from trade. I demonstrate this concept in the simplest possible way. I assume that there are only two people, Jane and Ken. I have C_{max} units of clothing and H_{max} units of housing. I want to demonstrate the proposition that if I allocate these units in any arbitrary way to Ken and Jane, they almost always will make each other better off by trading. To do this, I need to show Jane and Ken's indifference curves on the same picture. This is done through the use of an **Edgeworth box diagram**.

As a first step, I write Jane's indifference curves in figure 1-3, panel (*a*). I label C_{max} and H_{max} on the vertical and horizontal axis to remind myself that this is the maximum amount of clothing and housing available in the problem. In panel (*b*), I write Ken's indifference curves, but I do it in an odd way: I rotate it 180 degrees, so that his origin is diagonal to Jane's. In this picture, Ken has more clothing and housing as he moves away from his origin, as depicted by the arrows. Note that I also

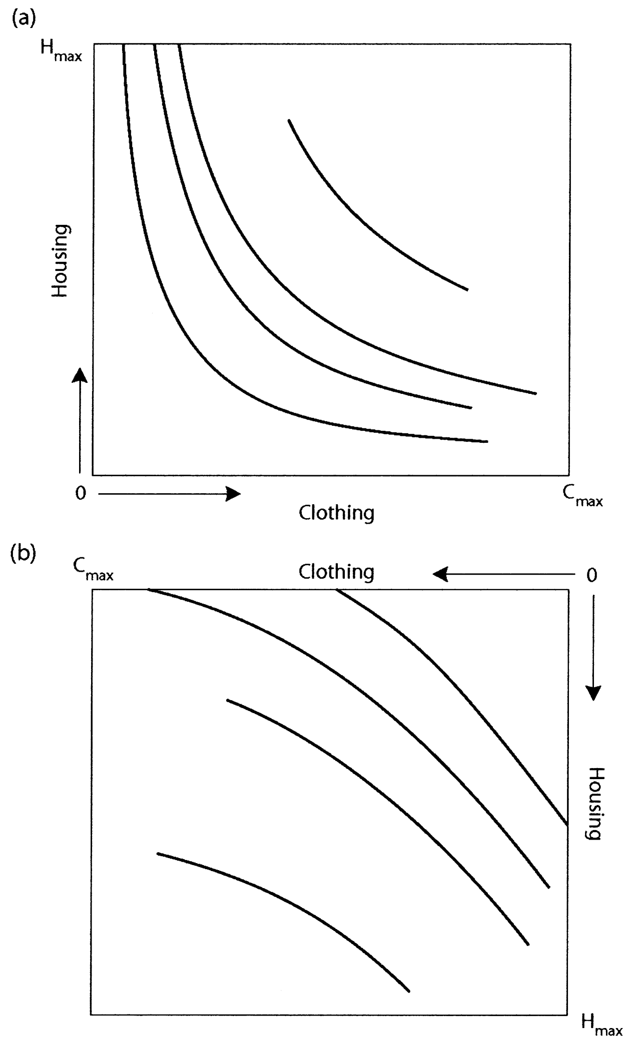

Figure 1-3. Jane's and Ken's Indifference Curve Maps

show $C_{\max}$ and $H_{\max}$ as the limits in this chart, so that the horizontal and vertical lengths of the axes are the same as Jane's.

> EXERCISE:
> *Step 1*: Draw two sets of indifference curves for Ken and Jane, both recognizing the maximum amount of housing and clothing. Draw them in seperate charts, but draw Ken's indifference map upside down.

A. CONSTRUCTION OF THE BOX

To create the "box," simply slide Ken's indifference curve map until it is superimposed onto Jane's. Note that the charts exactly fit together because the lengths of the axes are the same on Jane's and Ken's figures. I show these charts superimposed in figure 1-4. I label Ken's indifference curves K_i and Jane's J_i. Larger subscripts denote higher levels of utility. (Note that it is OK that Ken's and Jane's indifference curves cross each other, as long as Jane's and Ken's own indifference curves do not cross.)

> EXERCISE:
> *Step 2*: Slide the two indifference curve maps toward each other until they exactly overlap.

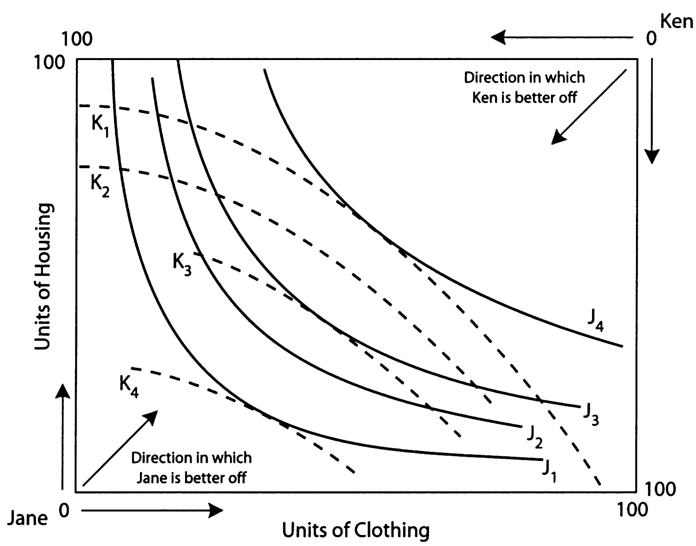

Figure 1-4. Edgeworth Box Diagram

I want to illustrate the initial amount of clothing and housing that Ken and Jane have to start with. I could portray this allocation anywhere in the box, because the axes have been drawn so that no matter where I plot a point, the total amount of clothing and housing must add to the maximum amounts. For illustration, I *arbitrarily* allocate these goods as described by point *A* as shown in panel (*a*) of figure 1-5. Jane has lots of housing and not much clothing, while Ken has lots of clothing and not much housing.

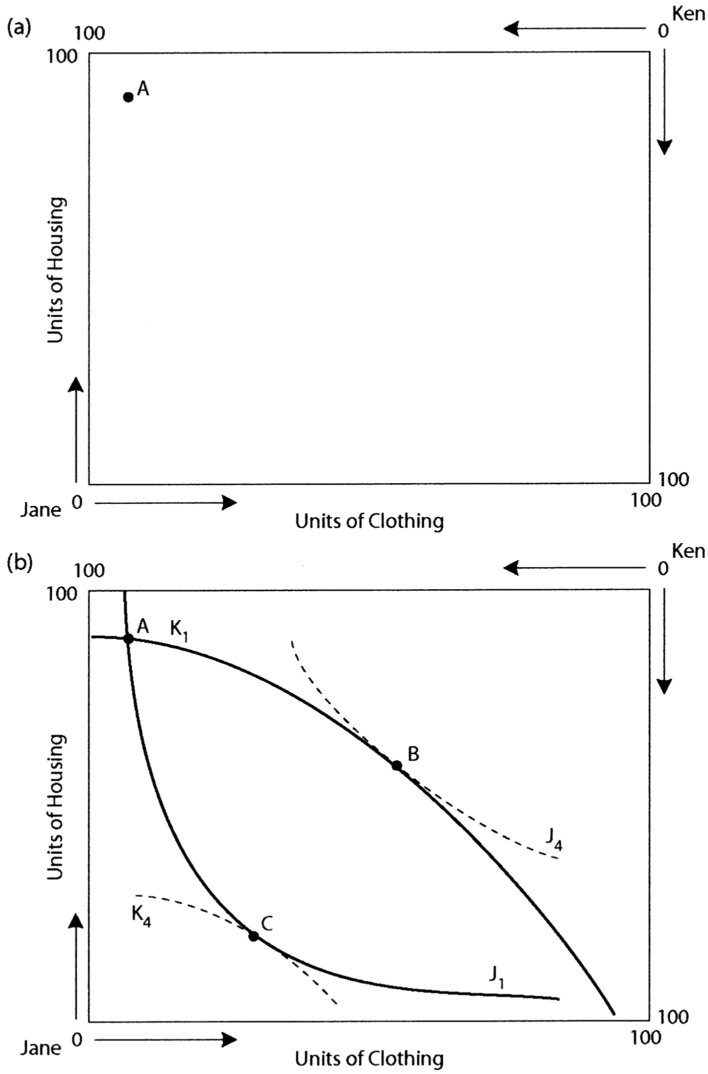

Figure 1-5. Initial Allocation to Ken and Jane

Exercise:
Step 3: Depict the initial allocation of housing and clothing to Ken and Jane. This allocation is arbitrary; it does not matter where in the box we start.

To solve the problem, I reintroduce some indifference curves. Recall that there are an infinite number of indifference curves for both Ken and Jane, and so by definition, we know that each has one curve passing through point *A*; and so I draw these curves as illustrated in panel (*b*), figure 1-5. Notice that these curves, when superimposed, look like a cigar. Ken's indifference curve is K_1 and Jane's is J_1.

Exercise:
Step 4: Draw Jane's and Ken's indifference curves through point *A*.

b. Pareto Superior Trades

It is immediately apparent that a trade could make either Ken or Jane or both better off without making either worse off. This trade involves Jane giving some housing to Ken, and Ken giving some clothing to Jane, meaning that the allocation moves in a southeast direction in the figure—that is, toward the fat part of the "cigar."

For example, suppose that Ken and Jane trade in a way that moves their allocation from *A* to *B*. In this case, Ken is no worse off than at *A*, because he is on the same indifference curve; but Jane is clearly better off because at *B* she is on a higher indifference curve compared to point *A* (compare J_4 to J_1). When a trade makes at least one participant better off and no participant is worse off, then it is said to be **Pareto superior**. Similarly, they could trade so that Jane is no worse off but Ken is better off. Ken gets the best deal without reducing Jane's utility at point *C*. The move from *A* to *C* also is Pareto superior. Many moves starting from *A* are Pareto superior.

