



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Overview

Chapter 18

Estimating the Cost of International Capital



Overview

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Overview

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?

Why Xrisk pops up in the InCAPM

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up



Overview

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Overview

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?

Why Xrisk pops up in the InCAPM

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up



Overview

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Overview

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?

Why Xrisk pops up in the InCAPM

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up



Overview

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Overview

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?

Why Xrisk pops up in the InCAPM

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up



CAPM issues in FDI

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

Suppose an Australian firm considers an investment in India. Issues:

- ▶ Can we assess NPV in INR, as locals would do?
 - would look simpler — no?
 - ... but works only if AU and IN are both (part of) an integrated market
- ▶ How does a multi-country market work, where investors “think in different currencies” depending on where they live?
 - risks and returns depend on the currency they are measured in,
 - ... which violates the “homogenous expectations” assumption of standard CAPM



CAPM issues in FDI

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

Suppose an Australian firm considers an investment in India. Issues:

- ▶ Can we assess NPV in INR, as locals would do?
 - would look simpler — no?
 - ... but works only if AU and IN are both (part of) an integrated market
- ▶ How does a multi-country market work, where investors “think in different currencies” depending on where they live?
 - risks and returns depend on the currency they are measured in,
 - ... which violates the “homogenous expectations” assumption of standard CAPM



CAPM issues in FDI

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

Suppose an Australian firm considers an investment in India. Issues:

- ▶ Can we assess NPV in INR, as locals would do?
 - would look simpler — no?
 - ... but works only if AU and IN are both (part of) an integrated market
- ▶ How does a multi-country market work, where investors “think in different currencies” depending on where they live?
 - risks and returns depend on the currency they are measured in,
 - ... which violates the “homogenous expectations” assumption of standard CAPM



Outline

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

Two procedures
When to do what?

The Single-Country
CAPM

The International
CAPM

Wrapping up

Translate first, then discount—or vv?

Two procedures
When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return
The tangency solution
How the weights affect mean and variance
How to make a portfolio efficient
The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?
Why Xrisk pops up in the InCAPM
Do assets have a financial nationality?
Aggregating the Efficiency Conditions
The InCAPM

Wrapping up



Two facts need reconciliation:

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

The International CAPM

Wrapping up

◇ **CF's are probably first computed in FC (INR):**

- ▷ cost data in INR; local sales “priced to market”
- ▷ all these prices are probably sticky in terms of INR, not AUD

◇ **Yet commonly used CoCa is in HC (AUD)**

So

- either we translate the cashflows into AUD
- or we shift to an INR CoCa



Two facts need reconciliation:

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

The International CAPM

Wrapping up

◇ **CF's are probably first computed in FC (INR):**

- ▷ cost data in INR; local sales “priced to market”
- ▷ all these prices are probably sticky in terms of INR, not AUD

◇ **Yet commonly used CoCa is in HC (AUD)**

So

- either we translate the cashflows into AUD
- or we shift to an INR CoCa



Two facts need reconciliation:

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

Two procedures

When to do what?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ **CF's are probably first computed in FC (INR):**

- ▷ cost data in INR; local sales “priced to market”
- ▷ all these prices are probably sticky in terms of INR, not AUD

◇ **Yet commonly used CoCa is in HC (AUD)**

So

- either we translate the cashflows into AUD
- or we shift to an INR CoCa



Two procedures

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

The International CAPM

Wrapping up

◇ ¿ “When in Rome, act like the Romans”?

- ▷ set CoCA on basis of INR risk-free rate, and add risk premium computed from INR stock returns (risk, price of risk)
- ▷ compute FCPV
- ▷ if desired, convert PV into HC

... or ...

◇ ¿ “My shareholders consume/think in AUD” ?

- ▷ translate expected CFs into AUD, including the covariance
- ▷ set CoCA on basis of AUD risk-free rate, and add risk premium computed from AUD stock returns (risk, price of risk)
- ▷ compute PV in HC



Two procedures

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

Two procedures

When to do what?

The Single-Country
CAPM

The International
CAPM

Wrapping up

- ◇ ¿ **“When in Rome, act like the Romans”?**
 - ▷ set CoCA on basis of INR risk-free rate, and add risk premium computed from INR stock returns (risk, price of risk)
 - ▷ compute FCPV
 - ▷ if desired, convert PV into HC

... or ...

