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Eytan Sheshinski: The Economic Theory of Annuities

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Introduction

“And All the days of Methuselah were nine hundred sixty and nine years: and he died” (Genesis 5:27).

AN ANNUITY IS A financial product that entitles the holder to a certain return per period for as long as the annuitant is alive. Annuities are typically sold to individuals by insurance firms at a price that depends on the payout stipulations and on individual characteristics, in particular, the age of the purchaser.¹

The demand for annuities is primarily based on the desire of individuals to insure a flow of income during retirement against longevity risks. In the United States today, a 65-year-old man and woman can expect to live to age 81 and 85, respectively, and there is a substantial variation in survival probabilities prior to and after these ages. Brown et al. (2001) report that at age 65, 12 percent of men and 8 percent of women will die prior to their 70th birthday, while 17.5 percent of men and 31.4 percent of women will live to age 90 or beyond.

Figure 1.1 exhibits the trend in age-dependent survival probabilities in the United States for cohorts from 1900 to those expected in 2100.

It is seen that while the hazards to survival at very young ages have been almost eliminated, increases in survival rates after age 60 have been slower, leaving substantial uncertainty about longevity for those who reach this age.

Uncertainty about the age of death poses for individuals a difficult problem of how to allocate their lifetime resources if they have no access to insurance markets. On the one hand, if they consume conservatively, they may leave substantial *unintended bequests* that in terms of forgone consumption are too high. Annuities and life insurance can jointly solve

¹ Annuities can be purchased or sold. Selling an annuity (going short on an annuity) means that the individual sells an income stream conditional on the seller's survival. Holding a negative annuity is an obligation by the holder to *pay* a return per period contingent on survival. Most loans to individuals are, at least partially, backed by nonannuitized assets (collateral), but some can be regarded as negative annuities. For example, credit card debts have a high default rate upon death because these debts are not backed by specific assets. As observed by Yaari (1965) and Bernheim (1991), the purchase of a pure life insurance policy can be regarded as a sale of an annuity. We discuss life insurance (bequest motive) in chapter 11.

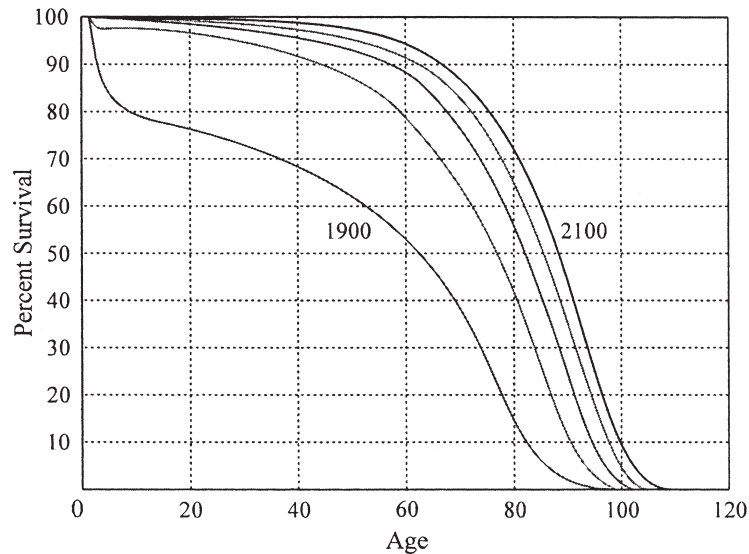


Figure 1.1. Survival functions for the social security population in the United States for selected calendar years (1900, 1950, 2000, 2050, 2100). (Source: F. Bell and M. Miller, *Life Tables for the United States Social Security Area, 1900–2100*, Social Security Actuarial Study No. 120, August 2005.)

this problem. A life insurance policy, by pooling many mortality ages, provides for a certain bequest whose value is independent of the age of death. Annuities, sometimes called *reverse life insurance*, also pool individual mortality risks, thereby ensuring a steady flow of consumption during life. As we shall show, access to these markets is extremely valuable to the welfare of individuals.

This stands in sharp contrast to the small private annuity markets in the United States and elsewhere. Several explanations have been offered for this *annuity puzzle*. One obvious explanation is that public social security (SS) systems, providing mandatory annuitized benefits, crowd out private markets. However, the SS system in the United States provides *replacement rates* (the ratio of retirement benefits to income prior to retirement) between 35 and 50 percent depending on income (higher rates for lower incomes). This should still leave a substantial demand for private annuities. Another potential explanation is annuity market imperfections. It was once argued that insurance firms offer annuities at higher than actuarially fair prices. This was largely refuted when annuitants' life tables, reflecting high survival probabilities, were used to calculate expected present values of benefits (Brown et al., 2001).