Exercise:
Step 5: Start trading so that the allocation of goods moves toward the center of the "cigar." We do not know how Ken and Jane will work the trade, but we know that both can be better off by some trade. Consider the extreme trades first, that is, those that make one consumer much better off but keep the other one at the same level of utility.

It is not possible to know exactly who is going to get the better deal in a trade. It depends on Jane's and Ken's relative bargaining power. Most likely, however, both will gain, and we can characterize the range of outcomes in which both can be better off compared to point *A*.

To do this, I add a few more indifference curves in the relevant range in panel (*a*) of figure 1-6. Consider a move from point *A* to point *D*. In

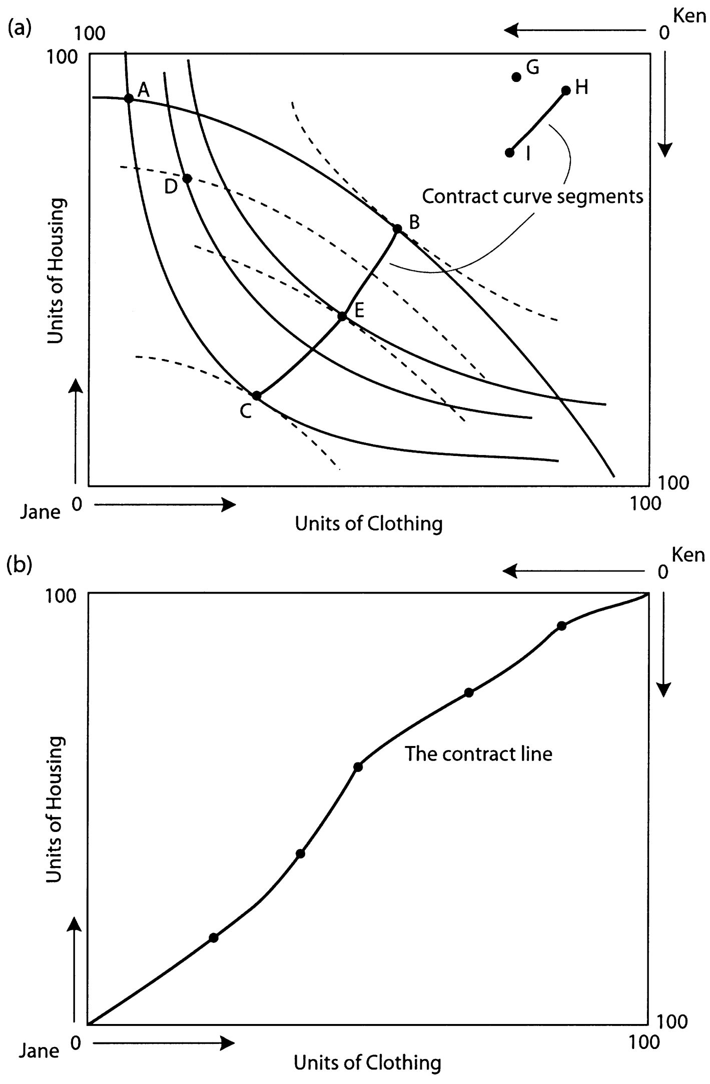

Figure 1-6. The Dynamics of Trading: Pareto Efficient Solutions

comparison to point *A*, both Ken and Jane each are on a higher indifference curve, and thus both have benefited from the trade. Clearly, the move from *A* to *D* represents a Pareto superior move. But at point *D*, both can trade again to further increase their utility. In general, as long as a "smaller cigar" can fit inside a "larger cigar" then in an Edgeworth box diagram, both consumers can be made better off by further trading. When does this process stop?

Exercise:
Step 6: Depict some arbitrary move toward the middle of the "cigar." Any such move will show that both Ken and Jane will be better off. Draw Ken's and Jane's indifference curves through this point.

C. The Contract Curve: Pareto Optimal Allocations

Once they reach a point where their indifference curves no longer form a "cigar" but are just tangent, then it is not possible for one to gain by further trade without making the other person worse off. One such outcome is depicted by point *E*. In general, this condition defines a **Pareto optimal allocation**. A Pareto optimal allocation exists when any possible move reduces the welfare of at least one person. Sometimes, a Pareto optimal solution is referred to as a **Pareto efficient allocation**. Likewise, a Pareto superior move sometimes is referred to as a Pareto efficient trade.

Exercise:
Step 7: The process ends when any further trade reduces the utility of at least one of the consumers. This occurs where two indifference curves just touch, or are tangent to, each other.

So far, I have portrayed a solution for one arbitrary initial allocation of clothing and housing, namely *A*. For this allocation, I have shown at least three possible trading outcomes, namely *B, C*, and *E* in panel (*a*), figure 1-6, whereby at least one consumer is better off and none is worse off. Depending on how Jane and Ken bargain, we could have a solution anywhere along the segment *CB* in the figure. Any point along this segment has the characteristic that Jane's and Ken's indifference curves are tangent.

What if the allocation we started with was not *A* but some other point in the Edgeworth box, for example, point *G* in panel (*a*)? Repeating the exercise for this allocation would lead us to some solution along the segment *IH*, which also is a segment along the contract curve.

If we completed many such exercises, we could find many solutions in the chart, all of which were characterized by the tangency of Ken's and Jane's indifference curves. I already have shown two segments along this line, namely, *CB* and *IH*. If we draw a line connecting all of these points, we have the **contract curve** in the Edgeworth box, which I show in panel (*b*), figure 1-6, by the diagonal line connecting the origins of Jane's and Ken's indifference curve maps. This line can be smooth or not so smooth, depending on how the participants' indifference curves look.

EXERCISE:
Step 8: Show the contract curve in the Edgeworth box, which depicts the bundles that are Pareto optimal outcomes, regardless of where the original allocation is depicted.

EXERCISE:
To test your understanding of the Edgeworth box, start with a replication of figure 1-5, panel (*a*). Put a dot anywhere in the Edgeworth box designating the initial allocation. Draw Ken's and Jane's indifference curves that pass through that point. Unless you know Ken's and Jane's utility function exactly, there is no way you can exactly represent where these indifference curves lie, but you can draw illustrative indifference curves for them, paying attention to the rules of indifference curves that you have learned. You can then bound the solution (best deal for Jane and best deal for Ken) and show the segment along the contract curve between these points that represents the range of possible solutions.

Finally, while we have not worried about where Ken and Jane end up on the contract curve, given their initial allocation, in reality it makes a difference to each participant. For example, starting from point *A* in panel (*a*), figure 1-6, it matters to Ken where along the segment *CB* he ends up; he is far better off at *C* than *B*. The opposite is true for Jane. The differences in outcomes is one reason why corporations spend large amounts of money trying to sway contracts in ways that are favorable to them, without at the same time making the deal unprofitable for the other party. Put simply, lawyers and other professionals are paid considerable sums to help influence outcomes along the contract curve.

Pareto superior: A trade that makes at least one party better off without making anyone worse off.
Pareto optimal allocation: Any outcome that cannot be altered without making at least one person worse off. Sometimes, a *Pareto optimal solution* is referred to as a *Pareto efficient allocation*. Likewise, a Pareto superior move sometimes is referred to as a *Pareto efficient trade*.
On the contract curve: A shorthand way of describing a Pareto optimum solution; its meaning derives from the Edgeworth box.

Gains from Trade: Lessons from the Edgeworth Box

Even though no new production takes place, Ken and Jane both improve their welfare by trading some units of clothing and housing. Both improve the welfare of their trading partner as a by-product of pursuing their own interests.

In reference to figure 1-6, panel (*a*), if the initial allocation is depicted by point *A* and the final allocation after trading by point *E*, then both Ken and Jane walk away from the transaction thinking they got a good deal. That is, trading is not a zero sum game: trading can improve the welfare of all the participants to the trade.

Owing to diminishing marginal utility and the fact that individuals do not all have the same preferences for goods, an arbitrary allocation of goods to individuals usually is not as good as the allocation that individuals choose if given the opportunity to trade.

III. The Budget Line: The Essence of the Economic Problem

In most market settings, individuals are not trading directly with each other but instead are faced with market prices that are beyond their influence. In addition, their income is given and limited. A consumer's problem is to allocate her income among available products and services to attain the highest level of utility. In our simple problem where there are only two goods and no taxes or savings, then we can depict the consumer's budget constraint as follows:

Budget line: $I = P_H H + P_C C$

The variable I is the consumer's income, P_H is the price of each unit of housing, P_C is the price per unit of clothing, and H and C are the units of housing and clothing that the consumer purchases.

