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**PAPER 2**



On the Motion of Small Particles  
Suspended in Liquids at Rest  
Required by the  
Molecular-Kinetic Theory of Heat

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IN THIS PAPER it will be shown that, according to the molecular-kinetic theory of heat, bodies of a microscopically visible size suspended in liquids must, as a result of thermal molecular motions, perform motions of such magnitude that they can be easily observed with a microscope. It is possible that the motions to be discussed here are identical with so-called Brownian molecular motion; however, the data available to me on the latter are so imprecise that I could not form a judgment on the question.

If the motion to be discussed here can actually be observed, together with the laws it is expected to obey, then classical thermodynamics can no longer be viewed as applying to regions that can be distinguished even with a microscope, and an exact determination of actual atomic sizes becomes possible. On the other hand, if the prediction of this motion were to be proved wrong, this fact would pro-

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vide a far-reaching argument against the molecular-kinetic conception of heat.

### 1. ON THE OSMOTIC PRESSURE TO BE ASCRIBED TO SUSPENDED PARTICLES

Let  $z$  gram-molecules of a non-electrolyte be dissolved in a part  $V^*$  of the total volume  $V$  of a liquid. If the volume  $V^*$  is separated from the pure solvent by a wall that is permeable to the solvent but not to the solute, then this wall is subjected to a so-called osmotic pressure, which for sufficiently large values of  $V^*/z$  satisfies the equation

$$pV^* = RTz.$$

But if instead of the solute, the partial volume  $V^*$  of the liquid contains small suspended bodies that also cannot pass through the solvent-permeable wall, then, according to the classical theory of thermodynamics, we should not expect—at least if we neglect the force of gravity, which does not interest us here—any pressure to be exerted on the wall; for according to the usual interpretation, the “free energy” of the system does not seem to depend on the position of the wall and of the suspended bodies, but only on the total mass and properties of the suspended substance, the liquid, and the wall, as well as on the pressure and temperature. To be sure, the energy and entropy of the interfaces (capillary forces) should also be considered when calculating the free energy; but we can disregard them here because changes in the position of the wall and suspended bodies will not cause changes in the size and state of the contact surfaces.

But a different interpretation arises from the standpoint of the molecular-kinetic theory of heat. According to this

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theory, a dissolved molecule differs from a suspended body *only* in size, and it is difficult to see why suspended bodies should not produce the same osmotic pressure as an equal number of dissolved molecules. We have to assume that the suspended bodies perform an irregular, albeit very slow, motion in the liquid due to the liquid's molecular motion; if prevented by the wall from leaving the volume  $V^*$ , they will exert pressure upon the wall just like molecules in solution. Thus, if  $n$  suspended bodies are present in the volume  $V^*$ , i.e.,  $n/V^* = \nu$  in a unit volume, and if neighboring bodies are sufficiently far separated from each other, there will be a corresponding osmotic pressure  $p$  of magnitude

$$p = \frac{RT}{V^*} \frac{n}{N} = \frac{RT}{N} \cdot \nu,$$

where  $N$  denotes the number of actual molecules per gram-molecule. It will be shown in the next section that the molecular-kinetic theory of heat does indeed lead to this broader interpretation of osmotic pressure.

## 2. OSMOTIC PRESSURE FROM THE STANDPOINT OF THE MOLECULAR-KINETIC THEORY OF HEAT<sup>1</sup>

If  $p_1 p_2 \dots p_l$  are state variables of a physical system that completely determine the system's instantaneous state (e.g., the coordinates and velocity components of all atoms of the

<sup>1</sup>In this section it is assumed that the reader is familiar with the author's papers on the foundations of thermodynamics (cf. *Ann. d. Phys.* 9 [1902]: 417 and 11 [1903]: 170). Knowledge of these papers and of this section of the present paper is not essential for an understanding of the results in the present paper.