Chapter 4

Understanding Forward Rates for Foreign Exchange

Overview

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Outline

How Forward Rates are Quoted

◇ **Quotes:** Two conventions: Outright (F) vs. swap rate ($F - S$)—see *e.g.* Globe and Mail

	\$1 U.S. in Cdn \$	Cdn \$1 in U.S.	swap rates (in cents US or Cdn)	
spot	1.3211	0.7569	—	—
1 month	1.3218	0.7565	+0.07	–0.04
2 months	1.3224	0.7562	+0.13	–0.07
3 months	1.3229	0.7559	+0.18	–0.10
6 months	1.3246	0.7549	+0.35	–0.20
12 months	1.3266	0.7538	+0.55	–0.31
…	…	…	…	…
5 years	1.3579	0.7364	+3.68	–2.05
…	…	…	…	…
10 years	1.5446	0.6875	+13.36	–6.94
	"at a premium", or "above par"	"at a discount" or "below par"	(a premium)	(a discount)

◇ **Which is used where?** Traders traditionally quoted swap rates. Newspapers have stopped the practice.

Sometimes one uses "p" (= premium), "d" (= discount) instead of "+", "–", or one entirely omits the indication.

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How we denote risk-free returns

- **Effective return** = simple percentage difference between start and end value, as % of start value

Examples

$T-t$	V_t	V_T	$r_{t,T}$
3 month	100	102	2%
5 years	1000	1500	50%

- **Interest rate** = annualized (“*p.a.*”) version of r. Needs to de-annualized into an effective return.

Examples: 3 months at 6% *p.a.* means ...

convention	formula	$r_{t,T}$
simple interest	$(1+3/12\times 0.06)-1=$	0.01500
comp., annual	$(1+0.06)^{3/12}-1=$	0.01467
comp., monthly	$(1+1/12\times 0.06)^{3}-1=$	0.01507
comp., daily	$(1+1/360\times 0.06)^{90}-1=$	0.01511
cont. comp	$e^{0.06\times 3/12}-1=$	0.01511
banker's discount	$(1-3/12\times 0.06)^{-1}-1=$	0.01523

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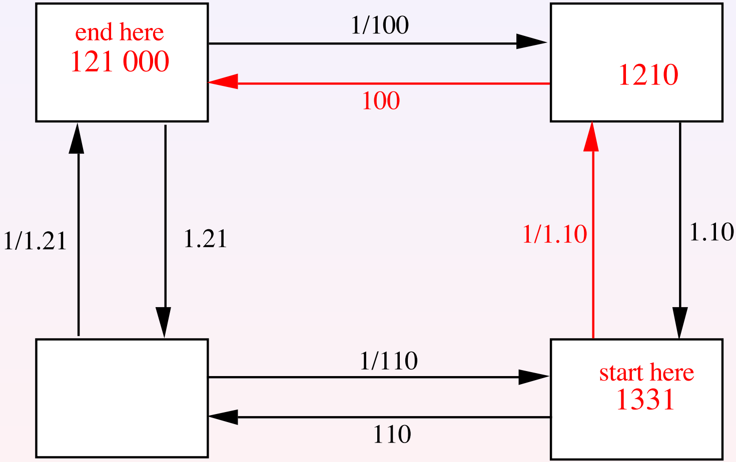
Outline

The ins & outs of FX & MM Transactions

- Assume perfect markets, in this chapter. (See next chapter for imperfections.)
- Time-subscripted *HC*, *FC* refer to amounts of a currency; t = now, T = future.
- 8 possible transactions in spot/forward/money markets:

	Output amount	= Input amount	× multiplic. factor
sell FX spot			
buy FX spot			
sell FX forward			
buy FX forward			
HC term deposit			
HC term loan			
HC term deposit			
HC term loan			

Getting our act together into a diagram



Outline

Two Key Results—for Perfect Markets

If Covered Interest Parity—CIP— holds, *ie*

$$F_{t,T} = S_t \frac{1 + r_{t,T}}{1 + r^*_{t,T}} \quad - \quad \text{(IRP or CIP)}$$

Then

- **[No-Arb:]** there are no arbitrage opportunities
  - (With spreads, this will be weakened into an inequality.)
- **[Shopping Around:]** shopping-around calculations are pointless
  - (Thus, shopping-around becomes relevant only because of market imperfections)