Figure 1-7, panel (*a*), illustrates the consumer's budget. To make the example concrete, I assume that the price of housing is \$1 per unit,

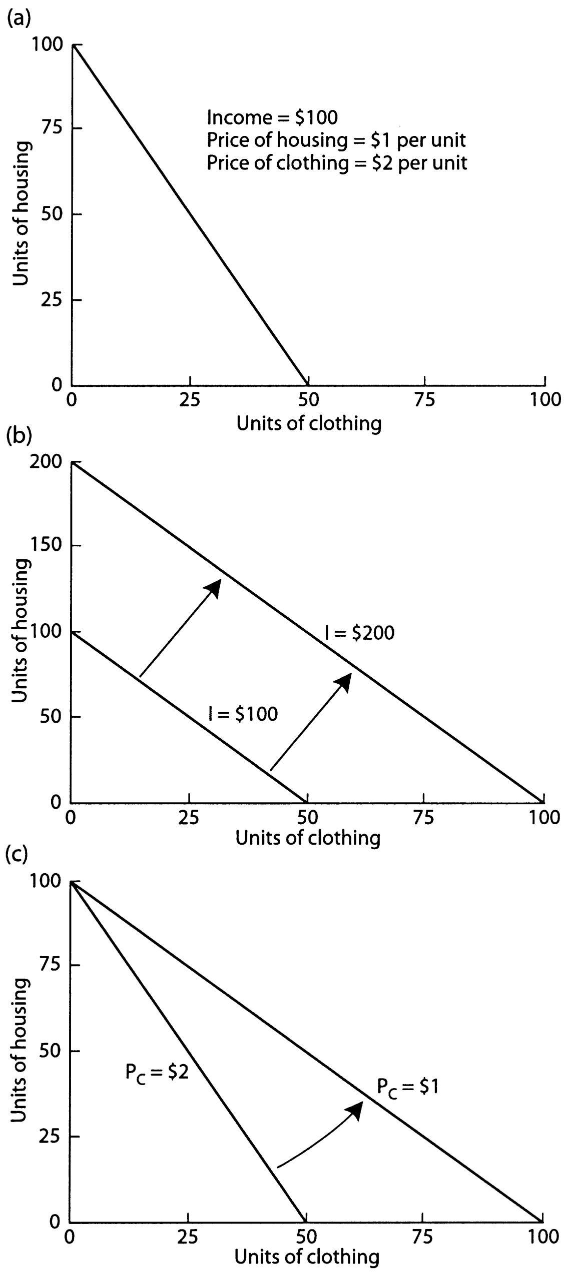

Figure 1-7. Mechanics of the Budget Line

while the price of clothing is $2 per unit. I also suppose that income is $100. The budget constraint tells us that if the consumer spends all of her money on housing, she can purchase 100 units; if she allocates all her money to clothing, then she can purchase 50 units. In addition, the budget constraint describes every other possible allocation of housing and clothing that she can afford. The linear segment in figure 1-7 represents this budget.

A. Impact of Income Changes

The particular budget shown in figure 1-7 assumes an income of $100. Suppose her income doubles to $200 but that the prices of clothing and housing remain the same. Then the budget line moves parallel to the right, as shown in panel (*b*). In this case, she now can purchase 200 units of housing and no clothing or 100 units of clothing and no housing, or any combination in between, as shown by the income line *I* = $200.

B. Impact of Price Changes

Alternatively, suppose that the price of clothing falls from $2 to $1, but everything else stays the same; that is, income is $100 and the price of housing is $1. Then, the maximum amount of housing that the consumer can purchase still is 100 units. But now she can purchase twice as many units of clothing if she allocates all her income to clothing. This means that the budget line rotates out in the direction of the price reduction, as illustrated in panel (*c*).

IV. Consumer Choice: The Optimum Use of a Limited Income

We are now ready to put our model together to determine how Jane allocates her income between clothing and housing. We merely superimpose Jane's budget constraint and indifference curves in the same chart, as shown in figure 1-8, panel (*a*).

A. Determining the Optimal Solution

We know that Jane must purchase a combination of housing and clothing that is consistent with her budget line; and thus, bundles like *A or F* in the figure are possible allocations. Suppose that Jane considers allocation *F*. This allocation is possible because it lies on her budget curve. She enjoys utility level U_1. Similarly, she could choose bundle *G* that also gives her utility U_1. But she can do better than either of these allocations.

In particular, at point *F*, Jane is willing to trade a substantial amount of housing to obtain some additional clothing, as depicted by the steepness

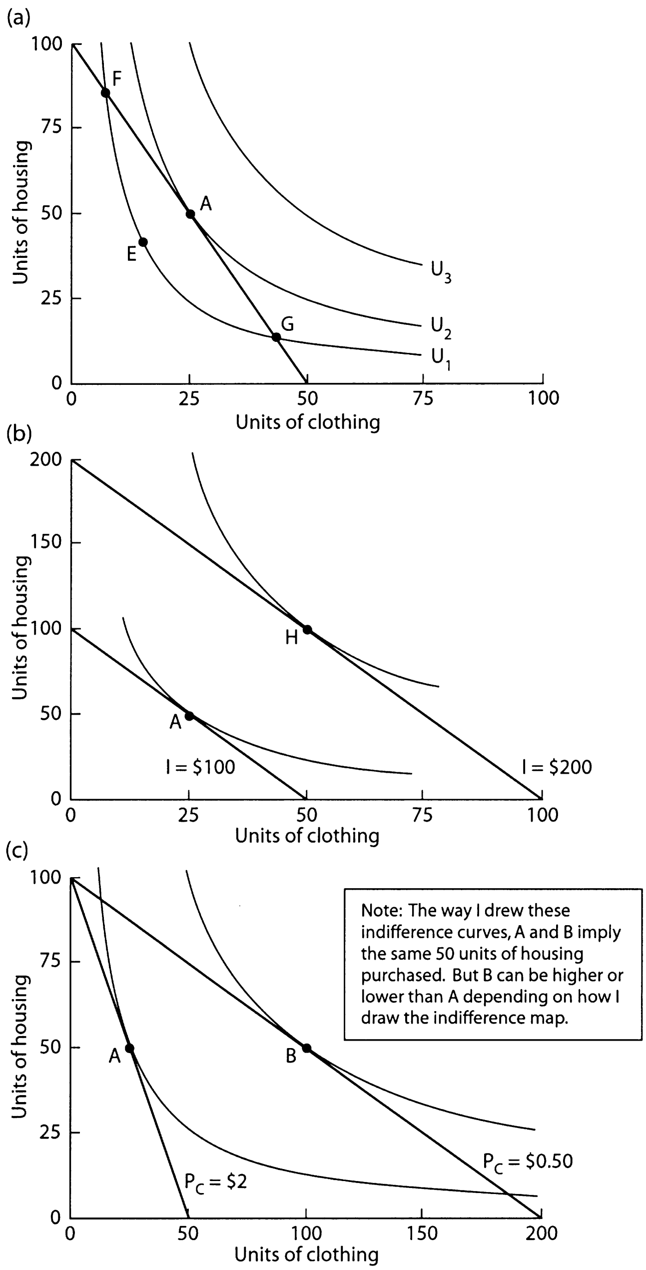

Figure 1-8. Optimal Allocation of Income

of her indifference curve around this point. The budget line is much flatter over this range. More specifically, around point F, in order to obtain one more unit of housing, Jane is *willing* to sacrifice about 10 units of housing. But the market prices are such that she is *able* to obtain 1 more unit of clothing in exchange for only 2 units of housing. So it appears like a good deal to continue giving up housing for clothing at these prices.

Alternatively, you can use the shortcut from the Edgeworth box. At point F, the area bounded by the indifference curve and the budget line (area FEG) looks like a cigar, and F is the tip of the cigar. You know that she needs to move toward the center of the cigar. As she trades housing for clothes at market prices, she goes to a higher utility curve and finds herself at the tip of ever-smaller cigars, until she attains the bundle where the budget line and indifference curve are just tangent. Point A describes this solution.

More simply still, Jane's optimal allocation is found by moving along her budget line until she attains her highest utility. It is apparent from inspection that this solution is depicted by point A in the figure, where Jane's budget line is tangent to her indifference curve. At point A, it is not possible for Jane to alter her allocation along her budget line without reducing her level of utility.

B. PORTRAYING AN EXACT SOLUTION

To obtain a specific solution for Jane, I assume that her utility is described by the following mathematical function, which is a common example used for illustration:

Assume that Jane's utility function is described as follows: $U = \sqrt{C}\sqrt{H}$

Jane's utility equals the square root of units of clothing consumed times the square root of units of housing consumed. For this utility function then, for any given value of U, say U_1, then setting C to a series of values from zero to some large number means that H must fall according to the shape of the indifference curve U_1.[2]

[2]This utility function is $U = C^{1/2} H^{1/2}$. At utility level U_1, then housing H and clothing C are related as follows $H = U_1^2/C$, which defines one indifference curve. To draw utility curves, you can draw out a 45-degrees curve from the origin using a chart like figure 1-8. For $H = C = 1$ then $U = 1$, which you can label U_1, then this indifference curve is defined by $H = 1/C$. Next, $H = C = 2$ so that $U_2 = 2$, where this indifference curve is defined by $H = 4/C$, and so on.

Assuming a particular utility function for Jane allows me to find exact solutions to Jane's allocation.[3] If I assume that her utility function is somewhat different, then I would find some other particular combination of clothing and housing that would maximize the value of her income. It turns out that given her tastes, Jane's optimum use of her \$100 is to buy 25 units of clothing and 50 units of housing.

At the optimal allocation at point *A* in panel (*a*), the slope of her indifference curves exactly matches the slope of her budget constraint. In equilibrium, Jane is *willing* to trade housing for clothing at exactly the same rate that she is *able* to at given market prices.[4] Since point *A* is on her budget curve, 25 units of clothing and 50 units of housing exactly exhaust her income.

Even if he had the same income as Jane, Ken's allocation likely would be different, unless he happened to have the same tastes for housing and clothing as Jane. For example, given his preferences, he might consume 70 units of housing and 15 units of clothing. That is, given the same income, consumers often find the highest value of their money by allocating it differently than other consumers.

C. HOW A CHANGE IN INCOME AFFECTS CHOICE

Now we can reconsider what happens to Jane's consumption if her income increases. Panel (*b*) in figure 1-8 demonstrates the solution when her income doubles to \$200. At the higher level of income, Jane searches for the allocation of clothing and housing that gives her the highest utility. This solution is depicted by bundle *H* in the figure. Notice that at the higher income level, Jane consumes more units of clothing and housing as compared with bundle *A*. In general, as long as a good is "normal," consumers will consume more of it at higher income levels.[5]

[3]If I maximize her utility function $U = C^{\alpha} H^{\beta}$ subject to her income constraint $I = P_C C + P_H H$, then I have the first-order condition $C = H\Phi$, where $\Phi = (P_H/P_C)(\alpha/\beta)$. Substituting for C in her budget constraint, I have $H^* = I/(P_H + \Phi P_C) = I/P_H(1 + \alpha/\beta)$. Thus, Jane consumes more housing the lower the price of housing relative to clothing, and the higher her income. Substituting H^* back into her income constraint, I have $C^* = (I - P_H H^*)/P_C$. Note that when α and β each are ½, then her optimal solution always occurs where she spends 50 percent of her income on each commodity.

