CHAPTER 1

Goods and Prices

1.1 Introduction

The aim of this chapter is to develop the main aspects of the economic environment in which economic agents operate. There are two categories of economic agents, consumers and firms. Consumers buy and sell goods with the ultimate goal of consuming those goods. Firms buy goods that they transform into other goods that they later sell. An economy is made up of these consumers and firms. After having developed models of the consumers and firms, we will combine them into a model of an economy with private ownership of production. Before developing these models, it is necessary to be somewhat more explicit about the economic goods and their prices that define the economic environment.

1.2 Goods

The characteristics of an economic good

Economic goods are defined by their "physical" characteristics or properties. These physical characteristics, which have to be taken in a very broad sense, may include all forms of services. But economic goods often feature other aspects than their physical properties. For example, the location and the date of delivery of an economic good are sufficiently important to be specified for each economic good. The conditions under which delivery takes place is also specified in many general equilibrium models, especially those that involve uncertainty. In these models, the delivery may depend on the realization of some state of nature. Such goods are known as contingent goods. To sum up, economic goods are defined by their characteristics, which may include many more things than just the physical properties of the goods.

The measurability requirement

Goods that have a price like all the goods traded in markets have to be measurable. Measurability is a concept that comes from physics. For an economic good to be measurable, it is necessary to have a meaningful

definition of the equality and the sum of two quantities of that good. In that regard, not all economic goods are measurable. For example, some public goods like national defense are typically not measurable because it is almost impossible to define the equality of two levels of national defense. Needless to say, the sum of two levels of national defense is even harder to conceive.

The goods considered in general equilibrium models are measurable. They are also divisible. Units are defined once and for all for each good in the economy.

THE COMMODITY SPACE

The number of goods is finite and denoted by ℓ, a number greater than or equal to 2. The commodity bundle $x = (x^1, \ldots, x^\ell)$ consists of x^1 units of the first good, x^2 units of the second good, up to x^ℓ units of the ℓ-th good. The *commodity space* is the Euclidean space $\mathbb{R}^\ell$. Quantities of goods in a commodity bundle like x can be positive, negative, or equal to zero.

1.3 Prices

We associate with every commodity j a price p_j. Prices are strictly positive. The price p_j of commodity j is actually the price of one *unit* of commodity j.

THE PRICE VECTOR

The price vector $p = (p_1, \ldots, p_j, \ldots, p_\ell) \in \mathbb{R}^\ell$ has for coordinates the prices $p_j > 0$ of the various goods.

VALUE OF A COMMODITY BUNDLE FOR A GIVEN PRICE SYSTEM

Let $x = (x^1, \ldots, x^\ell) \in \mathbb{R}^\ell$ be a commodity bundle and $p = (p_1, \ldots, p_\ell) \in \mathbb{R}^\ell_{++}$ some arbitrary price vector. The *value* of the commodity bundle $x \in \mathbb{R}^\ell$ given the price vector $p \in \mathbb{R}^\ell_{++}$ is equal to the inner product

$$w = p \cdot x = p_1 x^1 + \cdots + p_\ell x^\ell.$$

1.4 Relative Prices

Given the price vector $p = (p_1, \ldots, p_\ell) \in \mathbb{R}^\ell_{++}$, the price of good h relative to the price of good k is defined by the ratio p_h/p_k.

Relative prices depend only on the direction defined by the price vector $p = (p_1, \ldots, p_h, \ldots, p_k, \ldots, p_\ell) \in \mathbb{R}^\ell_{++}$, not on its length. An important

assumption that will be made in future chapters will be that consumption and production decisions depend only on relative prices or, in other words, on the direction defined by the price vector. It has therefore become customary to use some kind of normalization for the price vectors $p \in \mathbb{R}^{\ell}_{++}$.

1.5 Price Normalization

There are quite a few ways to normalize the price vector $p \in \mathbb{R}^{\ell}_{++}$. Mathematicians tend to choose some norm on the vector space $\mathbb{R}^{\ell}$ and to set that norm equal to one.

The Euclidean normalization

One example of such a normalization is to set the Euclidean norm $\|p\| = \left((p_1)^2 + \cdots + (p_\ell)^2\right)^{1/2}$ to one. Perfectly satisfactory from a mathematical perspective, this normalization has little economic appeal. We will not use this normalization in the book.

The simplex normalization

Another normalization that is particularly handy when dealing with the behavior of consumers and firms when some prices tend to zero is the *simplex normalization*.

Definition 1.1. *The price vector $p = (p_1, \ldots, p_\ell) \in \mathbb{R}^{\ell}_{++}$ is simplex normalized if $\sum_k p_k = 1$. We denote by S_Σ the set $\{p \in X \mid \sum_k p_k = 1\}$ of simplex normalized prices.*

Note that all prices are strictly positive for $p \in S_\Sigma$. The closed price simplex $\overline{S_\Sigma}$ consists of the price vector $p = (p_1, \ldots, p_\ell)$ where some coordinates can be equal to zero. The boundary $\partial S_\Sigma = \overline{S_\Sigma} \setminus S_\Sigma$ consists of the price vectors $p \in \overline{S_\Sigma}$ that have at least one coordinate equal to zero.

The numeraire normalization

Another price normalization has the favor of many economists. It consists in giving to some good the role played by money for expressing the prices of the other goods in quantities of that good. Such a good is known as the *numeraire*.

Numeraire is not money because money, paper money (also known as fiat money) is not an economic good in our sense. Paper money cannot be consumed physically like an orange or a banana. Paper money is not an argument of the utility functions considered by the classical theory of the consumer, a theory that we address in chapter 2. Similarly, paper money

is not a direct input of the production process in the classical theory of the firm considered in chapter 5. If the numeraire cannot be confused with paper money, gold has many attributes of a numeraire. In the times of the Gold Exchange Standard, prices were expressed in quantities of gold, an argument of consumers' utility functions and producers' production processes.

In practice, having a numeraire amounts to setting the price of the numeraire commodity to one. In this book, we choose the ℓ-th commodity to be the numeraire: $p_\ell = 1$.

Definition 1.2. *The price vector $p = (p_1, \ldots, p_\ell) \in \mathbb{R}^\ell_{++}$ is numeraire normalized if $p_\ell = 1$. We denote by S the set $\{p \in \mathbb{R}^\ell_{++} \mid p_\ell = 1\}$ of numeraire normalized prices.*

Unless the contrary is specified, all price vectors from now on are numeraire normalized. The main problem with the numeraire price normalization is that it treats goods asymmetrically, which may not be very satisfactory from a mathematical perspective. This defect is more than compensated by the relative simplicity it gives to the computation of derivatives of demand and supply functions with respect to prices, operations that we will do extensively in later chapters.

1.6 Notes and Comments

This introductory chapter is very similar to the first chapter of Debreu's *Theory of Value* [23].

The necessity of measurability for the goods that claim to have market prices is often neglected. Most public goods and environmental goods are not measurable. This has not prevented Samuelson and his followers to treat public goods as if they were measurable [7, 57].

Some mathematicians use the Euclidean normalization of the price vector because the price vector then belongs to the sphere of unit radius. Then, the aggregate excess demand of an exchange economy can be viewed as defining a vector field on the sphere, which enables the application of several powerful theorems of algebraic topology to the study of this vector field [26].