Davidoff, Brown, and Diamond (2005) suggest that a mismatch of the age profiles of benefits paid by annuities with individuals' consumption plans is a possible cause for partial annuitization. Bequest motives, shifting resources from annuities to life insurance or to other means for intergenerational transfers, have been offered as another explanation for the low demand for private annuities. It is difficult to rationalize, however, that this motive leads individuals to plan the drastic reductions in their standards of living implied by exclusive reliance on SS benefits (50 percent of the population in the United States has no pension beyond SS). Increasingly, *behavioral explanations*, based on *bounded rationality* (in particular, shortsightedness), are offered to explain the reluctance to purchase deferred annuities early in life.

While each of these explanations may have practical merit, we do not pursue them in this book for two major reasons. The first is methodological. Our objective is to analyze the demand for annuities by perfectly rational individuals and the functioning of competitive annuity and life insurance markets with only informational constraints. Analysis of such an idealized model economy is necessary in order to provide the background against which one can evaluate the impact of various practical constraints, behavioral or institutional, such as those outlined above. Second, many SS systems are currently being reformed to allow larger reliance on private savings accounts, which are expected to substantially increase the demand for private annuities. This lends urgency to the need to develop an understanding of the functioning of a competitive annuity market.

Among the arguments about annuity market imperfections that we do not incorporate into this analysis are those whose reason is not considered to be apparent. For example, annuity issuers seem to have no difficulty providing payout schemes that vary with age. If individuals are planning for rising or declining consumption with age, it can be expected that the market will provide annuities with a payout profile that matches these consumption plans.

On the other hand, we devote much attention in this book to the impact of *information* on the functioning of annuity markets, in particular, to the transmission of information to the issuers of annuities about changes in health and other factors that affect survival prospects.² As they age, individuals become better informed about future survival prospects, depending on factors such as health and occupation, and about the value of other needs and desirables, such as bequests. The

² Insurance firms that conduct medical tests on prospective clients sometimes find out information that is initially unknown to the subjects of the tests, but these subjects can soon be expected to become aware of the test results. It is interesting to speculate to what extent insurance firms have an interest in not fully informing clients, if so permitted.

uncertainties early in life create a demand by farsighted risk-averse individuals for insurance against different potential future outcomes. We analyze extensively to what extent a competitive annuity market can satisfy this demand by pooling individual risks.

When dealing with future longevity risks, market efficiency and its welfare implications depend critically on two considerations: first, whether there exist long-term annuities that yield returns as long as the holder is alive; second, whether information on each individual's survival probabilities does not remain *private information* but becomes known to annuity issuers. When this information is common knowledge, then the market for long-term annuities can provide efficient insurance against the arrival of information on survival probabilities (which we call *risk-class classification*). When the information on an individual's risk class is unknown to annuity sellers (and is not revealed by the individual's own actions), then annuities are sold at a common price to all individuals. The result is a *pooling equilibrium* that is characterized by *adverse selection*. That is, facing a common price, individuals with higher longevity purchase larger amounts of annuities, thereby driving the equilibrium price of annuities above prices based on the population's average longevity.

An issue not covered in this book is the relation between stock market risks and annuities. The issuers of private annuities, whether insurance firms, pension funds, or banks, have to choose a portfolio of assets that will best cover their annuity obligations. These assets, whether traded shares, bonds, or housing mortgages, fluctuate in value. Hence, optimum portfolio rules have to be formulated in order to reach a desirable balance between returns and risks.³ Typically, there is a link between these optimum portfolio rules and the flow of expected revenues and outlays on account of annuities. There is an extensive financial literature that deals with this and related issues. Analysis of this link and the functioning of a competitive annuity market is complex and seems largely separate from the issues discussed here.

In developed economies, the bulk of annuities are supplied to individuals by mandatory, government-run, SS systems that provide retirement benefits. The worldwide trends of population aging and lower birthrates created serious solvency problems for these systems, which are based on a pay-as-you-go principle. Much of the research in recent years has focused on the design of SS reforms aimed at closing these deficits. The issues involved are not only the economics of annuities but also much broader issues such as the effects on aggregate savings, labor incentives,

³ Unlike banks who face potential systemic risks (e.g., a “run on the bank”), annuity issuers can rely on fairly stable flows of outlays and revenues.

and distributional concerns (e.g., the effects of price or wage indexation on the level and distribution of benefits, the cross-subsidization implied by various defined benefits formulas, and the alleviation of poverty in old age), to name just a few. We think that it is best if these and other important policy issues are treated separately, confining our analysis to the more narrow but well-defined question of the functioning of a competitive annuity market. It is hoped that, this analysis can provide an underpinning for better SS reform designs.