[4]Economists sometimes refer to this condition as one where the ratio of the marginal utility of goods equals the ratio of prices, but I do not use this nomenclature in this book.

[5]There are exceptions; for example, perhaps if consumers have sufficient income, they do not purchase hamburger but replace it with steak. But these exceptions are not important for our purposes.

D. The Impact of a Price Change on the Optimum Solution

A change in prices also affects Jane's optimum consumption pattern. Suppose that the price of clothing falls from \$2 to 50¢ but everything else remains the same. Panel (*c*) depicts the problem. Point *A* denotes Jane's original allocation of income. Point *B* denotes her optimal allocation when the price of clothing falls. Not surprisingly, Jane ends up buying more units of clothing at the lower price, but it is interesting that given her particular utility function, Jane consumes the same 50 units of housing at the lower price of clothings.

If Jane had somewhat different tastes, meaning that her indifference curves looked somewhat different from those I depict in the figure, a reduction in the price of clothing might lead Jane to consume either more or fewer units of housing. It seems odd that if the price of clothing falls, Jane's consumption of both clothing *and* housing could increase; this sounds more like an outcome from an increase in income. This puzzle will be solved when we look more closely at the nature and consequences of the change in the price of clothing.

V. The Compension Principle: The Dollar Value of Changes in Utility

In this section, I want to look more closely at the price change depicted in panel (*c*), figure 1-8. The price reduction clearly makes Jane better off. Her utility increases as depicted. We know that Jane is better off at the higher utility, but by how much? While we do not know how to quantify "utils," it turns out that we can measure this utility change in dollars.

A. Valuing the Utility Change from a Price Reduction

The most obvious way to measure the dollar value of the price reduction is to reflect on the money Jane saves at the lower price. Jane is purchasing 25 units of clothing at price \$2. The price then falls from \$2 to 50¢. If she continues to consume 25 units of clothing, she has an additional \$37.50 to spend (\$1.50 times 25 units). In other words, Jane has to be better off by at least \$37.50. It turns out that she is even better off than this. To determine a more precise estimate, ask the following question: What is the maximum amount that Jane would pay Ken if he had the power to reduce the price of clothing from \$2 to 50¢?

Figure 1-9 demonstrates the solution. This figure reproduces panel (*c*) in figure 1-8, except that it adds two new budgets lines, one passing through bundle *A* and another tangent to bundle *C*. To determine the dollar value of the utility change, start at the new equilibrium denoted by point *B*. At this equilibrium, Jane enjoys utility level U_3. Then ask: At the new prices, how much income would Jane require to attain her *old* level of utility, U_1?

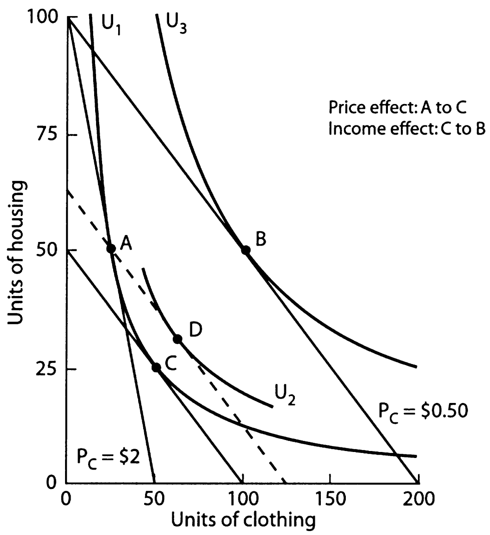

Figure 1-9. Effects from a Change in Price of Clothing

Put differently, how much income would we have to take away from Jane to put her on her old indifference curve? Income changes are represented by parallel shifts in budget lines. Start at point *B*. Drag Jane's budget line leftward in a parallel way until it just touches her old indifference curve—this is the budget line that passes through point *C* shown in the figure. It is tangent to U_1 at point *C*.

We now have sufficient information to obtain a dollar value of the utility change. At point *B*, Jane's income is \$100. You can read this income from figure 1-9. The budget line intersects the horizontal axis at 200 units. The price of clothing on this line is 50¢. Ergo, her budget is \$100.

Similarly, the budget line that passes through point *C* intersects the horizontal axis at 100 units of clothing. The price of clothing on this budget line also is 50¢. Hence, the income level that defines this budget line is \$50.

The dollar value of the increased utility from the price reduction is the difference between these two amounts, \$100 − \$50. Put differently, if Ken held the power to change the price of clothing, Jane would be willing to pay him an amount up to \$50. I obtained this estimate by applying the compensation principle; that is, I searched for that level of income that restores Jane's original level of utility.[6]

[6]For small price changes it turns out that we obtain a good approximation to this answer by simply calculating the product of the change in price, times quantity of clothing that Jane was consuming at the original \$2 price. For large price changes such as the one I show (from \$2 to \$0.50), this approximation is too crude. I pursue this issue more carefully in chapter 2.

Compensation principle: A change in utility brought about by either a change in price or other interference to the market can be translated into a dollar value by searching for the increment in income that restores the original level of utility.

Consider two jobs for lawyers. One is in the area of contracts, a job characterized by more or less predictable hours and a relatively low level of anxiety. The other is in the area of litigation, a job that involves tight deadlines, travel to various court venues, and a high degree of anxiety. Most lawyers require some pay premium (a "compensating differential") to do litigation over contracts that makes up for the reduction in utility caused by the rigors of the job. This is an application of the compensation principle.

Consider the situation in which a well-meaning mom forces her daughter, Jane, to attend a ballet performance. Jane does not pay anything for the ticket and wasn't going to do anything that night anyway. She is visibly upset during the performance and cannot describe how much she hated the experience. Upon leaving, in tears, she accuses her mom of "kidnapping" her and threatens legal action (life in the twenty-first century!). Mom knows that she has imposed substantial disutility on Jane and asks how much it would take (in dollars and cents) to make things right. Jane says that had mom asked her ahead of time how much she would have to pay Jane to accompany her to the ballet, Jane would have said $200. Assuming that Jane is honest, we know the dollar value of her reduction in utility. Damages after the fact often are illuminated by asking about the price of a contract that the plaintiff would have required to be exposed to the damages that resulted. This is an application of the compensation principle.

Why is the answer not $37.50? This is the dollar amount in Jane's pocket when the price change is announced. At the old prices, she purchased 25 units at $2 apiece. Now these units cost $12.50. Ergo, she has $37.50 still in her pocket to purchase more clothing and more housing. The reason this answer is incorrect is that if Jane had a budget of $62.50 at the new prices, she could attain a higher level of utility than U_1.

This alternative can be shown in figure 1-9 as follows. Drag Jane's budget line leftward from point *B*, but instead of continuing to point *C*, stop at point *A* as shown by the dotted-line budget schedule in the figure. This is the budget line at the new prices that permits Jane to purchase the bundle of goods that was optimal under the old prices.

It is evident, however, that faced with this budget constraint, Jane would not consume bundle *A*, but rather would proceed down the budget line to find a higher utility level, U_2. The optimum bundle is depicted at point *D*. Comparison of this budget line to her $100 income measures the difference in utility U_2 and U_3. We want the dollar value of the change in utility from U_1 to U_3. To find the true estimate, continue reducing Jane's income until it is just tangent to utility level U_1, which is shown by bundle *C* in the figure.

B. Anatomy of a Price Change: Income and "Price" Effects

The work we just did to value the change in Jane's utility also serves to illustrate the two components of any price change. First, the price of clothing falls relative to housing, meaning that Jane's new optimum allocation will be more favorable to clothing relative to housing. This is called either the **substitution effect** or, alternatively, the **price effect**, and is reflected by a move along a single indifference curve. Second, the lower price allows Jane to attain a higher level of utility. This is called the **income effect** and is shown by a parallel shift in budget lines between two indifference curves.

In terms of figure 1-9, the movement from bundle *A* to *C* describes the sole effect of the change in relative prices without commingling it with the effect of the change in utility. The movement from *A* to *C* describes "price effect," or, alternatively, the "substitution effect." More units of clothing are consumed and fewer units of housing, as is apparent from comparing points *A* and *C*. The price effect is always negative. That is, a reduction in the price of clothing leads to an increase in quantity consumed, after compensating for the income effect. The move from *C* to *B* denotes the income effect. Normally, the income effect is positive for both housing and consumption.

It is apparent by inspection that as a result of the price effect, Jane increases the quantity of clothes she consumes from 25 to 50 units. As a result of the income effect, she purchases an additional 50 units of clothes. In terms of housing, she reduces her quantity consumed from 50 units to 25 units, owing to the substitution effect. But she consumes 25 more units as a result of the income effect. The income effect exactly offsets the substitution effect, which is a result specific to her utility function. Other utility functions could generate situations in which either the substitution effect dominated the income effect or vice versa.

> *Anatomy of a price change*: If the price of *X* falls, then, owing to the substitution effect (also called the price effect), the quantity of *X* consumed increases. The price effect is always negative. Owing to the income effect, ruling out exceptions (which I do in this book), more of *X* is purchased as well as more of everything else.

VI. Applications of the Compensation Principle

A. Buckley's Tulips and Mums Problem

An illustration of the compensation principle is found in an example of the harm done by "detrimental reliance" from Frank Buckley's Contracts I class. My interpretation of the problem goes something like this. Frank likes to plant tulips and mums in his garden every year. Tulip bulbs are planted in the fall and bloom in the spring. Mum seeds are planted in the spring and bloom in the fall. For simplicity, suppose that Frank has $150 to spend on flowers, that there is no way his wife will give him more flower money, and that there is no chance Frank will spend less than his full budget. The price of mums and tulips is $1 per pot.