- ◇ ¿ **“My shareholders consume/think in AUD” ?**
 - ▷ translate expected CFs into AUD, including the covariance
 - ▷ set CoCA on basis of AUD risk-free rate, and add risk premium computed from AUD stock returns (risk, price of risk)
 - ▷ compute PV in HC

When to do what?



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

Two procedures

When to do what?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ Does the translation/discounting procedure matter?

- ▶ **In practice:** the “local” version looks easier (but only if we cut corners, as we’ll see)
- ▶ **In principle:** no if mkts are integrated, yes if mkts aren’t
 - Australians care about the value to them, not about how Indians would value the project. The two are not the same in segmented markets.
 - We can “see” the Australian’s required return in their own capital market. So we value “à l’Australienne”

◇ Can there be a CAPM-type no-PPP equilibrium?

- ▶ presence of (real) exchange risk means expectations cannot be homogenous: (real) expected returns and risks for asset j differ across investors
- ▶ But this special type of investors heterogeneity is easily incorporated (InCAPM instead of CAPM)



When to do what?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

Two procedures

When to do what?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ Does the translation/discounting procedure matter?

- ▶ **In practice:** the “local” version looks easier (but only if we cut corners, as we’ll see)
- ▶ **In principle:** no if mkts are integrated, yes if mkts aren’t
 - Australians care about the value to them, not about how Indians would value the project. The two are not the same in segmented markets.
 - We can “see” the Australian’s required return in their own capital market. So we value “à l’Australienne”

◇ Can there be a CAPM-type no-PPP equilibrium?

- ▶ presence of (real) exchange risk means expectations cannot be homogenous: (real) expected returns and risks for asset j differ across investors
- ▶ But this special type of investors heterogeneity is easily incorporated (InCAPM instead of CAPM)



When to do what?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

Two procedures

When to do what?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ Does the translation/discounting procedure matter?

- ▶ **In practice:** the “local” version looks easier (but only if we cut corners, as we’ll see)
- ▶ **In principle:** no if mkts are integrated, yes if mkts aren’t
 - Australians care about the value to them, not about how Indians would value the project. The two are not the same in segmented markets.
 - We can “see” the Australian’s required return in their own capital market. So we value “à l’Australienne”

◇ Can there be a CAPM-type no-PPP equilibrium?

- ▶ presence of (real) exchange risk means expectations cannot be homogenous: (real) expected returns and risks for asset j differ across investors
- ▶ But this special type of investors heterogeneity is easily incorporated (InCAPM instead of CAPM)



When to do what? Overview

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

The International CAPM

Wrapping up

	CoCa model	currency of calculations
1. Foreign investments:		
• home and host financially integrated	inCAPM	FC & HC
• home and host financially segmented		
• home country part of larger financial market	inCAPM	HC only
• home country totally isolated	CAPM	HC only
2. domestic investments		
• home country part of larger financial market	inCAPM	HC only
• home country totally isolated	CAPM	HC only



Outline

The Cost of International Capital

P. Sercu,
International

Finance: Theory into Practice

Translate first, then discount—or vv ?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

Translate first, then discount—or vv ?

Two procedures

When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?

Why Xrisk pops up in the InCAPM

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up



From Asset Returns to Portfolio Return

The Cost of International Capital

P. Sercu,
International

Finance: Theory into Practice

Key relation—stated in excess return terms, here—between portfolio return \tilde{r}_p and asset returns \tilde{r}_j & weights x_j :

$$\tilde{r}_p - r = \sum_{j=1}^N x_j (\tilde{r}_j - r)$$

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

Example

		time-0 data and decisions				time-1 result		(excess) rates of return		
	j	$V_{j,0}$	n_j	$n_j V_{j,0}$	x_j	$V_{j,1}$	$n_j V_{j,1}$	r_j	$r_j - r$	$x_j (r_j - r)$
<i>risky:</i>	1	100	4	400	0.40	120	480	0.20	0.15	0.060
	2	50	4	200	0.20	70	280	0.40	0.35	0.070
	3	25	12	300	0.30	20	240	-0.20	-0.25	-0.075
subtotal				=900	=0.90					=0.055
<i>riskfree</i>	0			+100	+0.10		105			+0.050
total				=1000	=1.00		=1105			=0.105



