

# Chapter 6

# The Market for Currency Futures



# Overview



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Handling Default Risk in Forward Markets: Old & New Tricks

How Futures Contracts Differ from Forwards

Effect of Marking to Market on Futures Prices

Hedging with Futures Contracts

- The general MinVar problem
- The delta hedge
- The cross hedge

Conclusion: pros and cons



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# Handling Default Risk in Forward Markets





## ◇ Issues

- ▷ Default risk—limited by right of offset to $\tilde{S}_T - F_{t,T}$
- ▷ Illiquidity: early settlement is a favor, not a right

## ◇ Forwards: Standard Ways of Reducing Default Risk

- ▷ towards firms: credit agreements, security
- ▷ towards firms: restricted use
- ▷ towards banks: credit lines
- ▷ towards all: short lives; rolling over

## ◇ New gimmicks

- ▷ start with small collateral, covering 1-day risk instead of N
- ▷ [variable collateral:] every day: ask new collateral (or release old) depending on change mkt value—OR:
- ▷ [mk2mkt:] every day, settle yesterday's contract, "buy" a new one

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# Variable collateral / Mk2Mkt: Example



| data | Variable Collateral | Periodic Recontracting |
|---|---|---|
| **time 0:** | | |
| $F_{0,3} = 40$<br>$r_{0,3} = 3\%$ | Smitha buys forward USD 1m at $F_{0,3} = 40$ | Smitha buys forward USD 1m at $F_{0,3} = 40$ |
| **time 1:** | | |
| $F_{1,3} = 38$<br>$r_{1,3} = 2\%$ | Market value of old contract is $\frac{38m-40m}{1.02} = -1.961m$<br><br>Smitha puts up T-bills worth at least 1.961m | Market value of old contract is $\frac{38m-40m}{1.02} = -1.961m$<br><br>Smitha buys back the old contract for 1.961m and signs a new contract at $F_{1,3} = 38$. |
| **time 2:** | | |
| $F_{1,3} = 36$<br>$r_{2,3} = 1\%$ | Market value of old contract is $\frac{36m-40m}{1.01} = -3.960m$<br><br>Smitha increases the T-bills put up to at least 3.960m | Market value of old contract is $\frac{36m-38m}{1.01} = -1.980m$<br><br>Smitha buys back the old contract for 1.980m and signs a new contract at $F_{2,3} = 36$. |
| **time 3:** | | |
| $F_{3,3} = S_3$<br>$= 34$<br>$r_{3,3} = 0\%$ | Smitha pays the promised INR 40m for the USD 1m, and gets back her T-bills | Smitha pays the promised INR 36m for the USD 1m |
| **total paid:** | | |
| | INR 40m | (adjusted for time value:)<br>- time 3: 36m<br>- time 2: 1.980 × 1.01= 2m<br>- time 1: 1.961 × 1.02= 2m<br>- total: 40m |



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# How Futures Differ from Forwards



## Feature#1: Marking to Market

- Mk2Mkt using undiscounted change of price

### Example:

| price; $r$ | 40; $r$=0.03 | 38; $r$=0.02 | 36; $r$=0.01 | 34; $r$=0.00 |
|---|---|---|---|---|
| fwd, mk2mk | – | $\frac{38-40}{1.02} = -1.961$ | $\frac{36-38}{1.01} = -1.980$ | buy at 36 |
| futures | – | $38 - 40 = -2.000$ | $36 - 38 = -2.000$ | $34 - 36 = -2.000$ and then buy at 34 |

- loser pays winner via margin accounts held with broker / clearing members
- payment based on settlement price – or trade price in case of exit/entry during the day
- reduces loser's incentive to run away, and counterpart's loss if loser still runs away

## Feature#2: Clearing Corporation

- central counterpart between buyer and seller ⇒ guarantor
- also nets a player's purchases against sales

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- **Feature#3: initial margin; maintenance**
  - initial margin—interest bearing
  - small losses can accumulate until maintenance margin is reached; then a margin call is issued
  - failure to pay up = order to close out

## Example

Nick Leeson had accumulated losses *ad* GBP 800m—Barings' entire equity—but the Singapore Exchange lost "only" 50m:
- 500m was paid as m-to-m with Barings' money
- 250m was paid as m-to-m with other customers' money

The balance was lost by Simex while liquidating (immense price pressure).