I portray Frank's indifference curves in figure 1-10, panel (*a*). Given his particular set of indifference curves, it turns out that if the prices of mums and tulips are the same (which they are in this problem), Frank's optimal flower bundle has an equal number of mums and tulips. I have drawn a 45-degree line from the origin to make sure that I portray his optimal solution along this line (the 45-degree line describes equal numbers of the two kinds of flowers in the garden). Frank's usual allocation is depicted by point *A*, where his garden has 75 tulips and 75 mums.

For Frank's birthday one year (his birthday is in a winter month), Frank's uncle Dick feels generous and promises Frank an extra $100 to finance the planting of more flowers in the coming year. Dick promises to give Frank the gift after he receives his tax refund. Anticipating the extra $100, Frank now has a $250 budget to purchase flowers. I depict the higher budget line in the figure. Naturally, given Frank's preference for symmetry, as reflected in the neat-looking indifference map, he wants to plant 125 mums and 125 tulips with the higher income (point *D*).

Anticipating the gift, Frank orders 125 mums in the winter for spring planting. After he plants them, Uncle Dick calls, saying that on account of an unusually small refund this year, he is changing the amount of Frank's gift from $100 to zero. But since Frank already has committed $125 of his flower budget, he has only $25 to spend on tulips.

Frank immediately threatens to sue his uncle Dick, claiming substantial harm. Dick replies, "How can there be any harm? You still have the $150 you always had." But harm was imposed. *Assuming that we know exactly how to calculate Frank's indifference curves*, we can calculate the amount of the harm. Since I assume a particular utility function for Frank, I can find the answer mathematically. You cannot know the exact

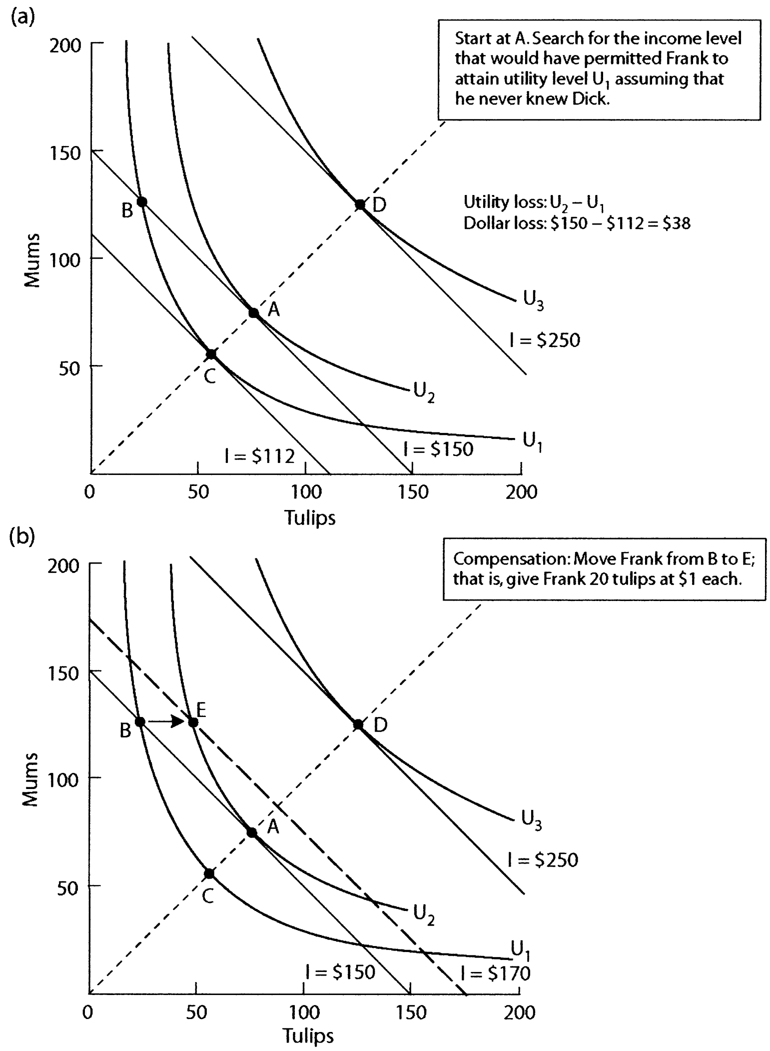

Figure 1-10. Buckley's Tulips and Mums Problem

answer by eyeballing panel (*a*), figure 1-10, but you can show the harm qualitatively.[7]

[7]In the example, I suppose that Frank's utility curve is described by $U = M^{1/2}T^{1/2}$. His budget line is described by $\$150 = P_M M + P_T T$, where P_M and P_T are \$1. Note that this is the same utility function that I assigned to Jane, and thus I can use the derivation in note 3 to show that when prices are equal, Frank always chooses a 50:50 allocation of mums and tulips in his garden.

1. Using the Compensation Principle to Calculate Damages

I suppose that after the flower seasons are over by late fall, Uncle Dick thinks about the harm he imposed on his nephew and decides to make Frank "whole." We need to figure out how much Dick owes Frank to accomplish this outcome. To do this, note that the problem arises because Frank relies on his uncle's promise. But owing to Dick's unreliable character, Frank ends up with 125 mums and 25 tulips; that is, he still has $150 to spend, but he has committed to 125 mums (they cannot be returned after they have been planted), thereby leaving him with only $25 to purchase tulips. This outcome is depicted by point *B* in panel (*a*).

While at first it seems odd that harm has occurred even though no monetary damages are observed ($150 is $150 according to Uncle Dick), it is apparent that harm has been imposed, because consumers are not indifferent to arbitrary allocations of their income. Harm has occurred because Frank's optimal 50-50 allocation of flowers is more valuable than the lopsided look caused by Uncle Dick reneging on his promise.

Frank evaluated point *B* when he decided on the original allocation of his income, but rejected it in favor of point *A*. By choosing point *A*, Frank enjoys U_2 units of utility, whereas point *B* yields only U_1 units of utility. Because of his uncle's promise and subsequent reneging, Frank is stuck at point *B*. This explains why Frank is mad, but does not provide a dollar value of the loss.

We can determine this amount by using the compensation principle. We ask the question: at what level of income could Frank have attained utility level U_1? To find out, simply drag the $150 income budget line parallel to the left until it is tangent to utility level U_1. This solution is found at point *C* (where the budget line is tangent to the indifference curve U_1). Since I have assumed a particular utility function for Frank, it turns out that this budget line is equivalent to $112.[8]

In effect Uncle Dick's unreliability reduced Frank's income from $150 to $112. Put simply, Uncle Dick owes his nephew $38 to compensate him for the harm done by his untrustworthy act.

In reality, we could not know Frank's exact indifference curve mapping. But Frank surely does. And so we could imagine Uncle Dick

[8]The solution is easily determined. Because of the special utility curve I assume for Frank (see prior note), we know that $U_1 = 125^{1/2}\, 25^{1/2} = 56$. How much income does Frank need to attain 56 utils? We know that if the prices of mums and tulips are the same, then he always chooses an equal number of tulips and mums. Call this variable x. Solve for the value of x that gives 56 utils: $U_1 = 56 = x^{1/2}x^{1/2}$. Solving for x times the solutions, $x = 56$. Since the price of mums and tulips is $1, then 56 tulips and 56 mums can be obtained with $112. That is, if Frank has $112 he would allocate half to mums and tulips and attain U_1. With $150 in income, we know that Frank attains U_2. The dollar difference in these two utility levels is $38.

asking Frank in September after winning some money at the racetrack, "Frank, how much would it take to offset the harm I imposed on you last spring?" Supposing that Frank was totally honest with his uncle, then if his indifference curves are like those drawn in figure 1-10, he would answer "\$38." Dick forks over this amount. A family quarrel is settled.

The point is general. *Anytime that a consumer is pushed away from his optimal allocation of income, harm is imposed. In principle, money damages from this harm are calculable.* While it generally is impossible to know this number (since the harmed party might overstate his loss), it explains one reason why your client might be sued for damages even when no dollar loss is visible. Two additional issues, however, must be addressed before leaving this problem. First, there is often more than one way to compensate for harm, and thus, it is worth looking for the cheapest settlement amount. Second, we can at least bound the damages done to Frank, even if we cannot know his indifference curves.

2. A Second Way to Compensate Frank

I have supposed that Uncle Dick procrastinates until late fall to compensate Frank. We have calculated this amount to be \$38. Suppose, however, that Uncle Dick understands the harm he has done in August, when there still is time for Frank to plant more tulips in the fall. In this case, it might be cheaper for Uncle Dick to get Frank back onto his utility curve U_2 right away.

Consider panel (*b*) in figure 1-10. Dick can do this by purchasing enough extra tulips to move Frank from point *B* to point *E*. It turns out that given the particular indifference curves that I have assumed for Frank, an additional 20 tulips would accomplish this outcome.[9]

In this case, Uncle Dick makes things right for only \$20 instead of \$38. This solution is cheaper because Frank attaches some value to the extra 50 mums that Dick induced Frank to purchase, and so adding a few tulips to this burgeoning mums collection is sufficient to make Frank whole right away. In this case, waiting to settle up would cost Uncle Dick an extra \$18. If, of course, Frank is so mad at Uncle Dick that he refuses Dick's offer of \$20 worth of tulips right away, then

[9]Using the particular utility function for Frank as specified in note 7, we know that absent interference from Dick, Frank would have purchased 75 tulips and 75 mums, and thus, would have attained utility level U_2 , which equals $U_2 = 75^{1/2}75^{1/2} = 75$. At point *B*, we know that Frank is at utility level U_1, which as I showed in note 8, equals 56 ($U_1 = 56$). We know that at *B*, Frank has 125 mums. If he had *T* tulips, then, together with 125 mums, he could attain utility U_2 as described at point *E*. We need to solve for *T*: $U_2 = 75 = 125^{1/2}T^{1/2}$, which implies $T = 45$. Since Frank already has 25 tulips at point *B*, this means that Dick can get Frank to point *E* by giving him 20 more tulips.

