

A Counterexample to Fernandez-Martinez's Conjecture ¹

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Abstract Motivated by robustness problem in stable transfer functions under positive polynomial perturbations, Fernandez and Martinez presented a conjecture on stability of composite polynomials. In this note, we present a simple counterexample to Fernandez-Martinez's Conjecture.

Keywords Stability, Robustness, Stable Polynomial, Fernandez-Martinez's Conjecture, Counterexample

Motivated by robustness problem in stable transfer functions under positive polynomial perturbations [1, 2], Fernandez and Martinez presented the following conjecture [3]:

Conjecture [3] Given any stable polynomial:

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$$

and given any polynomial $q(s)$ with non-negative coefficients, then the polynomial:

$$q(a_n) s^n + q(a_{n-1}) s^{n-1} + \cdots + q(a_1) s + q(a_0)$$

is stable.

In this note, we present a counterexample to the conjecture above.

Example 1 Consider the following polynomial:

$$s^3 + 2s^2 + 3s + 5$$

Obviously, this polynomial is stable since $2 \times 3 - 5 = 1 > 0$.

Let $q(s) = s + 2$ with non-negative coefficients, then the composite polynomial:

$$q(1)s^3 + q(2)s^2 + q(3)s + q(5) = 3s^3 + 4s^2 + 5s + 7$$

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is not stable since $q(2)q(3) - q(1)q(5) = -1 < 0$.

Remark It should be noted that the conjecture above is true only for the special case that the degree of the given stable polynomial (with all positive coefficients) is less than or equal to 2. If the given stable polynomial is allowed to have (all) negative coefficients, then it is much easier to construct a counterexample of order 1 or order 2 to the conjecture.

If the given stable polynomial is allowed to have (all) negative coefficients, Dr. Roland Hildebrand provided the following counterexample to Fernandez-Martinez's Conjecture.

Example 2 Consider the following polynomial:

$$-s^2 - 3s - 1$$

Obviously, this polynomial is stable.

Let $q(s) = s + 2$ with non-negative coefficients, then the composite polynomial:

$$q(-1)s^2 + q(-3)s + q(-1) = s^2 - s + 1$$

is not stable.

References

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