- **Feature#4: organized markets**
  - Fwd: OTC—so no info on prices, volumes; just an informal snapshot around noon
  - Futures: formal exchanges. CME/CBOT, Eurex, LIFFE, etc
  - More and more via computerized PLOB or mixed system

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# How Futures Differ from Forwards



**FUTURES PRICES [...] CURRENCY**

| | Open | High | Low | Settle | Change | Lifetime High | Lifetime Low | Open Interest |
|---|---|---|---|---|---|---|---|---|
| **JAPAN YEN (CME) — 12.5 million yen ; $ per yen (.00)** | | | | | | | | |
| Sept | .9458 | .9466 | .9386 | .9389 | – .0046 | .9540 | .7945 | 73,221 |
| Dec | .9425 | .9470 | .9393 | .9396 | – .0049 | .9529 | .7970 | 3,455 |
| Mr94 | | | | .9417 | – .0051 | .9490 | .8700 | 318 |

Est vol 28,844; vol Wed 36,595; open int 77,028, + 1.820

◇ **Feature#5: Standardized contracts** to stop fragmentation and facilitate secondary dealing

- ▷ contract size (far smaller than OTC currency)
- ▷ expiry dates: e.g. monthly ($\leq 3mo$), mar/jun/sept/dec ($< 12mo$), annual (12 to 4 mo)

| Rate | at | Q (FC) | Other exchanges |
|---|---|---|---|
| USD:GBP | IMM | 62,500 | PBOT, LIFFE, SIMEX, MACE |
| USD:EUR | IMM | 125,000 | LIFFE, PBOT, SIMEX, MACE, FINEX |
| EUR:USD | OM-S | 50,000 | EUREX |
| USD:CHF | IMM | 125,000 | LIFFE, MACE, PBOT |
| USD:AUD | IMM | 100,000 | PBOT, EUREX |
| NZD:USD | NZFE | 50,000 | |
| USD:NZD | NZFE | 100,000 | |
| USD:JPY | IMM | 12,500,000 | LIFFE, TIFFE, MACE, PBOT, SIMEX |
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# Effect of Mk2Mkt on Futures Prices



Q: Does Mk2Mkt drive a wedge between futures and forward prices?

A: Yes, downward—but it's absolutely tiny.

## Example (data):

3 dates (0, 1, T=2). $F_{0,2} = 100, F_{1,2} = \begin{cases} 105, & p = 1/2 \\ 95, & p = 1/2 \end{cases}, \tilde{F}_{2,2} = \tilde{f}_{2,2} = \tilde{S}_3.$

– $f_{1,2}$ must be equal to $T_{1,2}$ because ...

– Q: is $f_{0,2} = F_{1,2}$?? We ¿verify/falsify? this conjecture.

| $F_{1,2}$ | HC flows: futures time 1 | HC flows: futures time 2 | HC flows: forward time 1 | HC flows: forward time 2 | difference time 1 | difference time 2 |
|---|---|---|---|---|---|---|
| 105 | $105 - 100 = +5$ | $-105$ | 0 | $-100$ | +5 | -5 |
| 95 | $95 - 100 = -5$ | $-95$ | 0 | $-100$ | -5 | +5 |

– Q rephrazed: do investors mind/love/loathe the no-interest loan/deposit in the up/down state, resp.?

– assume risk neutrality

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| | HC flows: futures | | HC flows: forward | | difference | |
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# Effect of Mk2Mkt on Futures Prices

Undo Mk2Mkt by borrowing 5 (down) or lending 5 (down). Costly? Profitable?

## Example

| | case 1 | | case 2 | | case 3 | |
|---|---|---|---|---|---|---|
| state | $r$ | net time value | $r$ | net time value at $T$ | $r$ | net time value at $T$ |
| up | 0 | $5 - 5 = 0.00$ | 10 | $5 \times 1.10 - 5 = \quad 0.50$ | 12 | $5 \times 1.08 - 5 = \quad 0.40$ |
| down | 0 | $-5 + 5 = 0.00$ | 10 | $-5 \times 1.10 + 5 = -0.50$ | 8 | $-5 \times 1.12 + 5 = -0.60$ |
| E(.) | | $0.00$ | | $0.00$ | | $-0.10$ |

- **Case 1: market interest rate is zero.** You don't even notice interest-free deposits/loans. Conjecture acceptable.
- **Case 2: market interest rate is a positive "constant".** You still don't mind. Conjecture acceptable.
- **Case 3: outflows financed at high rates, inflows deposited at low rates.** You now dislike interest-free deposits/loans. Conjecture falsified.
  - $f$ bust be below $F$
  - but general-eq simulations all suggest that effect is tiny

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# The long & short of it:

## In a nutshell:

| interest | prices | Mk2Mkt means ... | concl |
|---|---|---|---|
| down? | prices rise ($\pm$ ) | inflows, to be deposited | deposit at low rates |
| up? | prices fall ($\pm$ ) | outflows, to be financed | finance at high rates |

To induce investors to hold futures contracts, futures prices must be lower than forward prices.