From Asset Returns to Portfolio Return

The Cost of International Capital

P. Sercu,
International

Finance: Theory into
Practice

Translate first, then
discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International CAPM

Wrapping up

Key relation—stated in excess return terms, here—between portfolio return \tilde{r}_p and asset returns \tilde{r}_j & weights x_j :

$$\tilde{r}_p - r = \sum_{j=1}^N x_j (\tilde{r}_j - r)$$

Example

	j	time-0 data and decisions				time-1 result		(excess) rates of return		
		$V_{j,0}$	n_j	$n_j V_{j,0}$	x_j	$V_{j,1}$	$n_j V_{j,1}$	r_j	$r_j - r$	$x_j(r_j - r)$
<i>risky</i> :	1	100	4	400	0.40	120	480	0.20	0.15	0.060
	2	50	4	200	0.20	70	280	0.40	0.35	0.070
	3	25	12	300	0.30	20	240	-0.20	-0.25	-0.075
subtotal				=900	=0.90					=0.055
<i>riskfree</i>	0			+100	+0.10		105			+0.050
total				=1000	=1.00		=1105			=0.105

Two assets—one risk-free, one risky



The Cost of International Capital

P. Sercu,
International

Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

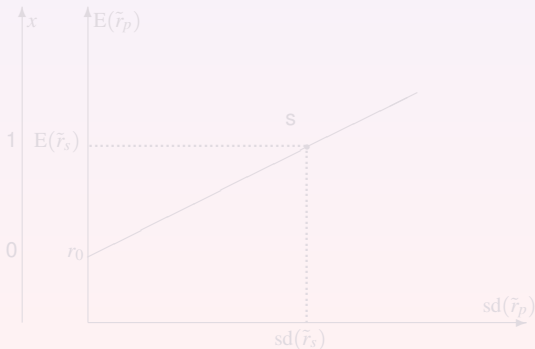
How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

$$\begin{aligned}\tilde{r}_p &= x\tilde{r}_s + (1-x)r_0 = r_0 + x(\tilde{r}_s - r_0) \\ \Rightarrow &\begin{cases} E(\tilde{r}_p) = r_0 + xE(\tilde{r}_s - r_0), \\ \text{sd}(\tilde{r}_p) = |x| \text{sd}(\tilde{r}_s) \end{cases}\end{aligned}$$



Two assets—one risk-free, one risky



The Cost of International Capital

P. Sercu,
International

Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

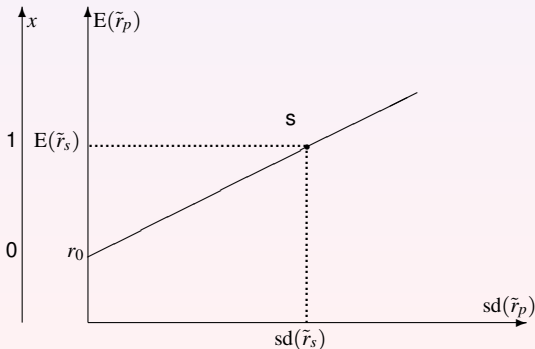
How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

$$\begin{aligned}\tilde{r}_p &= x\tilde{r}_s + (1-x)r_0 = r_0 + x(\tilde{r}_s - r_0) \\ \Rightarrow \begin{cases} E(\tilde{r}_p) = r_0 + xE(\tilde{r}_s - r_0), \\ \text{sd}(\tilde{r}_p) = |x| \text{sd}(\tilde{r}_s) \end{cases}\end{aligned}$$