1.1 BRIEF OUTLINE OF THE BOOK

The purpose of chapter 2 is to demonstrate in a simple deterministic setting the interaction among longevity, retirement age, and the savings required to maintain a steady lifetime consumption flow. Benchmark calculations show the need for significant savings toward retirement (beyond retirement benefits provided by social security) and the required response of consumption to changes in longevity or in retirement age.

Chapter 3 defines the technical concepts used throughout, in particular the definitions of survival functions and hazard rates. It also defines the precise meaning given to terms such as “more risky” or “higher longevity,” using the concepts of *stochastic dominance* developed in the theory of finance. In particular, it describes the possible effects of changes in longevity on survival probabilities at different ages.

Chapter 4 lays out the basic model from which the demand for annuities is derived. Individuals who face longevity risks and have access to a perfectly competitive annuity market choose a lifetime consumption path and an age of retirement. A *no-arbitrage condition* for a competitive equilibrium is shown to equate the rate of return on annuities to the *hazard rate* (plus the interest rate on non-annuitized assets).

Having derived the demand for annuities, optimum savings, and the age of retirement, chapter 5 performs comparative statics calculations showing how these choices are affected by changes in income and by changes in longevity. The effects of a positive *subjective time preference* by individuals and positive interest rates are also analyzed. The classic Yaari (1965) result is demonstrated: When faced with longevity risks, it is optimum for individuals to annuitize all savings.

The time preferences of individuals are shown to be important in determining the dependence of optimum retirement age on longevity (a much discussed question in the design of SS reforms).

Chapter 5 also analyzes the behavior of an individual who has no access to an annuity insurance market. In the face of uninsured longevity

risks, it is shown that varied attitudes toward risk preclude many of the predictions about individual responses derived in chapter 4.

In chapter 6 we analyze the implications of deviations in subjective beliefs about survival probabilities from observed market survival frequencies. Some empirical studies (e.g., Manski, 1993; Gan, Hurd, and McFadden, 2003) found only small such overall deviations, though these are more pronounced at older ages. The recent literature on quasi-hyperbolic discounting (Laibson, 1997), positing decreasing discount rates over time (age) (viewed as a problem of self-control), can be interpreted in our context as the adoption of excessively pessimistic survival prospects when deciding on optimum consumption and retirement age. This game-theoretic inconsistency conflict leads sophisticated individuals to use the purchase of annuities early in life as an instrument to steer later “selves” in a desirable direction.

Chapter 7 analyzes the potential distortions created by *moral hazard*. This occurs when individuals can invest resources (such as medical care and healthy nutrition) to raise survival probabilities. In a competitive annuity market, this leads to an inefficient resource allocation due to overinvestment in life extension. This is a well-known result in insurance markets: The cause of the inefficiency is that individuals disregard the effect of their actions on the equilibrium rate of return on annuities. Essentially, these distortions are due to *asymmetric information* because if the issuers of annuities could ascertain medical and other expenses that enhance longevity, prices could be conditioned on these expenses, thereby eliminating the distortions.

Chapter 8 tackles a particularly important issue and lays the ground for discussions in subsequent chapters: When faced with uncertainty about future survival functions early in life, to what extent can the annuity market provide the insurance desired by risk-averse individuals against alternative realizations. It is shown that this is not possible if there are available only *short-term* annuities. It is then demonstrated that when *long-term* annuities are available and the information about individuals’ risk classes is common knowledge, then the competitive annuity market equilibrium is *first best*. The discussion includes a derivation of the equilibrium rate of return on annuities purchased prior to the realization of heterogeneous risk classes.

Chapter 9 analyzes the characteristics of a pooling equilibrium, where individuals’ risk-class identities are unknown to annuity issuers. In particular, it is demonstrated that because of adverse selection, the pooling equilibrium price of annuities is higher than a price based on average longevity in the population.

Complementary to uncertainty about future survival probabilities, chapter 10 considers the effects of (uninsurable) uncertainty about future

incomes. How does this uncertainty affect the demand for annuities early in life and the ex post chosen age(s) of retirement?