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The Row to Hoe

- Previous data—$S_t = 100$, $r_{t,T} = 0.21$, $r^*_{t,T} = 0.10$. From CIP, we should have
$$F_{t,T} = 100\frac{1.21}{1.10} = 110$$
- (next 3 slides;) if not, there is an arb opp
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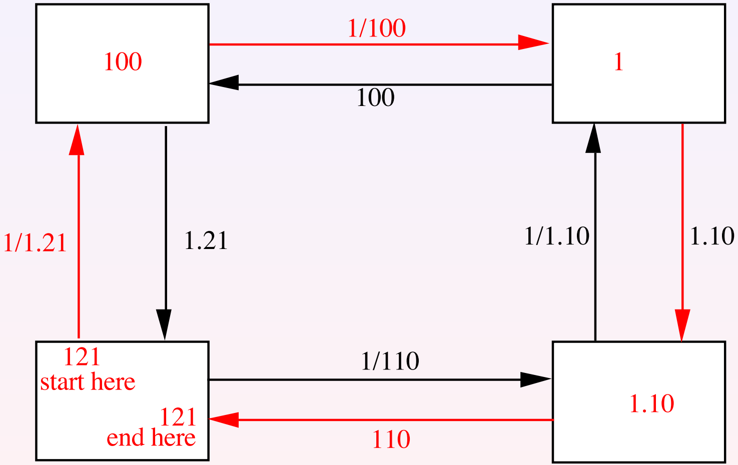
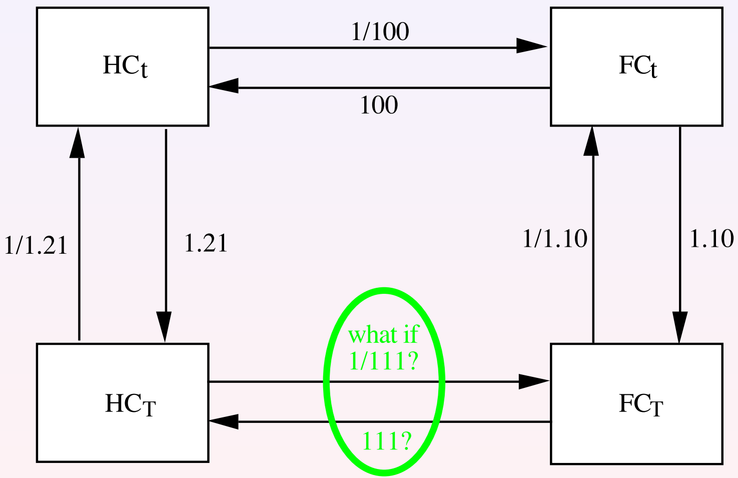
No-arb 1: with CIP, a roundtrip breaks even

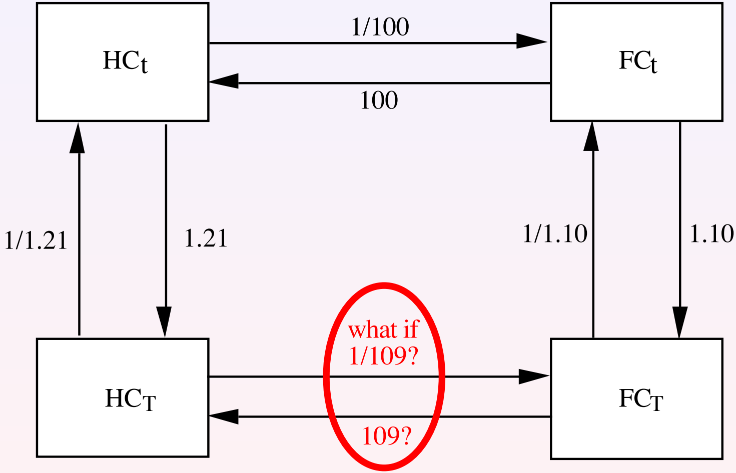


Note: the first three steps form a synthetic forward purchase.