Two imperfectly correlated risky assets

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

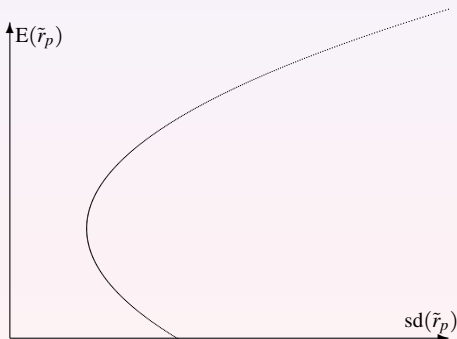
The International CAPM

Wrapping up

$$\tilde{r}_p = x_1 \tilde{r}_1 + (1 - x_1) \tilde{r}_2; \Rightarrow$$

$$\left\{ \begin{array}{l} E(\tilde{r}_p) = E(\tilde{r}_2) + x [E(\tilde{r}_1) - E(\tilde{r}_2)], \\ \end{array} \right.$$

$$\left\{ \begin{array}{l} \text{sd}(\tilde{r}_p) = \sqrt{x_1^2 \text{var}(\tilde{r}_1) + 2x_1(1 - x_1) \text{cov}(\tilde{r}_1, \tilde{r}_2) + (1 - x_1)^2 \text{var}(\tilde{r}_2)} \end{array} \right.$$





Many risky assets and one risk-free

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

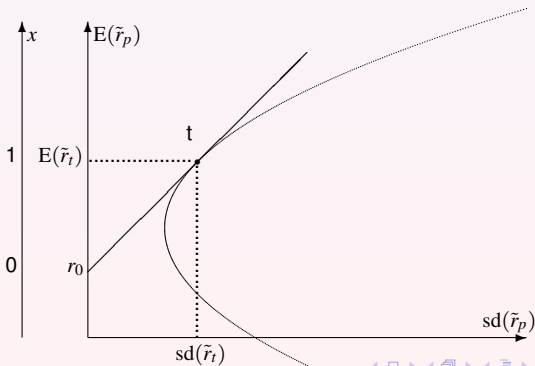
Wrapping up

◇ many risky assets:

- bound is similar, but most portfolios are now inside the bound
- only portfolios on the upper half o/t bound are efficient

◇ add a risk-free asset

- best risky portfolio is the tangency one:





Many risky assets and one risk-free

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

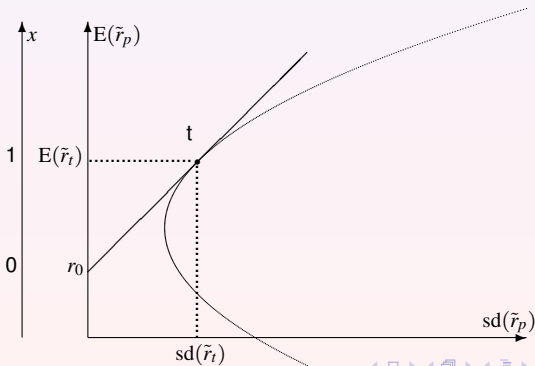
Wrapping up

◇ many risky assets:

- bound is similar, but most portfolios are now inside the bound
- only portfolios on the upper half o/t bound are efficient

◇ add a risk-free asset

- best risky portfolio is the tangency one:





How weights affect mean and variance (1)

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

From $\tilde{r}_p = r + \sum_{j=1}^N x_j (\tilde{r}_j - r)$:

$$E(\tilde{r}_p) = r + \sum_{j=1}^N x_j E(\tilde{r}_j - r),$$

$$\text{var}(\tilde{r}_p) = \sum_{j=1}^N x_j \sum_{k=1}^N x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$

Understanding the variance formula:

- portfolio variance is a weighted average of each asset's covariance with the entire portfolio:

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = \text{cov}\left(\sum_{k=1}^N x_k \tilde{r}_k, \tilde{r}_p\right) = \sum_{k=1}^N x_k \text{cov}(\tilde{r}_k, \tilde{r}_p).$$

- each of these assets' portfolio covariances is, in turn, a weighted average of the asset's covariance with all components of the portfolio:

$$\text{cov}(\tilde{r}_j, \tilde{r}_p) = \text{cov}\left(\tilde{r}_j, \sum_{k=1}^N x_k \tilde{r}_k\right) = \sum_{k=1}^N x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$



How weights affect mean and variance (1)

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

From $\tilde{r}_p = r + \sum_{j=1}^N x_j (\tilde{r}_j - r)$:

$$E(\tilde{r}_p) = r + \sum_{j=1}^N x_j E(\tilde{r}_j - r),$$

$$\text{var}(\tilde{r}_p) = \sum_{j=1}^N x_j \sum_{k=1}^N x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$

