Continuity (1974-75)

Calculus thrives on continuity. At its core is the assumption that things change smoothly, that everything is only infinitesimally different from what it was a moment before. Like a movie, calculus reimagines reality as a series of snapshots, and then recombines them, instant by instant, frame by frame, the succession of imperceptible changes creating an illusion of seamless flow.

This way of understanding change has proven to be powerful beyond words—perhaps the greatest idea that humanity has ever had. Calculus enables us to travel to the moon, communicate at the speed of light, build bridges across miles of river, halt the spread of epidemics. Without calculus, modern life would be impossible.

Yet in another way, calculus is fundamentally naive, almost childish in its optimism. Experience teaches us that change can be sudden, discontinuous, and wrenching. Calculus draws its power by refusing to see that. It insists on a world without accidents, where one thing leads logically to another. Give me the initial conditions and the law of motion, and with calculus I can predict the future—or better yet, reconstruct the past.

I wish I could do that now. Unfortunately, my correspondence with Mr. Joffray is riddled with discontinuities. Letters were lost or discarded. Those that remain are fragmentary and emotionally muted, and sometimes prone to half-truths, silver linings, and deliberate omissions.

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It was my sophomore year, spring term 1974. I was taking precalculus with a different teacher, Mr. Johnson, an MIT graduate, a tall, stern man, about 35 or 40, very fair but not given to smiling.

Some of my friends were in Mr. Joffray's section of the same class. I'd never talked to him and did not know much about him. There were rumors he'd been the national champion in whitewater kayaking. He was physically impressive—anyone could see that—big chest, muscular arms and legs, close-cropped hair. He looked like a stronger version of Lee Marvin, whom I'd seen in lots of war movies.

When we were learning about the rigorous definition of continuity—a very fundamental, difficult concept in calculus—Mr. Johnson told us something I'd never heard a teacher say before. It was ominous. He said he was going to present some ideas we wouldn't understand, but we had to go through them anyway. He was referring to the $\varepsilon - \delta$ definition of continuity:

> A function f is continuous at a point x if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $|x - y| < \delta$, then $|f(x) - f(y)| < \varepsilon$.

He said we'd need to see this four or five times in our education and that we'd understand it a little better each time, but there has to be a first time, so let's start.

It *was* difficult. Kids in our class were having a lot of trouble following the logic of these $\varepsilon - \delta$ arguments.

And then word filtered back to us that Mr. Joffray was doing things very differently in his class. He wasn't even trying to explain ε and δ. He'd defined a continuous function as one whose graph you could draw without lifting your pencil from the paper.

That told me a lot. Of course that was the intuition of what "continuity" must mean. But to leave it at that struck me, with my sophomore mentality, as taking the easy way out. It was soft. It was avoiding the issue. And so I began with a suspicion about Mr. Joffray, that he wasn't really hard core. I was glad I was in Mr. Johnson's class.

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The following year, Mr. Joffray was my teacher. Now I was able to take the measure of the man up close. Again I was impressed by his sheer physicality. His hands were the biggest I'd ever shaken—my hand was engulfed by his. And when he'd write on the blackboard, the chalk would pulverize with each stroke. Shards and splinters would fly off. Smithereens and dust all over him by the end of class.

He seemed to be very outdoorsy (something that had never appealed to me—I played tennis and basketball but never liked the woods—too many bugs—or canoeing or backpacking or any of that). A yearbook photo captured Mr. Joffray in his preferred habitat: high in a tree, inspecting a birdhouse he'd built. He was also faculty advisor to a group called the Darwin Club. I have no idea what they did, but it was outdoorsy nature stuff.

Anyway, what was his class like? Fun and pleasant. Low key. He was a happy man, friendly, always enthusiastic, though about strange things. He'd stride in and start talking about a goat that is tethered to a tree by a long rope. The stubborn animal pulls the rope taut and tries to walk away from the tree but ends up wrapping itself around the tree in

a tighter and tighter spiral. And then he'd ask us to find an equation for the goat's spiraling path.

I didn't know what to make of it. This wasn't suave, serious Mr. Johnson with his impressive MIT background. I felt like I was being taught by some sort of person I couldn't recognize.

But he was certainly jovial, so it was not an issue.

The math itself was interesting and came easily. I could learn it all from the book. His class did not add much, except for the weird nature problems.

On those signature occasions when he'd interrupt the class to tell us about his best former students, invariably he'd be in the middle of a calculation and then lapse into a reverie, with a faraway look in his eye and a smile breaking out. Then, in a hushed tone, he'd tell us about the time that Jamie Williams wrote down a formula for the nth term of the Fibonacci sequence.

Actually, that achievement did deserve reverence. As you may remember, the Fibonacci sequence is 0, 1, 1, 2, 3, 5, 8, 13, 21, 34 . . . . The sequence starts with the numbers 0 and 1, and after that each number is the sum of the two before it. The problem is, if $F_0 = 0$ and $F_1 = 1$, find a formula for F_n, the nth term. You'd want such a formula if you were interested in F_{100} or F_{1000}; you wouldn't want to have to add up a hundred or a thousand intermediate terms to get the answer. So is there a short-cut formula that expresses F_n directly in terms of n? The answer is amazing:

$$F_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

How did Jamie Williams ever come up with that?

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With the passage of time I see now that I was like the goat tethered to the tree—and Mr. Joffray was the tree. I pulled taut on the rope and tried to get away from him, but only ended up wrapping myself closer and closer to him, all these years.

How did that happen? It wasn't because he taught me so much in the usual sense. No, his approach was so humble and unconventional, it confused me. It made me feel superior to him. I'm embarrassed to admit that, but it's true.

Here's what he'd do.

He'd suggest a problem, very gently, not at all insistent, and then he'd step aside. Often Ben Fine and I would compete to see who could solve it. Or if we could both solve it, who could solve it *better*.

Ben was a brilliant kid, a year younger than me, owlish and small, with sophisticated interests. (I often felt cloddish next to him.) And his math style was that of a languorous genius. He'd ponder the question without writing anything—he was a *philosophe*. Then, with a lightning stroke, he'd write a few lines of equations, render a poor sketch or two, and bam! He'd solved it.

Whereas I was a grinder. Not nearly as clever as Ben (and in retrospect, I see that he had much more talent for math than I did). My style was brutal. I'd look for a method to crack the problem. If it was ugly or laborious, with hours of algebra ahead, I didn't mind because the right answer was guaranteed to emerge at the end of the honest toil. In fact, I loved that aspect of math. It had justice built into it. If you started right and worked hard and did everything correctly, it might be a slog but you were assured by logic to win in the end. The solution would be your reward.

It gave me great pleasure to see the algebraic smoke clear.

And there was another reward. Mr. Joffray was an incredible cheerleader. He'd sometimes watch me and Ben, the tortoise and the hare, with a look of such admiration, almost awe, and happiness too.

At the end of my junior year, the school held its annual award ceremony. My name was called when they announced the Rensselaer prize for the top junior in math and science, and if I remember right, Mr. Joffray made a speech about me. He portrayed me as a mountain climber, ascending the mathematical peaks and then returning with tales of what I'd seen.

He made me sound generous, and heroic.