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**Steven J. Brams: Mathematics and Democracy**

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## Electing a Single Winner: Approval Voting in Practice

### 1.1. INTRODUCTION

It may come as a surprise to some that there is a science of elections, whose provenance can be traced back to the Marquis de Condorcet in eighteenth-century France, Charles Dodgson (Lewis Carroll) in nineteenth-century England, and Kenneth Arrow in twentieth-century America. Since Arrow published his seminal book, *Social Choice and Individual Values*, more than fifty years ago (Arrow, 1951, 1963)—for which in large part he received the Nobel Memorial Prize in Economics in 1972—there have been thousands of articles and hundreds of books published on everything from the mathematical properties of voting systems to empirical tests of the propensity of different systems to elect centrist candidates.<sup>1</sup>

The 2000 U.S. presidential election highlighted, among other things, the frailties of voting machines and the seeming arbitrariness of such venerable U.S. institutions as the Electoral College and the Supreme Court. Political commentary has focused on these aspects but given very little attention to alternative voting systems, about which the science of elections has much to say.

Several alternative systems for electing a single winner have been shown to be far superior to *plurality voting* (PV)—the most common voting system used in the United States as well as in many other places—in terms of a number of criteria. PV, which allows citizens to vote for only one candidate, suffers from a dismaying flaw. In any race with more than two candidates, PV may elect the candidate least acceptable to the majority of voters. This frequently happens in a three-way contest, when the majority splits its votes between two centrist candidates, enabling a candidate on the left or right to defeat both centrists. PV also

*Note:* This chapter is adapted from Brams and Fishburn (2005) with permission of Springer Science and Business Media; see also Brams (2002, 2006b) and Brams and Fishburn (1992a).

<sup>1</sup>For background on the formal theory underlying social choice and collective decision-making, see Austen-Smith and Banks (1999, 2005) and Arrow, Sen, and Suzumura (2002). Recent texts include Grillidi Cortono, Manzi, Pennisi, Ricca, and Simeone (1999), Hodge and Klima (2005), Gaertner (2006), and Nurmi (2006).

forces minor-party candidates into the role of spoilers, as was demonstrated in the 2000 presidential election with the candidacy of Ralph Nader. Nader received only 2.7 percent of the popular vote, but this percentage was decisive in an extremely close contest between the two major-party candidates.

Of the alternatives to PV, I recommend *approval voting* (AV), on both practical and theoretical grounds, in single-winner elections. Proposed independently by several analysts in the 1970s (Brams and Fishburn, 1983, 2007, ch. 1), AV is a voting procedure in which voters can vote for, or approve of, as many candidates as they wish in multicandidate elections—that is, elections with more than two candidates. Each approved candidate receives one vote, and the candidate with the most votes wins.

The candidate with the most votes need *not* win in an election. Merrill and Nagel (1987) make the useful distinction between a balloting method, which describes how voters can legally vote (e.g., for one candidate or for more than one), and a decision rule that determines a winner (e.g., the candidate with a plurality wins, or the candidate preferred to all others in pairwise comparisons wins). For convenience, I use the shorthand of AV to mean approval balloting with a plurality decision rule, but I consider other ways of aggregating approval votes later.

In the United States, the case for AV seems particularly strong in primary and nonpartisan elections, which often draw large fields of candidates. Here are some commonsensical arguments for AV that have been made:

1. *It gives voters more flexible options.* They can do exactly what they can under PV—vote for a single favorite—but if they have no strong preference for one candidate, they can express this fact by voting for all candidates they find acceptable. In addition, if a voter's most preferred candidate has little chance of winning, that voter can vote for both a first choice and a more viable candidate without worrying about wasting his or her vote on the less popular candidate.

2. *It helps elect the strongest candidate.* Today the candidate supported by the largest minority often wins, or at least makes the runoff if there is one. Under AV, by contrast, the candidate with the greatest overall support will generally win. In particular, *Condorcet winners*, who can defeat every other candidate in separate pairwise contests, almost always win under AV, whereas under PV they often lose because they split the vote with one or more other centrist candidates.

3. *It will reduce negative campaigning.* AV induces candidates to try to mirror the views of a majority of voters, not just cater to minorities whose voters could give them a slight edge in a crowded plurality contest. It is thus likely to cut down on negative campaigning, because candidates will have an incentive to try to broaden their appeals by reaching out for approval to voters who might have a different first choice. Lambasting such a choice would risk alienating this candidate's supporters and losing their approval.





















**TABLE 1.1**  
PV and AV Vote Totals in 1985 TIMS Election

Candidates	Official PV	Actual AV	Extrapolated AV
A	166	417	486
B	827	1,038	1,224
C	835	908	1,054
Total	1,828	2,363	2,764
No. of Voters	1,828	1,567	1,828

1. 70 provided rankings in the order ABC.
2. 66 provided rankings in the order ACB.
3. 3 provided no rankings but approved both A and B.
4. 27 made no distinction between B and C by rankings or approval.

In the B-versus-C comparison, it is reasonable to credit (1) and (3) to B (73 votes), (2) to C (66 votes), and (4) to neither candidate. When added to the PV totals, these credits give C (901 votes) exactly one more vote than B (900 votes). However, assuming the 27 voters in (4) split their votes between B and C in the pattern of the 139 voters (70 + 66 + 3) who ranked A first and also expressed a preference between B and C, B would pick up an additional vote (rounded to the nearest vote), resulting in a 914–914 tie.

This extrapolation indicates that there is not a single *Condorcet winner*, who can defeat all other candidates in pairwise contests. The usual reason one does not exist is that there is a *Condorcet paradox*, whereby majorities cycle. To illustrate, if there are three voters with preferences for candidates {X, Y, Z} of (1) XYZ, (2) YZX, and (3) ZXY, two out of three voters (voters 1 and 3) prefer X to Y, two out of three (voters 1 and 2) prefer Y to Z, and two out of three (voters 2 and 3) prefer Z to Y, showing that there is no candidate that is preferred to all others. Instead, each candidate can be defeated by one other, so majorities cycle.

In this election, however, it is a projected tie that precludes one candidate from defeating the others in pairwise contests. That there is no cycle, and that A in fact would lose to both B and C, is shown by ranking data in Fishburn and Little (1988).

While surprising, the lack of a single Condorcet winner should not obscure the fact that 170 more voters approved of B rather than C in the extrapolated AV returns, albeit C won the PV contest by eight votes. The reason for this discrepancy between the AV and PV results is that whereas C has slightly more *stalwart* supporters (i.e., those who vote only for one candidate) than B, supporters of the third candidate, A, somewhat more approved of B than C (44 percent to 40 percent). But B's big boost comes from the fact that substantially