Understanding the variance formula:

- portfolio variance is a weighted average of each asset's covariance with the entire portfolio:

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = \text{cov}\left(\sum_{k=1}^N x_k \tilde{r}_k, \tilde{r}_p\right) = \sum_{k=1}^N x_k \text{cov}(\tilde{r}_k, \tilde{r}_p).$$

- each of these assets' portfolio covariances is, in turn, a weighted average of the asset's covariance with all components of the portfolio:

$$\text{cov}(\tilde{r}_j, \tilde{r}_p) = \text{cov}\left(\tilde{r}_j, \sum_{k=1}^N x_k \tilde{r}_k\right) = \sum_{k=1}^N x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$



How weights affect mean and variance (1)

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

From $\tilde{r}_p = r + \sum_{j=1}^N x_j (\tilde{r}_j - r)$:

$$E(\tilde{r}_p) = r + \sum_{j=1}^N x_j E(\tilde{r}_j - r),$$

$$\text{var}(\tilde{r}_p) = \sum_{j=1}^N x_j \sum_{k=1}^N x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$

Understanding the variance formula:

- portfolio variance is a weighted average of each asset's covariance with the entire portfolio:

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = \text{cov}\left(\sum_{k=1}^N x_k \tilde{r}_k, \tilde{r}_p\right) = \sum_{k=1}^N x_k \text{cov}(\tilde{r}_k, \tilde{r}_p).$$

- each of these assets' portfolio covariances is, in turn, a weighted average of the asset's covariance with all components of the portfolio:

$$\text{cov}(\tilde{r}_j, \tilde{r}_p) = \text{cov}\left(\tilde{r}_j, \sum_{k=1}^N x_k \tilde{r}_k\right) = \sum_{k=1}^N x_k \text{cov}(\tilde{r}_j, \tilde{r}_k).$$



How weights affect mean and variance (2)

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Example:

	$E(\tilde{r}_j - r)$	$\text{cov}(\tilde{r}_j, \tilde{r}_1)$	$\text{cov}(\tilde{r}_j, \tilde{r}_2)$	x_j
1	0.200	0.16	0.05	0.50
2	0.122	0.05	0.09	0.40

$$E(\tilde{r}_p - r) = 0.50 \times 0.200 + 0.40 \times 0.122 = 0.1488$$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.50 \times 0.160 + 0.40 \times 0.050 = 0.1000$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.50 \times 0.050 + 0.40 \times 0.090 = 0.0610$$

$$\Rightarrow \text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.50 \times 0.100 + 0.40 \times 0.061 = 0.0744$$

Marginal effect of x_j on mean and variance



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Three-asset example:

- Asset 1's marginal contribution to portfolio expected excess return is **its own expected excess return**:

$$\begin{aligned} E(\tilde{r}_p - r) &= x_1 E(\tilde{r}_1 - r) + x_2 E(\tilde{r}_2 - r); \\ \Rightarrow \frac{\partial E(\tilde{r}_p - r)}{\partial x_1} &= E(\tilde{r}_1 - r). \end{aligned}$$

- Asset 1's marginal contribution to portfolio variance is (twice) **its covariance with the portfolio return**:

$$\begin{aligned} \text{var}(\tilde{r}_p) &= x_1^2 \text{var}(\tilde{r}_1) + 2x_1 x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2) + x_2^2 \text{var}(\tilde{r}_2); \\ \Rightarrow \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} &= 2\text{cov}(\tilde{r}_1, \tilde{r}_p). \end{aligned}$$

$$\begin{aligned} \text{Proof: } \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} &= 2x_1 \text{var}(\tilde{r}_1) + 2x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2), \\ &= 2[x_1 \text{cov}(\tilde{r}_1, \tilde{r}_1) + x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2)], \\ &= 2[\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1) + \text{cov}(\tilde{r}_1, x_2 \tilde{r}_2)], \\ &= 2\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1 + x_2 \tilde{r}_2), \end{aligned}$$

Marginal effect of x_j on mean and variance



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Three-asset example:

- Asset 1's marginal contribution to portfolio expected excess return is **its own expected excess return**:

$$\begin{aligned} E(\tilde{r}_p - r) &= x_1 E(\tilde{r}_1 - r) + x_2 E(\tilde{r}_2 - r); \\ \Rightarrow \frac{\partial E(\tilde{r}_p - r)}{\partial x_1} &= E(\tilde{r}_1 - r). \end{aligned}$$

- Asset 1's marginal contribution to portfolio variance is (twice) **its covariance with the portfolio return**:

$$\begin{aligned} \text{var}(\tilde{r}_p) &= x_1^2 \text{var}(\tilde{r}_1) + 2x_1 x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2) + x_2^2 \text{var}(\tilde{r}_2); \\ \Rightarrow \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} &= 2\text{cov}(\tilde{r}_1, \tilde{r}_p). \end{aligned}$$

$$\begin{aligned} \text{Proof: } \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} &= 2x_1 \text{var}(\tilde{r}_1) + 2x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2), \\ &= 2[x_1 \text{cov}(\tilde{r}_1, \tilde{r}_1) + x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2)], \\ &= 2[\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1) + \text{cov}(\tilde{r}_1, x_2 \tilde{r}_2)], \\ &= 2\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1 + x_2 \tilde{r}_2), \end{aligned}$$

Marginal effect of x_j on mean and variance



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Three-asset example:

- Asset 1's marginal contribution to portfolio expected excess return is **its own expected excess return**:

$$\begin{aligned} E(\tilde{r}_p - r) &= x_1 E(\tilde{r}_1 - r) + x_2 E(\tilde{r}_2 - r); \\ \Rightarrow \frac{\partial E(\tilde{r}_p - r)}{\partial x_1} &= E(\tilde{r}_1 - r). \end{aligned}$$

- Asset 1's marginal contribution to portfolio variance is (twice) **its covariance with the portfolio return**:

$$\begin{aligned} \text{var}(\tilde{r}_p) &= x_1^2 \text{var}(\tilde{r}_1) + 2x_1 x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2) + x_2^2 \text{var}(\tilde{r}_2); \\ \Rightarrow \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} &= 2\text{cov}(\tilde{r}_1, \tilde{r}_p). \end{aligned}$$

$$\begin{aligned} \text{Proof: } \frac{\partial \text{var}(\tilde{r}_p)}{\partial x_1} &= 2x_1 \text{var}(\tilde{r}_1) + 2x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2), \\ &= 2[x_1 \text{cov}(\tilde{r}_1, \tilde{r}_1) + x_2 \text{cov}(\tilde{r}_1, \tilde{r}_2)], \\ &= 2[\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1) + \text{cov}(\tilde{r}_1, x_2 \tilde{r}_2)], \\ &= 2\text{cov}(\tilde{r}_1, x_1 \tilde{r}_1 + x_2 \tilde{r}_2), \end{aligned}$$



Effect of w on $var(\tilde{r}_p)$: example

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

Example

	$E(\tilde{r}_j - r)$	$\text{cov}(\tilde{r}_j, \tilde{r}_1)$	$\text{cov}(\tilde{r}_j, \tilde{r}_2)$
1	0.200	0.04	0.05
2	0.122	0.05	0.09

portfolio 1: $x_1 = .40, x_2 = .40, x_0 = .20$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.200 \times 0.40 + 0.04 \times 0.40 = 0.036000,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.122 \times 0.40 + 0.05 \times 0.40 = 0.056000,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.200 \times 0.20 + 0.04 \times 0.40 + 0.05 \times 0.40 = 0.036800,$$

portfolio 2: $x_1 = .41, x_2 = .40, x_0 = .19$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.200 \times 0.41 + 0.04 \times 0.40 = 0.036400,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.122 \times 0.41 + 0.05 \times 0.40 = 0.056500,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.200 \times 0.19 + 0.04 \times 0.41 + 0.05 \times 0.40 = 0.037524,$$

$$\frac{\Delta \text{var}}{\Delta x_1} = \frac{.037524 - .036800}{0.01} = \frac{.000724}{0.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).$$