No-arb 2a: without CIP, there's an arb opp



No-arb 2b: without CIP, there's an arb opp



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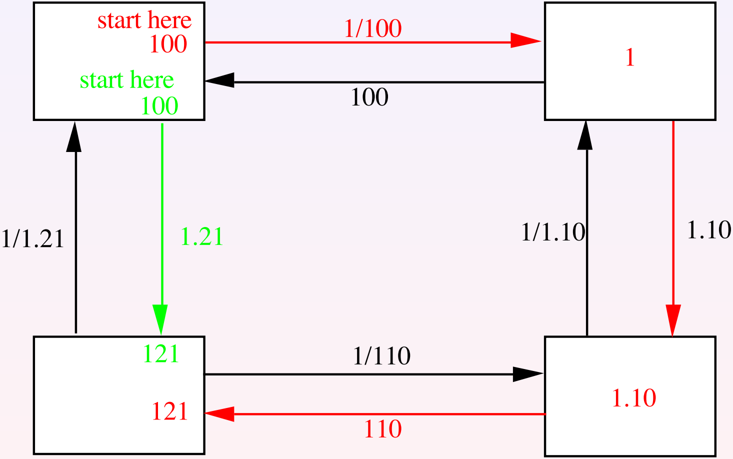

Deposits: HC v swapped FC. (This is why CIP is called CIP.)

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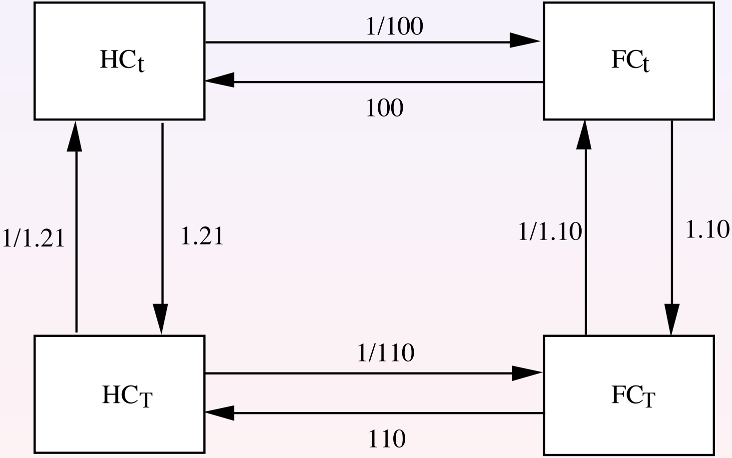


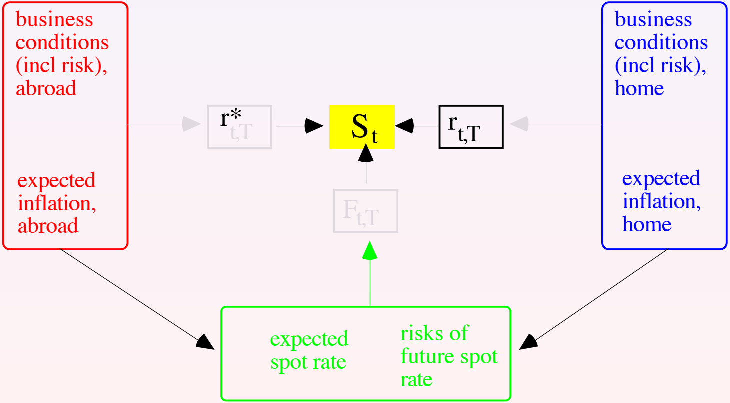
Try out all 4×3 trips!

Causality?

CIP in itself has no causality, but you can append stories:

- interest rates: Fisher's story
- forward rate: (risk-adjusted) expected future spot rate

... and end with a theory on the spot rate:



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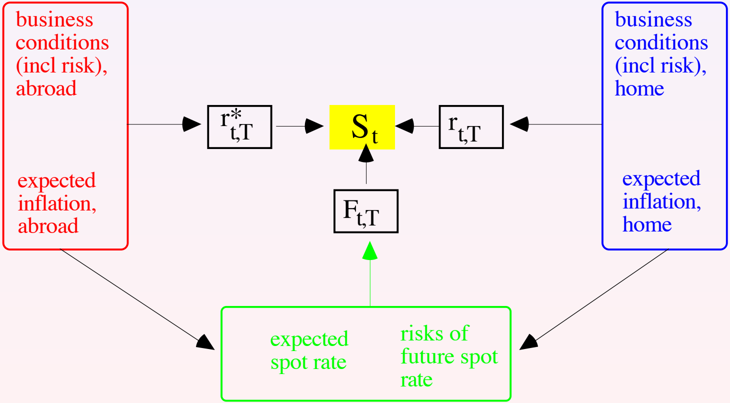