Frank's claim on Dick is limited to \$20.[10] So, if Frank cools down only next fall, and then demands \$38 from Dick, then Dick should give him only the \$20 he originally offered. Frank's irrational behavior cost him the \$18 difference.

3. Bounding the Solution When Dick Doesn't Know Frank's Utility Function

I have supposed that either I know Frank's indifference curve map or he accurately portrayed it for me. But suppose that I do not know his mapping and he is not volunteering the information. He just demands damages in the amount \$$Z$. How can I bound a reasonable estimate on harm imposed by Uncle Dick? One way is to assume that Frank is absolutely committed to symmetry in his garden. I defined this condition as follows: Frank always chooses the same number of each flower for his garden, *regardless of the relative prices between tulips and mums*. In this case, an additional tulip is worthless unless it is accompanied by an additional mum and vice versa.[11]

Figure 1-11, panel (*a*), portrays these indifference curves. Each utility curve is a right angle denoting the idea that given some level of mums, say at point A, no additional number of tulips will add any utility to Frank unless they are accompanied in exact proportion by more mums. Recall that Frank starts out with \$150, which corresponds to the purchase of 75 tulips and 75 mums in the example (denoted by point A).

> *Zero substitution:* Two goods have zero substitution when, in a compensated sense, the consumer always chooses the same bundle of these two goods regardless of price.

In the Mums and Tulips problem, Uncle Dick promises Frank an additional \$100, thereby inducing Frank to purchase 125 mums. So, Frank anticipates attaining utility U_3 at point D. After Dick reneges, Frank finds himself at point B in the figure, which describes the purchase of 25 tulips and 125 mums. Since he has only 25 tulips, then only 25 mums have any value to Frank; the remaining 100 might just as well be thrown in the trash. Frank is on indifference curve U_1.

[10]It does not matter whether Dick gives Frank \$20 in cash or 20 tulips. In the former case, Frank's budget shifts outward, but he cannot follow the new budget to find the optimal combination of flowers because he is stuck at 125 mums, and the best he can do is stop at this point E, which allows him to enjoy his original level of utility.

[11]This utility function is given by $U = a \min(T, M)$, where a is some arbitrary constant. Frank only attaches value to the minimum number of tulips or mums in his garden. If he has 50 tulips and 30 mums, then any tulips beyond 30 are worthless to him.

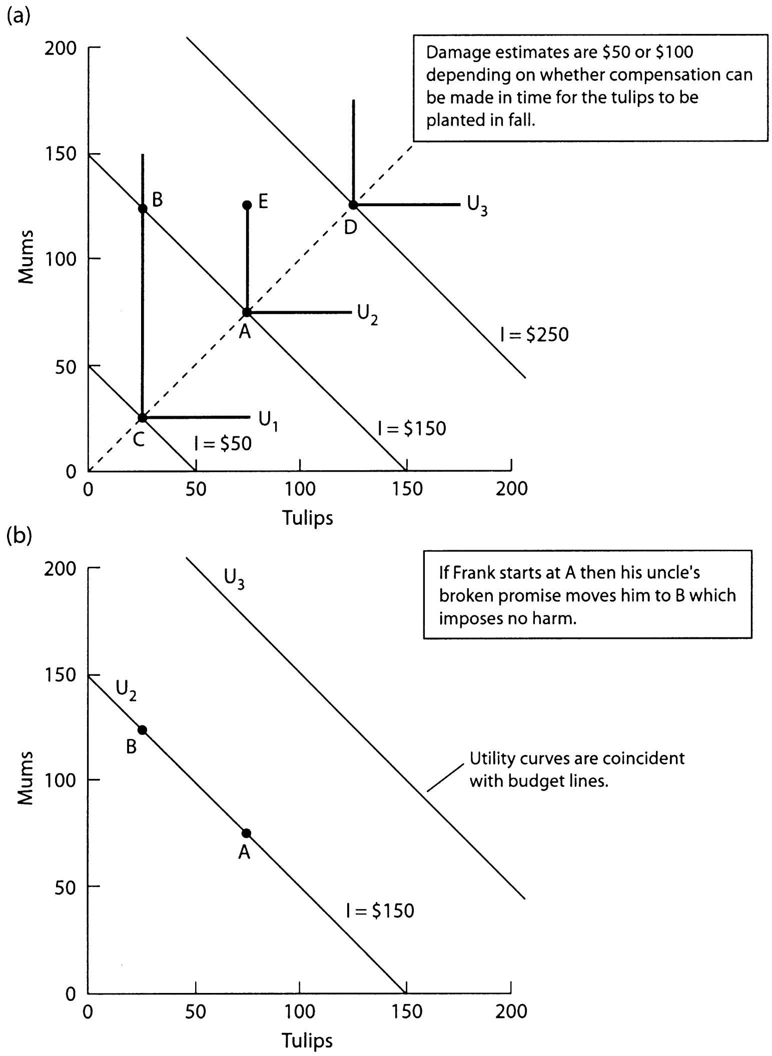

Figure 1-11. Bounding the Damage Amount

It is apparent that Uncle Dick's untrustworthy behavior has done the equivalent of reducing Frank's income to $50. At this income level, Frank could have purchased 25 tulips and 25 mums, which would have put him at point C in the figure, which has the same utility as point *B*. In this case, Dick owes Frank $100: this is the difference between his original budget curve ($150) and the budget line labeled I = $50 in panel (*a*).

Note that it is coincidental that the $100 that Dick owes Frank when the indifference curves are right angles is the same as the amount of the gift that Dick promised Frank in the first place. If the price of tulips and mums were different, then the amount that Dick owes Frank would be different from the amount promised.[12]

If Uncle Dick compensates Frank by buying more tulips before the ensuing fall, then he could get Frank back onto his original indifference curve by giving him 50 tulips. This compensation moves Frank from *B* to *E* in the figure. In this scenario, Uncle Dick salvages 50 of the 125 mums to which Frank currently attaches no value, by matching them with 50 tulips. This solution costs Uncle Dick $50.

Even though we do not know Frank's indifference curves, we know that if Frank attaches a *zero value* to mums without matching tulips, the short-run cost is bounded by $50 if Uncle Dick acts before tulip-planting season and by $100 if he waits until after the fall season to compensate Frank. This is the upper bound cost of the harm imposed upon Frank.[13]

The prior analysis gives us the upper bounds on damages. How can we find the lower bound cost of the harm? We do this by assuming that tulips and mums are perfect substitutes, that is, that Frank enjoys the same utility no matter what combination of tulips and mums he buys. He is just as happy to have 150 mums and no tulips as he is to have 75 tulips and 75 mums. In this case, his indifference curves are 45-degree downward-sloping lines from left to right, as shown by the downward-sloping lines in figure 1-11, panel (*b*). In this case, one of his utility curves is coincident with his budget line. Any point along this curve gives Frank the same utility. Suppose he chooses point *A* arbitrarily.

The fact that his uncle's promise induces him to purchase 125 mums is of no consequence, because having 25 tulips and 125 mums is the same as having 75 of each kind. This solution is depicted by point *B* in panel (*b*), figure 1-11. He has suffered zero utility loss from his uncle's untrustworthiness. Thus, the lower bound harm is zero.

[12]For example, if the price of tulips is $2 and the price of mums is $1, then if Frank's indifference curves are right angles his optimal allocation is 50 tulips and 50 mums based on his $150 flower budget. Let Dick promise Frank $75 and then renege. In this case, you should be able to show that Frank ends up with 37.5 tulips and 75 mums. He only attaches value to the first 37.5 mums. He could have purchased 37.5 of each flower for $112.50, and so the harm imposed by Dick on Frank is $37.50 (= $150 − $112.50). In this case, the harm is only half of the amount Dick promised Frank.

[13]There is one other possibility that I have not considered. What if Frank not only attaches zero utility to each mum that comes up without a matching tulip, but also experiences disutility from it? One utility function that corresponds with this idea is as follows: $U = a \min(T, M) - b|T - M|$, which says that Frank obtains utility from the minimum number of tulips or mums in his garden and attaches a negative utility to the absolute difference in their numbers. The values of a and b measure the intensity of Frank's utility and disutility. In this case, the indifference curves are no longer right angles but evince an angle of less than 45 degrees, forming a kind of "arrow" look, where the arrows are pointed toward the origin.

By bounding the damages, Uncle Dick is in a better position to strike a deal with Frank to make him whole. If the argument takes place while there is still time for Frank to buy more tulips, then if Frank is asking for damages in excess of $50, Dick knows that his nephew is trying to pull a fast one. If the argument takes place after the tulip-planting season, then Dick knows that any claim for damages by Frank beyond $100 is a fabrication. Dick might split the difference between the upper and lower bound and offer Frank either $25 before or $50 after the tulip-planting season, depending on when he decides to settle the issue.

THE CASE FOR A $50 UPPER LIMIT ON THE DAMAGE AMOUNT, REGARDLESS OF SETTLEMENT TIME

In the context of the L-shaped indifference curves, I made the argument that if Dick settles up with Frank after the tulip-planting season, he owes Frank $100. But in fact as tulip-planting season approaches, Frank could have purchased an extra $50 of tulips on his own, and in so doing, reattain his old level of utility for a total cost of $50. His failure to do so increases the value of his losses to $100. Hence, an argument can be made that Dick owes Frank the lesser amount and that Frank himself is responsible for the remainder of his losses.

Alternatively, suppose that Frank's wife will not give him $50, but just prior to the tulip-planting season, Frank calls Dick to explain that an immediate payment of $50 will settle the dispute. Dick ignores Frank until after the tulip-planting season. In this case, Dick is responsible for the $100 losses that develop after the tulip-planting season.

We will revisit this issue in chapter 8 when the notions of contributory negligence and comparative negligence are introduced.