## But ...

– correlation between $\Delta r$ and $\Delta f$ is lowish

– GenEq models suggest lowish price of risk

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# What's special?: Approximate hedging



- **Limited choice:**
  - only a few currencies (or grades / delivery points, for commodities)
  - only a few expiry dates

  Almost surely there is no perfect hedge

- **How to deal with it?**
  - Chose a hedge ratio that reduces the remaining risk to a minimum
  - Let $T_1$ be the firm's hedging horizon
  - Let $\tilde{y} := \tilde{C}_{T_1}$ be the firm's cash flow at $T_1$, in HC
  - Let $T_2 (\geq T_1)$ be the expiry date of the hedge
  - Then: chose $B_{t,T_1}$ so as to minimize $\text{var}(\tilde{y} - B_{t,T_1}\underbrace{[\tilde{f}_{T_1,T_2} - f_{t,T_2}]}_{=:\,\tilde{x}})$

- **Solution**
  - This is a LS problem: $\min_B \text{var}(\tilde{e})$ with $\tilde{e} := \tilde{y} - B\tilde{x}$
  - So we get a regression-coeff solution: $B = \frac{\text{cov}(\tilde{y},\tilde{x})}{\text{var}(\tilde{x})}$

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  - So we get a regression-coeff solution: $B = \frac{\text{cov}(\tilde{y},\tilde{x})}{\text{var}(\tilde{x})}$



# Notes on the solution

- **Linear regression?**
  - We do not assume $\tilde{y}$ is linear in $\tilde{f}$. Rather, we extract the linear component in a almost always nonlinear relation, because the hedge's pay-off is linear in $\tilde{f}$.
  - Time series regression will not do, in general:
    - relation is changing over time
    - ($\tilde{y}$ and) $\tilde{f}$ are long-memory processes ($\pm$ random walks)

- **Narrowed-down problem:** a contract with $\tilde{y} = n_{t,T_1}\tilde{S}_{T_1}$
  - you should generally look at net exposure of *all* contracts
  - even this broader view ignores correlations of non-contractual flows

**Example**

A firm once commissioned a hedging program for its commodity purchases, ignoring the fact that price increases were passed on to the consumers with a delay of about 2 weeks.

Their sales price was, to a large extent, a natural hedge. Their proposed policy would probably have *increased* risk.

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# The narrow contractual-exposure problem

## ◇ A standardized problem

- ▷ a unit inflow of FC $e$ at $T_1$ — e.g SEK (in USD/SEK)
- ▷ hedge contract is available for FC $h$, contract size one unit — e.g. EUR (in USD/EUR)
- ▷ Solution: $B = \frac{\text{cov}(\tilde{f}^{(h)}_{T_1,T_2}, \tilde{S}^{(e)}_{T_1})}{\text{var}(\tilde{f}^{(h)}_{T_1,T_2})}$.

## ◇ Case 1: perfect hedge: $e = h$, $T_1 = T_2$ so $\tilde{f}^{(h)}_{T_1,T_2} = \tilde{S}^{(e)}_{T_1}$

- ▷ regress $\tilde{y}$ on itself, so $B = 1$



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# The Delta hedge: $e = h$, $T_2 > T_1$





Regression: $\tilde{S}^{(e)}_{T_1} = A + B\,\tilde{f}^{(e)}_{T_1,T_2} + e$ where $\tilde{f}^{(e)}_{T_1,T_2} \approx \tilde{F}^{(e)}_{T_1,T_2} = \tilde{S}^{(e)}_{T_1}\frac{1+\tilde{r}_{T_1,T_2}}{1+\tilde{r}^{(e)}_{T_1,T_2}}$

- Rule-of-thumb solution for regression:
  - If time-$T_1$ interest rates were known in advance, then so would be $B$:

$$\underbrace{S^{(e)}_{T_1}}_{\text{LHSvarbl}} = \underbrace{\frac{1+r^{(e)}_{T_1}}{1+r_{T_1}}}_{\text{=B!}} \underbrace{\frac{1+r_{T_1}}{1+r^{(e)}_{T_1}} S^{(e)}_{T_1}}_{\text{RHSvarbl}}$$

  - quick-and-hardly-dirty solution: ignore variability, set B at the current level of $(1+r^*)(1+r)$ or even to unity

- If you do run time-series regression, do not use levels: long-memory variables!
  - either regress $\Delta\tilde{S} = a + B\,\Delta\tilde{f} + \nu$
  - or regress $\Delta\tilde{S}/S = c + b\,\Delta\tilde{f}/f + \epsilon$ and compute $B = b\,f/S$

- If you do run TS regression, use computed $F$'s for the horizon you need, not observed $f$'s
  - easier to get synchronized data
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# Outline

# Conclusion: pros and cons





- **Advantages of futures** include
  - Easy access: less (initial) margin is required than for forwards.
  - Low commissions, because of standardization.
  - Secondary market: Futures positions can be easily closed out.
- **Disadvantages of futures** include
  - Standardization. Choice: imperfect but cheap futures hedge $v$ more expensive, tailor-made forward hedge?
  - Ruin risk. Mk2Mkt can create severe short-term cash flow problems for a hedger.
  - Interest rate risk. The daily cash flows must be financed/deposited in the money markets at interest rates that are not known when the hedge is set up.
  - Limited menu of futures contracts.
  - Short maturities—less than one year.

⇒ Forward markets are still widely and mostly used by corporate hedgers, while futures *tend to* attract *relatively more* speculators.

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