CIP and the swap rate

- **Fact 1:** Sign of swap rate depends just on $r - r^*$:
- **Fact 2:** Swap rate has low sensitivity to S:

$$
\begin{aligned}
F_{t,T} - S_t &= S_t\left[\frac{1+r_{t,T}}{1+r^*_{t,T}} - 1\right], \\
&= S_t\left[\frac{1+r_{t,T}}{1+r^*_{t,T}} - \frac{1+r^*_{t,T}}{1+r^*_{t,T}}\right], \\
&= S_t\left[\frac{r_{t,T} - r^*_{t,T}}{1+r^*_{t,T}}\right]; \\
\Rightarrow \frac{\partial \cdot}{\partial S_t} &= \left[\frac{r_{t,T} - r^*_{t,T}}{1+r^*_{t,T}}\right] \approx r_{t,T} - r^*_{t,T}.
\end{aligned}
$$

Traditionally, $r - r^*$ is small: short $T - t$, low p.a. interest.

This is why traders used to quote swap rates: if you change S, the required change in the swap rate is tiny relative to spreads.

CIP and the swap rate

Example

$T - t$= 1/12, *p.a.* simple interest 4% (home), 3% (foreign)

	spot		forward	swap rate
level_0	100.0	$100.0 \times \frac{1.003333}{1.002500}$ =	100.0831	0.0831
level_1	100.5	$100.5 \times \frac{1.003333}{1.002500}$ =	100.5835	0.0835
change	0.5		0.5004	0.0004

Analytically:

$$\text{change} \approx (0.003333 - 0.002500) \times 0.5 = 0.000416.$$

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CIP and taxes

◇ ”Neutral” taxes do not affect decisions

▷ neutral: there is just one income number, including interest income plus capgains, minus interest paid and caplosses

Example (S=100, r=0.21, r^*=0.10)

	Invest CLP 100		Invest NOK 1 hedged	
initial investment		100	$1 \times 100 =$	100
final value	$100 \times 1.21 =$	121	$[1 \times 1.10] \times 110 =$	121
income		21		21
interest		21	$[1 \times 0.10] \times 110 =$	11
capgain		0	$110 - 100 =$	10
taxable		21		21
tax (33.33 %)		7		7
after-tax income		14		14

From CIP: $$F \times (1 + r^*) = S \times (1 + r),$$

capgain + foreign interest = $$F - S + Fr^* = Sr$$ = domestic interest.

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Who wants to know a contract's MktVal?

◇ **Why do we care?**

- ▷ Valuation in financial statements or internal reports
- ▷ Negotiating an early termination:
  - speculator—wants to lock in gains, or cut losses
  - hedger—underlying hedged position is gone
  - default—file claim for damages
- ▷ Theory of options:
  - value of unconditional purchase (sale) is lower bound for value option to purchase (sell)
  - needed to explain early exercise issue in American-style option

NOTE: "a contract" means a purchase of FC 1.

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The valuation formula

- A forward contract has two legs, each of which can be thought of as a promissory note:
  - (asset:) you receive a PN from the bank *ad* FC 1
  - (liability:) you write a PN to the bank *ad* HC $F_{t_0,T}$
- So the contract's value is equal to the net value of this small portfolio.

Example (S=100, r=0.21, r^*=0.10; $F_{t_0,T}$=115)

	value in NOK	value in CLP
asset (NOK 1 at T)	1/1.10 = 0.90909	0.9090909 × 100 = 90.909
liability (CLP 115 at T)		115/1.21 = 95.041
net		–4.132

- Generalisation

$$\begin{bmatrix}\text{Market value of}\\ \text{forward purchase}\\ \text{at } F_{t_0,T}\end{bmatrix} = \underbrace{\overbrace{\frac{1}{1+r^*_{t,T}}}^{\text{PV* of FC 1, the asset}} \times S_t}_{\text{translated value of FC asset}} - \underbrace{\frac{F_{t_0,T}}{1+r_{t,T}}}_{\text{PV of HC liability}} .$$

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Alternative version of the formula

PV is just the PV of $F_{t,T} - F_{t_0,T}$, the locked-in "gain ($\pm$)" from a reversed purchase:

Example (S=100, r=0.21, r^*=0.10; $F_{t_0,T}$=115)