Question 1: Forget about Uncle Dick and suppose that next season, the price of mums is $2 and the price of tulips is $1. Frank has $150 to spend. If his preferences are described in panel (*a*), figure 1-11, how many tulips does he buy? How many mums?[14]

Question 2: Same problem except now assume that Frank's indifference curves are described by those in panel (*b*), figure 1-11.[15]

[14]With L-shaped indifference curves, Frank always chooses the same number of tulips, T, and mums, M. His budget constraint is $\$2M + \$1T = \$150$. Since $M = T$, then Frank buys 50 mums and 50 tulips.

[15]Since Frank is indifferent between tulips and mums, then he will purchase 150 tulips and zero mums. That is, if both are interchangeable to Frank, he simply spends all his flower money on the cheaper alternative.

4. The Second-Round Cost of Untrustworthiness

Assume that Frank and his uncle settle up for last year's unfortunate events. And suppose that next winter, Uncle Dick again promises Frank a $100 gift; in fact, he promises to show up with 100 mums for spring planting. He says, "Don't worry, Frank: this time, I am good for the $100 because I know I am getting a big tax refund." Figure 1-12 depicts the emerging problem.

If Frank believes his uncle, then he anticipates a total flower budget of $250, comprising his regular allocation of $150 plus Dick's contribution. Frank plans on locating at point *D* in figure 1-12 next season, which corresponds to 125 tulips and mums and utility level U_3. But Frank no longer trusts his uncle, and so he attaches zero merit to the promise. So Frank buys 75 mums for spring planting, leaving him with sufficient funds to plant 75 tulips in the fall. He figures on locating at point *A* in the figure.

When spring arrives, however, Uncle Dick comes through with the $100, and in fact, he brings 100 mums for Frank. Now Frank has 175 mums and 75 tulips for planting in the fall, which is depicted by point *C* in figure 1-12. Surely, Frank is better off than if his uncle did not bring more flowers, but he is not as well off as he would have been had he

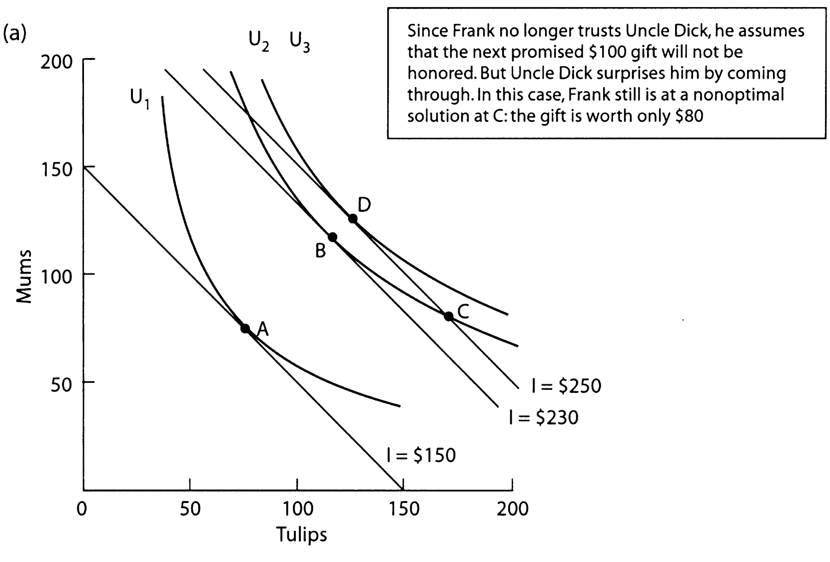

Figure 1-12. Cost of Unreliability

trusted his uncle. Indeed, at point *C*, Frank is on indifference curve U_2, which is lower than U_3.

Recall my assumption that because I know Frank's utility function, I can figure out the cost of Dick's unreliability. It is obvious from the figure that Frank could have attained utility level U_2 by purchasing an equal number of tulips and mums, which would put him at point *B* in the figure. Using Frank's utility function, I determined that he could obtain this allocation with only \$230. Note that his money budget amounts to \$250. Thus, Uncle Dick's \$100 gift is worth only \$80.[16] This explains why, when he sees his uncle coming up the driveway with the additional mums, Frank does not flash a \$100 smile, but rather shows what his uncle perceives to be four-fifths of a \$100 smile.

The wedge between the money Dick spent on mums for Frank and the value that Frank attached to them is a measure of the cost of untrustworthiness.

> *First lesson about reputation value*: There is a cost to reneging on a contract because it builds a reputation for unreliability. The benefits of dealing with an unreliable person are lower than those of dealing with someone with a reputation for honesty and trustworthiness. Put differently, the gains from trade between parties are higher if both have a reputation for trustworthiness than if one has a reputation for untrustworthiness. Hence, we should expect the asset value of reputation to be positive. I will return to this theme later in the book.

B. Dominic's Report Card and Computer Games

Consider the following problem. Dominic is a pretty good student but is a better student when he gets paid to do well in school. In particular, if paid nothing for good grades, he turns in a 3.0 grade point average (GPA). For \$300, he works sufficiently hard to earn a 3.5 GPA. For \$400, he turns in a 4.0 GPA. He reacts the same way every quarter. His economist father decides that the \$400 is worth the extra GPA and decides to pay this amount. Sure enough, the next quarter, Dominic brings home the desired 4.0.

[16]Using the utility function I have assumed for Frank as shown in note 7, utility at point *D* is $U_3 = 125^{1/2}125^{1/2} = 125$. Utility level at point *C* is $U_2 = 75^{1/2}175^{1/2} = 115$. Thus, if Frank has x number of tulips and mums, he could attain U_2 with the value of x that satisfies: $U_2 = x^{1/2}x^{1/2} = 115 \Rightarrow x = 115$. Since tulips and mums cost \$1 apiece, then Frank can attain this utility with \$230 in income.

1. The Solution at First

Dominic decides to allocate the money in the following way: six computer games at $50 each, with the remaining money spent on everything else. Suppose that we create a *composite good* to represent "everything else" and arbitrarily attach a price of $1 per unit. Thus, with $400, Dominic buys six computer games and 100 units of "everything else." Assume that Dominic will repeat this allocation every time he gets his pay-for-performance money.

This allocation is depicted in figure 1-13, panel (*a*). Note that Dominic's budget line is labeled $400 and that his optimal consumption is denoted by point *A*, which reflects six computer games. So far, everything is working out to the satisfaction of Dominic and Dad. It looks like a Pareto optimum solution.

2. The Problem That Arises

A problem arises, however, when Mom discovers how Dominic is spending his money. She feels strongly that he should be limited to three computer games per quarter and that he is better off purchasing 250 units of other things, like shirts and books. Dad tries to change her mind, explaining that this could turn out to be an expensive restriction, but to no avail. The restriction stands.

When he learns of the newfound restriction, Dominic retorts that effectively Dad is reducing his money payment to $300, even though Dad forks over $400. Accordingly, Dominic delivers a 3.5 GPA next time.

The three-game restriction effectively moves Dominic from point *A* to point *B*. That is, he still is on his $400 budget line, but he is restricted to only three computer games, leaving him with 250 units of "everything else." Notice that at this point, he no longer is on utility curve U_2 but instead attains utility level U_1. To put a dollar value on this lower utility, Dominic can ask: what level of income would allow me to attain this

Question: What value did Dominic attach to the restriction of three computer games per quarter? Put differently, how much more cash is Dad paying Dominic to obtain a 3.5 GPA compared to a world in which no restrictions are imposed on his spending?

Answer: Given the previous information, it is apparent that Dominic attaches a value of $100 to the three-game restriction. Put differently, to obtain a 3.5 GPA from Dominic, Dad must pay him an extra $100 above the amount that he would have paid for a 3.5 had Mom not imposed a restriction.

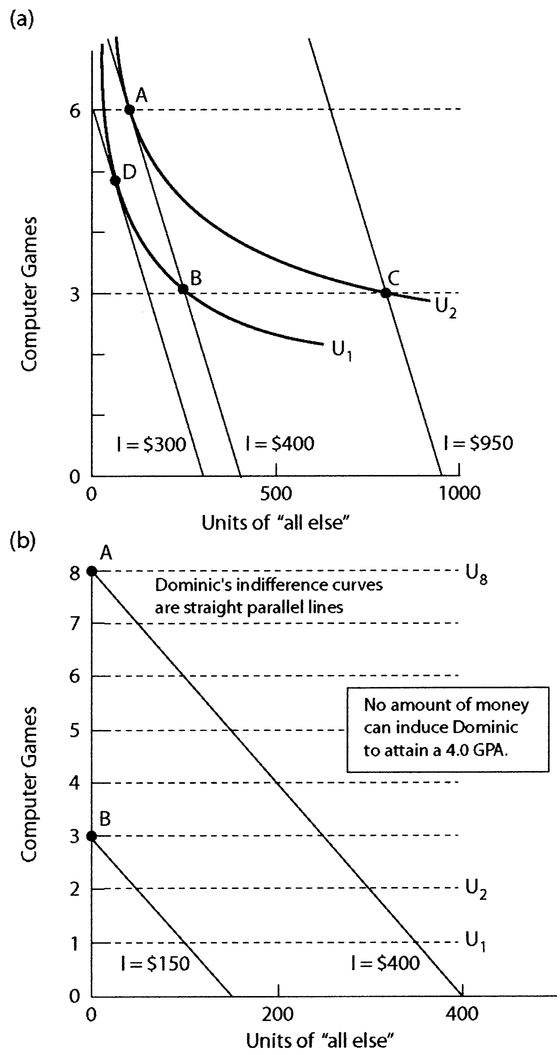

Figure 1-13. Report Cards and Computer Games

utility curve without any restrictions on my spending? The answer is found by dragging his budget line to the left in a parallel way until it is just tangent to utility curve U_1, which is denoted by point D. Since this budget line intersects the vertical axis at six games (which cost \$50 each), we can infer that this new budget line must be \$300, which squares with Dominic's response.