Chapter 11 incorporates a bequest motive into individuals' lifetime plans, leading to the purchase of life insurance. Particular consideration is given to *period-certain* annuities (e.g., 10-year-certain), which provide not only a flow of income during life but also payment of a lump sum to a designated beneficiary if death occurs soon after annuitization. The optimum allocation of resources between annuities and a life insurance policy is derived for a population with heterogeneous life expectancies (a continuum of risk classes). Again, the focus is on timing and asymmetric information. When individuals make decisions prior to longevity realizations, a competitive annuity and life insurance market equilibrium attains the first-best allocation. In such an equilibrium, regular annuities and life insurance dominate the holding of period-certain annuities. In contrast, a pooling equilibrium opens up the possibility for a variety of types of annuities to be sold in the market. Specifically, it is shown that in a typical pooling equilibrium individuals with high longevities hold regular annuities and life insurance, those with low longevities hold period-certain annuities and life insurance, and those with intermediate longevities hold both types of annuities and life insurance.

Through their effects on individual behavior, one can trace the macroeconomic implications of annuity markets. Chapter 12 examines the transmission to aggregate savings of changes in individual savings due to changes in longevities. The analysis incorporates the induced long-term changes in the population's age density function due to the changes in longevities. Conditions that ensure that *steady-state aggregate savings* increase with longevities are derived. The results refute some recent empirical studies predicting that the increase in aggregate savings observed in many fast-growing economies will eventually be dissipated by the dissavings of the larger fraction of old individuals.

Prices of annuities implicit in social security systems invariably imply cross-subsidization between different risk classes (e.g., males/females). What are the guiding principles for a tax/subsidy policy that improves social welfare? Chapter 13 applies the theory of optimum commodity taxation in examining the utilitarian social welfare approach to the pricing of annuities under full information. It is shown that second-best optimum pricing depends on the joint distribution of survival probabilities and incomes in the population. Specifically, a low (high) correlation between survival probabilities and incomes leads, under utilitarianism, to subsidization (taxation) of individuals with high (low) survival probabilities.

The setting for the standard theory of optimum commodity taxation (Ramsey, 1927; Diamond and Mirrlees, 1971) is a competitive

equilibrium that attains efficient resource allocation. In contrast, annuity and other insurance markets with asymmetric information are characterized by non-Pareto-optimum pooling equilibria. Chapter 14 analyzes the conditions for optimum taxation in pooling equilibria. We focus on the general equilibrium effects of each tax in such equilibria and derive modified *Ramsey–Boiteux conditions*. These conditions involve the additional consideration of how individuals who purchase different amounts because of adverse selection react to a price increase resulting from a marginal tax rise.

It is well known that monopolists who sell a number of products may find it profitable to “bundle” the sale of some of these products, selling them jointly with fixed quantity weights. Bundling cannot occur in competitive equilibria when products are sold at marginal costs. This conclusion, however, has to be modified under asymmetric information. Chapter 15 demonstrates that pooling equilibria, characteristic of insurance markets, may typically have such bundling. These bundles are composed of goods whose unit costs are *negatively correlated* when sold to different risk classes, leading to a smaller variation of the bundled costs with individual attributes. This tends to reduce adverse selection and hence leads to lower prices. Annuities bundled with medical care or with life insurance are prime examples, and there is some evidence that in the United Kingdom some insurance firms link the sales of annuities and medical care.

Chapter 16 provides a general analysis of sequential annuity markets and proposes a new financial instrument. The analysis generalizes the discussion in chapter 8, allowing for uncertainty early in life about longevity and future income. It was previously shown that when uncertainty is confined to longevity, the early purchase of long-term annuities can provide perfect insurance, implying no annuity purchases later in life (and therefore no adverse selection). This is in stark contrast to evidence that most private annuities are purchased at advanced ages. It is demonstrated in this chapter that allowing for uninsurable income uncertainty leads to a sequential annuity market equilibrium with an active “residual” market for short-term annuities.

In the general case analyzed in chapter 16 the competitive annuity market equilibrium is second best. This raises the question whether there are other financial instruments that, if available, could be welfare-enhancing. We answer this question in the affirmative, proposing a new type of *refundable annuities*. These are annuities that can be refunded at a *predetermined price*. It is argued that a portfolio of such annuities with varying refund prices may enable individuals to better adjust their consumption paths to information about longevity and income that unfold over a lifetime. It is shown that these annuities are equivalent

to *annuity options*, available in the United Kingdom, which allow their holders to purchase annuities at a later date at a predetermined price. Such options may be attractive for behavioral reasons (procrastination, hyperbolic discounting) that are discussed at length in current economic literature.