You bought at 115 and now you reverse (close out) at 110. Flows at T:

	flows in NOK	flows in CLP
original purchase	NOK 1	CLP < 115 >
new sale	< NOK 1 >	CLP 110
net	NOK 0	< CLP 5 >

$$\begin{bmatrix}\text{Market value of forward} \\ \text{purchase at } F_{t_0,T}\end{bmatrix} = \frac{1}{1+r_{t,T}} \underbrace{\frac{1+r_{t,T}}{1+r^*_{t,T}} S_t}_{=F_{t,T},\ \text{(CIP)}} - \frac{F_{t_0,T}}{1+r_{t,T}}$$

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new sale	$<$ NOK $1 >$	CLP 110
net	NOK 0	$<$ CLP $5 >$

$$\begin{bmatrix}\text{Market value of forward} \\ \text{purchase at } F_{t_0,T}\end{bmatrix} = \frac{1}{1+r_{t,T}} \underbrace{\frac{1+r_{t,T}}{1+r^*_{t,T}} S_t}_{=F_{t,T},\ \text{(CIP)}} - \frac{F_{t_0,T}}{1+r_{t,T}}$$

$$= \frac{F_{t,T} - F_{t_0,T}}{1+r_{t,T}}$$

Implication 1: Value at Maturity

If $t = T$, then $r_{T,T} = \ldots = r^*_{T,T}$; so

$$\frac{S_t}{1 + r^*_{t,T}} - \frac{F_{t_0,T}}{1 + r_{t,T}} \overset{\text{t=T}}{=}$$

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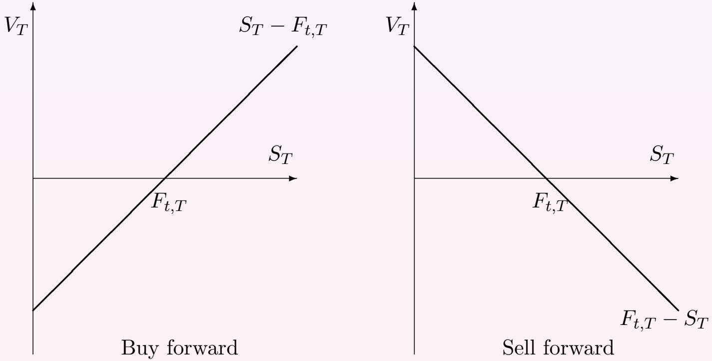

Implication 2: Value at Inception

If $t_0 = t$, then $\frac{F_{t,T} - F_{t_0,T}}{1 + r_{t,T}} \overset{t_0 = t}{=}$

NOTES

- holds only at the moment the contract is signed—otherwise the contract would be pointless
- Major!! implication: at the moment of hedging, the value of an asset *per se* is the same as the value of the hedged asset

$$\text{PV}(A + B \times \tilde{S}_T) = \frac{A + B \times F_{t,T}}{1 + r_{t,T}}\ ;$$

- we replace $\tilde{S}_T$ by $F_{t,T}$, and
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- ... meaning F is the certainty equivalent ("$F_{t,T} = \text{CEQ}_t(\tilde{S}_T)$")

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Implic3: F is a risk-adjusted expectation

- Two ways to value a unit FC TBill:
  - Way #1: General asset pricing approach:

$$\mathrm{PV}_t(\tilde{S}_T) = \frac{\mathrm{E}_t(\tilde{S}_T)}{1 + \mathrm{E}_t(\tilde{r}_{\tilde{S},t,T})}$$

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Implication 4: (ir)relevance of hedging?

Does the zero initial value mean that hedging adds no value?

- Criterion of firm's MktVal as the yardstick of relevance:
  - takes into account effects of hedging on expected cash flow *and* risk
  - MM's criterion
- At t, adding a hedge does not add/destroy any value *provided the firm's other cash flows are unaffected*.
- In many cases, the firm's other cash flows are likely to be affected:
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Outline

What have we learned in this chapter?

- Forward quotes can be *outright* or in *swap-rate* format.
- The Matrix:
  - Spot, forward, and money markets are so closely related that we have to study them together.
  - In a perfect market one could eliminate one of them and not lose anything.
- The perfect replicability implies a no-arb result, Covered Interest Parity—no causality suggested, here:

$$F_{t,T} = S_t \frac{1 + r_{t,T}}{1 + r^*_{t,T}}.$$

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