A Short Primer on Notation in *Engineering Dynamics: A Comprehensive Introduction*

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In this short note, we discuss the main elements of the notational tools used in *Engineering Dynamics: A Comprehensive Introduction* by Kasdin & Paley. Our objective is to show how the various notational elements serve our pedagogical purposes and likewise how they provide unambiguous methods for solving real-world, complex problems. We also provide references for our particular notational choices as well as alternatives in the literature.

First and foremost, our explicit and rigorous notation is a pedagogical tool. There are several key physical ideas that form the foundation of Newtonian mechanics, and we find that explicitly highlighting them through notation forces the student to continually call upon these underlying concepts. For example, understanding the importance of inertial frames and the usefulness of different reference frames for describing relative motion, the fact that angular momentum is a function of a particular reference point, that kinetic energy is referred to an inertial frame and origin, and that the direction cosine matrix describes the relative orientation of two frames all are represented by explicit notational tools. Modern cognitive research into learning has shown that *spaced repetition*—that is, re-exposing students to the same information and ideas spaced throughout the semester—has a profound impact on retention and comprehension. Our notational approach continually reinforces these important ideas and forces the student to confront prior knowledge as each new concept is introduced.

It is certainly true that, in some problems, the notation could be simplified without losing clarity; for example, problems that use only a single particle in a single inertial frame do not need the subscript identifiers or the inertial frame markings. In many books, the extra notational tools are only introduced when there is some ambiguity (for instance, the author might only indicate the frame in a derivative when there is a need to distinguish two derivatives, such as in the transport equation). We instead opted for a consistent notation throughout the book in every occurrence of a particular operation or variable. In our view, it is better for students to develop the habits of consistency first and later adopt these simplifications and short cuts only after having practiced with the more general and specific notation, thus understanding precisely what simplifications are being made.

Second, our educational objective is not only to provide a basic foundation in Newtonian mechanics, but also to provide effective real world approaches that can be used by the practicing engineer. Many problems in mechanical and aerospace engineering involve multiple frames of reference, some inertial and some not, with multiple particles and/or rigid bodies. Notational tools are necessary for describing the relative motion and the appropriate frames. For instance, many introductory texts might use the subscript *rel* to indicate relative motion to a different, usually non-inertial frame. This is effective at introducing the concept of relative motion, but in practice more precise notational tools are needed when many rotating frames can be present. For example, a complicated robot arm problem can involve many different interconnected frames.

In the remainder of this primer we review the key notational elements and refer to other sources from which we benefited.

i) We adopted the practice of noting both the starting and ending point of position vectors using a subscript with each point separated by a slash, $\mathbf{r}_{P/O}$. The provides an unambiguous and clear notation for differentiating among multiple vectors and frames with different origins. (Traditional texts often use different letters, such as lower case $\mathbf{r}$ and upper case $\mathbf{R}$, or they might use primes for the different vectors.) We adopted our notation from Kane [6]. It is also used in Tongue & Sheppard [12], Rao [10], and Ginsberg [3] among others. We maintain the subscript on the velocity and acceleration, ${}^{\mathcal{I}}\mathbf{v}_{P/O}$ and ${}^{\mathcal{I}}\mathbf{a}_{P/O}$, to visually note that this is just a shorthand for the derivative of the original vector. This is followed in Kane and Rao, for instance. Tongue & Sheppard and Hibbeler [4] use only the single letter when the reference point is O but keep both letters when it is any other point (such as the origin of a second frame).

ii) We always explicitly note the reference frame within which a vector derivative is taken. We do so by writing the frame letter in the upper left of the derivative or the upper left of the shorthand variable (velocity, acceleration, or angular momentum). We adopted this notation from that used by Kane; it also appears Rao, Josephs & Huston [5] and others. Many introductory texts assume that a derivative is taken with respect to some inertial frame if there is no explicit notation for the frame. We find it pedagogically more consistent to always indicate the frame, particularly since there are many instances with multiple frames.

There are also a variety of notations in use in the literature for indicating the frame derivative. These include letters above the variables instead of the traditional dot (e.g., Cannon [2]), parentheses (e.g., Meriam & Kraige [8] and Boresi [1]) or braces (e.g., Wie [13]) around the derivative with a subscript indicating the frame, or a vertical line next to the derivative with a subscript indicating the frame (e.g., Tongue & Sheppard).

iii) Almost all texts and working engineers use some indication in the representation of the direction cosine or transformation matrix for the frames between which it is representing the relative orientation. We adopted Kane's [7] approach of using superscripts on the right and left. Other common notations use subscripts or superscripts (e.g., Wie) with a slash between frame labels, a sub- and superscript on the right for each frame (e.g. Stengel [11]), or a sub- and superscript with brackets (e.g., Rao). Some books use no frame indication (.e.g, O'Reilly [9] and Ginsberg) when treating only a pair frames or rigid bodies. As in our other notational choices, we opt for always indicating frames, even when there is no ambiguity, to maintain consistency and enhance pedagogy. We also find it helps student recall which direction of rotation is being indicated, reducing errors when the transpose is called for.

iv) In introducing the angular velocity, most texts do not indicate the frames for which the rotation is being defined. We adopt the notation using superscripts on the right and left to indicate that the angular velocity represents the rate of rotation of one frame relative to another. When there is no ambiguity, some texts will often drop the superscripts; we keep them throughout for consistency and reinforcement. Our angular velocity notation is also consistent with the notation used on the direction cosine matrices.

v) Our use of matrices, and the distinction between them and vectors, also follows the practice in Kane. Because of our emphasis on the distinction between coordinates and unit vectors, and the importance of distinguishing between frames of reference, we adopted the subscript notation on matrices to indicate the frame associated with the component magnitudes. Our use of the transformation array for unit vectors and matrices for component magnitudes is adopted from Kane and also used by Tongue & Sheppard and Rao. Part of our motivation here is to introduce students early in their career to the distinction between coordinates, vectors, and frames so that when they later study analytical mechanics the concept of generalized coordinates will be a natural extension. (We introduce this idea in Chapter 13.)

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