3. The Second-round Solution

Dad really wants that 4.0, and so he tries to figure out how to strike a deal with Dominic that respects Mom's wishes but still results in the desired GPA.

> *Question*: How much cash does Dad have to pay Dominic to induce him to produce the 4.0 GPA without violating the three-game restriction? While you do not have Dominic's utility function, you can read the answer from the information given in panel (*a*), figure 1-13.

We need to find a point where Dominic is on utility curve U_2 *and* on his budget line *and* satisfies the three-game restriction. Clearly, we are not looking for a tangency solution, because the restriction inhibits us from finding an optimum solution.

To find the answer, we need to shift Dominic's budget line to the right along the three-game line until the intersection of the budget line and the three-game limit touches his original indifference curve U_2. In panel (*a*), figure 1-13, this outcome is denoted by point *C*. Note that at this outcome, Dominic has only three games, so he is not in violation of the restriction, but he now has a sufficiently high income to regain utility level U_2. Dad knows that if Dominic attains U_2, he gets the 4.0 GPA on the next report card.

> *Answer*: The answer requires knowledge of Dominic's utility function. It turns out, given the function I have assumed for him, that Dad must give Dominic \$950 in cash to push Dominic back to the utility curve that was attainable with only \$400 in cash with no restrictions. We know that this is the budget line because it intersects the horizontal axis at 950 units of the composite good that costs \$1 per unit.

> Dominic views "other things" as poor substitutes for computer games, and so requires a large number of additional units of other things to make him just as happy as he would be with six computer games and only 100 units of everything else. Thus, it turns out that Mom's restriction cost Dad \$550. Dominic is just as well off as he was when the first contract was made, but now Dad is \$550 worse off without making Dominic any better off, a clearly inefficient solution from their perspective.

4. The Extreme Case: A Corner Solution

I can expand on this example to illustrate the extreme case of preference. Suppose that Dominic's preference for computer games is absolute in the sense that no matter what the price of games is relative to "all else," Dominic always prefers spending all his money on games. This is an even more extreme case than the one depicted in figure 1-11, panel (*a*), for Frank's L-shaped utility curves, because the only combination that gives Dominic utility is zero units of "other things" and as many games as his budget permits him to purchase. These utility "curves" are portrayed in panel (*b*) of figure 1-13 by straight dashed horizontal lines, only some of which I have labeled.

In this utility mapping, more games yield more utility, but no amount of additional units of "all else" confers any utility. Thus, when confronted with his $400 budget line, Dominic chooses eight games and zero of everything else. He has a **corner solution** depicted by point *A*.

Corner solution: When considering two goods, a corner solution occurs when the highest level of utility is found by setting consumption of one of the goods to zero. These solutions tend to occur when the consumer views a competing good as a poor substitute for a favored good. In figure 1-13, panel (*b*), we have both a corner solution and zero substitution, meaning that the optimal solution is zero for everything else.

It is interesting to reconsider his economist dad's scheme together with his mom's restriction. If the restriction is three games, then this puts Dominic at point *B* in the figure. Dominic values this as the equivalent of $150, because he has no use for the additional $250 in cash. In this case, he will not deliver even the 3.5 GPA, let alone the 4.0. Moreover, as long as Mom's restriction holds, no amount of money will induce Dominic to budge from his 3.0 GPA. The marginal dollars of cash beyond the $150 he is allowed to spend on games has no value to him. This is an extreme case of the general lesson that income has more value to individuals if they have no restrictions on how to spend it.

Exercise:
Consider the real-life case reported in the *Washington Post* in August 2001. A mentally ill person who could neither hear nor talk was imprisoned pending a hearing on a trespass case. Let me call this person Mr. Smith. The judge dismissed the case, but owing to a paperwork mishap, the D.C. prison never released Mr. Smith. They kept him in the mentally ill section of the D.C. prison for two years until they realized their error, whereupon he was released. You are his

continued . . .

EXERCISE: *Continued*

lawyer. Assume that the court agrees that the District of Columbia was grossly negligent in this case. How much does the District of Columbia owe Mr. Smith? What principle will you invoke to try to convince a jury that Mr. Smith is owed some amount, *X*? Keep in mind that Mr. Smith does not work and has never worked. Moreover, no one was dependent on him; indeed, no one ever missed him for two years. Mr. Smith had a private cell, and so you can assume that he was free of the possibility of attack or personal injury while in jail.[17]

ECONOMICS IN A SHORT STORY: A PARETO OPTIMUM TRADE

To appreciate economics, and the gains from trade that lie at its core, it is not important to understand complex mathematics and graphical analysis, nor is it important to understand the business transactions involving major corporations or world trade treaties.

continued . . .

[17]One obvious approach is to invoke the compensation principle. Ask the following question to the jury: "Ex ante, how much would *you* have required in compensation to have the opportunity to be locked up in a prison for fully two years, without contact with the outside world?" This is not a silly question. It goes to the amount of money that makes two options equally valuable: (1) freedom with all its attendant benefits (and costs) and (2) going to prison for two years and receiving some amount of compensation in the amount *X*.

Presumably, there is some value of *X* that Mr. Smith would have agreed to accept in order to take prison over freedom. Ask yourself the question: would $200,000 make *you* willing to accept the jail option? If not, would $500,000 do the trick? Sooner or later, I will come to a number that will make the jail option appealing. Ideally, we want the honest number that Mr. Smith would have chosen. But he is not capable of telling us. But perhaps we can ask the jury to come up with their own estimates based on the mental exercise that you ask them to undertake. Will we obtain a perfect number? No. Consider the alternative: the District of Columbia owes Mr. Smith nothing because he does not work and no one depends on him. If you know that the latter answer cannot be correct, then you are beginning to understand the compensation principle.

A case that invokes this approach is *United States v. McNulty* (446 F.Supp. 90). In 1973, Mr. McNulty won about $530,000 (valued in 2001 dollars) in the Irish Sweepstakes, whereupon he deposited his winnings on the Isle of Jersey where secrecy laws put the monjes beyond the reach of the Internal Revenue Service. He deliberately did not pay taxes. He apparently had no other assets and no significant income. The IRS brought him to court, whereupon he received a jail sentence for tax evasion. It is unclear when his sentence began, but it ended on January 23, 1978. At this point, the IRS brought him to civil court to obtain an order for Mr. McNulty to repatriate the taxes owed (about $250,000 in 2001 dollars, including interest and penalties). McNulty refused and was imprisoned for contempt of court for five months, whereupon he was released, free from further legal action owing to double jeopardy. In exchange for these five months, plus the perhaps three or so years he served for tax evasion, McNulty won the rights to the $250,000 he otherwise would have paid the IRS.

Economics in a Short Story: *Continued*

A Christmas Memory, an autobiographical essay by Truman Capote, conveys its essence. The story is about poor folk who live in Alabama in the midst of the Depression.

In the story, Buddy, who portrays Capote as a boy, lives with his underprivileged adult cousin, Miss Sook Falk. She has been endowed with neither wealth, education, nor ability, but she has lots of love to give Buddy. Upon entering the holiday season, Buddy and his cousin decide to make pecan fruitcakes for about two dozen Americans they admire, most of whom they do not know. These people not only include great Americans like the president of the United States, but also include people who crossed their lives, like the family whose car broke down in front of their house last year and with whom Miss Falk had a very interesting conversation. After collecting their spare dimes, nickels, and pennies, they set out to collect their supplies—some at the store, where real money is required, and some that require climbing over fences to collect pecans that have fallen from trees in a local orchard. But the critical input to the fruitcakes is more difficult to come by—legally anyhow.

So, they set out for a strange place in the middle of the woods, owned by one Mr. Haha Jones, who has some clear stuff in used-over bottles. Upon coming to the door, a giant man howls, "What you want?" Buddy hides behind his cousin, who says, "Uh, Mr. Haha Jones, we need some of your best, uh, stuff." "What fer?" says Mr. Haha Jones. "Well, we are making fruitcakes, and we need the finest ingredients to make them taste just right." Mr. Jones returns with a bottle. Holding it in one hand and holding out the other, he demands, "That'll be two dollars."[18]

"Count it out, Buddy," says Miss Falk, whereupon Buddy starts counting out the amount, one dime, nickel, and penny at a time. Finally, Mr. Haha Jones, who looks like he's eaten nothing homemade in years, at least not anything made by a competent cook, says, "Tell ya what, ma'am, you take this here bottle [and just as she was about to refuse on account of she doesn't take charity, he continues] . . . you just give me one of those pecan fruitcakes when you make 'em." Whereupon, Ms. Falk's eyes light up. A great smile comes upon her face as she says, "You mean like a *Trade*?" "Yes 'um, ma'am, I reckon so," says Mr. Jones with a slightly less hardened look on his face.

continued . . .

[18]Quoted remarks are my memory of the exchange. Actual language in the book may differ. See Truman Capote, *A Christmas Memory* (New York: Scholastic, 1997).

Economics in a Short Story: *Continued*

And so, the bottle passes hands. Buddy and Sook, who have so little cash but are about to be so rich in fruitcakes, have made a deal with Mr. Haha Jones, who has plenty of two-dollar bills and clear liquid, but so little good food. The trade has made both better off.

As the scene closes, we see Mr. Haha Jones cracking a small smile in the background as Buddy and his cousin laugh, dance, and sing their way off his property with their prize, thankful that they still have enough change to pay the postage that will be due on all the fruitcakes they need to send, except of course the one for Mr. Jones, which she tells Buddy will have "an extra cup of raisons" and no doubt will be delivered in person.

Most everyone tries to make economics too complicated. The essence of all economics is contained in the trade between Miss Sook Falk and Mr. Haha Jones. It's all about creating surplus through trades. If you keep this in mind, economics will never be hard.