1.2 SHORT HISTORY OF ANNUITY MARKETS

Annuities, in one form or another, have been around for more than two thousand years. In Roman times, speculators sold financial instruments called *annua*, or annual stipends. In return for a lump-sum payment, these contracts promised to pay the buyer a fixed yearly payment for life, or sometimes for a specified period or term. The Roman Domitius Ulpianus was one of the first annuity dealers and is credited with creating the first life expectancy table.

During the Middle Ages, lifetime annuities purchased with a single premium became a popular method of funding the nearly constant war that characterized that period. There are records of a form of annuity called a *tontine*, which was an annuity pool in which participants purchased a share and in return received a life annuity. As participants died off, each survivor received a larger payment until, finally, the last survivor received the remaining principal. Part annuity, part lottery, the tontine offered not only security but also a chance to win a handsome jackpot. For a delightful description of tontines see Jennings and Trout (1982).

During the eighteenth century, many European governments sold annuities that provided the security of a lifetime income guaranteed by the state. In England, Parliament enacted hundreds of laws providing for the sale of annuities to fund wars, to provide a stipend to the royal family, and to reward those loyal to it. Fans of Charles Dickens and Jane Austen will know that in the 1700s and 1800s annuities were all the rage in European high society.

The annuity market grew very slowly in the United States. Annuities were mainly purchased to provide income in situations where no other means of providing support were available. They were mostly purchased by lawyers or executors of estates who needed to provide income to a beneficiary as described in a last will and testament.

Until the Great Depression, annuities represented a miniscule share of the total insurance market (only 1.5 percent of life insurance premiums collected in the United States between 1866 and 1920). During and after the Great Depression, investors regarded annuities as relatively safe assets (see, “annuities” in *Wikipedia*).

Today, annuities represent an important line of business for U.S. insurance companies. While annuity payouts represented less than 10 percent of the combined payouts on life insurance and annuities before World War II, they have climbed to about 40 percent today (Brown et al., 2001, chap. 2). The growth of individual annuities has exceeded that of group annuities, reflecting the decline in defined benefits pension plans and the rapid expansion of variable annuities. The growth of variable annuities was accompanied by an expansion of investment options: Starting with diversified common stock portfolios, policies now offer a variety of specialized portfolios of bonds and securities. With the exception of the United Kingdom, European private annuity markets lag behind those in the United States, presumably reflecting a crowding out of private markets by generous public social security systems.

As described in a recent survey of annuity pricing (Cannon and Tonks, 2005), the 1956 Finance Act in the United Kingdom required that accumulated pension assets be converted to annuities upon retirement. This act expanded the annuity market in the United Kingdom (called the *voluntary purchase* market) because of favorable tax treatment and created a much larger *compulsory purchase* annuity market for individuals who opted for a defined contributions personal pension. The prices of annuities are higher in the voluntary market (valued at \$111 million) than in the compulsory market (valued at \$15 billion), reflecting the higher average age of annuitants in this market. The United Kingdom has annuities whose payout is indexed to consumer prices, as well as nonindexed annuities. Annuities are sold at discounted prices to those who can prove they have “severely impaired lives” (Finkelstein and Poterba, 2002, 2004).

A recent survey of annuity markets around the world (James and Song, 2001) reports increasing and thriving annuity markets in developed and developing countries and calculates the *money’s worth* of annuities (expected present value of payouts relative to the annuity’s price), indicating surprisingly low adverse selection.

Private annuities in the United States and the United Kingdom are sold by insurance companies. Supply conditions depend upon the availability of assets that provide satisfactory coverage for the obligations of these firms. In both countries, there is a high degree of concentration of annuity providers (Prudential, for example, accounts for 40 percent of new annuity sales in the United Kingdom), but according to Cannon and Tonks (2006), based on money’s worth calculations, there is no evidence of monopolistic profits. Life insurers and annuity providers can reduce their exposure to cohort longevity risk by buying longevity bonds (whose coupons fall in line with longevity) or by reinsurance. So far, the supply of longevity bonds is rather limited, but reinsurance is widely practiced.

1.3 REFERENCES TO ACTUARIAL FINANCE

An encyclopedic book of actuarial calculations with different mortality functions is Bowers et al. (1997), published by the Society of Actuaries. Duncan (1952) and Biggs (1969) provide formulas for variable annuities, that is, for annuities with stochastic returns. For an overview of life insurance formulas, see Baldwin (2002). Another useful book with rigorous mathematical derivations is Gerber (1990).

Milevsky's (2006) recent book contains many useful actuarial formulas for specific mortality functions (such as the Gompertz–Makeham function) that provide a good fit with the data. It also considers the implications of stochastic investment returns for annuity pricing, a topic not discussed in this book.