Effect of w on $var(\tilde{r}_p)$: example

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

Example

	$E(\tilde{r}_j - r)$	$\text{cov}(\tilde{r}_j, \tilde{r}_1)$	$\text{cov}(\tilde{r}_j, \tilde{r}_2)$
1	0.200	0.04	0.05
2	0.122	0.05	0.09

portfolio 1: $x_1 = .40, x_2 = .40, x_0 = .20$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.036\,000,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.056\,000,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.036\,800,$$

portfolio 2: $x_1 = .41, x_2 = .40, x_0 = .19$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.036\,400,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.056\,500,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.037\,524,$$

$$\frac{\Delta \text{var}}{\Delta x_1} = \frac{.037\,524 - .036\,800}{0.01} = \frac{.000724}{0.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).$$



Effect of w on $var(\tilde{r}_p)$: example

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Example

	$E(\tilde{r}_j - r)$	$\text{cov}(\tilde{r}_j, \tilde{r}_1)$	$\text{cov}(\tilde{r}_j, \tilde{r}_2)$
1	0.200	0.04	0.05
2	0.122	0.05	0.09

portfolio 1: $x_1 = .40, x_2 = .40, x_0 = .20$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.036\,000,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.056\,000,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.036\,800,$$

portfolio 2: $x_1 = .41, x_2 = .40, x_0 = .19$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.036\,400,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.056\,500,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0. \dots \times 0. \dots + 0. \dots \times 0. \dots = 0.037\,524,$$

$$\frac{\Delta \text{var}}{\Delta x_1} = \frac{.037\,524 - .036\,800}{0.01} = \frac{.000724}{0.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).$$



Effect of w on $var(\tilde{r}_p)$: example

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

Example

	$E(\tilde{r}_j - r)$	$\text{cov}(\tilde{r}_j, \tilde{r}_1)$	$\text{cov}(\tilde{r}_j, \tilde{r}_2)$
1	0.200	0.04	0.05
2	0.122	0.05	0.09

portfolio 1: $x_1 = .40, x_2 = .40, x_0 = .20$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.200 \times 0.40 + 0.04 \times 0.40 = 0.036000,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.122 \times 0.40 + 0.05 \times 0.40 = 0.056000,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.200 \times 0.20 + 0.04 \times 0.40 + 0.05 \times 0.40 = 0.036800,$$

portfolio 2: $x_1 = .41, x_2 = .40, x_0 = .19$

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.200 \times 0.41 + 0.04 \times 0.40 = 0.036400,$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.122 \times 0.41 + 0.05 \times 0.40 = 0.056500,$$

$$\text{cov}(\tilde{r}_p, \tilde{r}_p) = 0.200 \times 0.19 + 0.04 \times 0.41 + 0.05 \times 0.40 = 0.037524,$$

$$\frac{\Delta \text{var}}{\Delta x_1} = \frac{.037524 - .036800}{0.01} = \frac{.000724}{0.01} = 0.0724 \approx 2 \times \text{cov}(\tilde{r}_1, \tilde{r}_p).$$

How to make a portfolio efficient



The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

- ◇ **Micro economics:** budget allocation problem has as efficiency condition that

$$\frac{\text{marginal utility}_j}{\text{marginal cost}_j} = \text{same across all goods } j$$

- ◇ **Mean-Variance:** the “good” side is not utility but expected return; and the bad side is variance. So the efficiency condition is

$$\frac{j\text{'s contribution to } E(\tilde{r}_p)}{j\text{'s contribution to } \text{var}(\tilde{r}_p)} = \text{same across all assets } j$$

$$\frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_p)} = \lambda, \text{ for all risky assets } j=1, \dots, N$$

How to make a portfolio efficient



The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

- ◇ **Micro economics:** budget allocation problem has as efficiency condition that

$$\frac{\text{marginal utility}_j}{\text{marginal cost}_j} = \text{same across all goods } j$$

- ◇ **Mean-Variance:** the “good” side is not utility but expected return; and the bad side is variance. So the efficiency condition is

$$\frac{j\text{'s contribution to } E(\tilde{r}_p)}{j\text{'s contribution to } \text{var}(\tilde{r}_p)} = \text{same across all assets } j$$

$$\frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_p)} = \lambda, \text{ for all risky assets } j=1, \dots, N$$

How to make a portfolio efficient



The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

- ◇ **Micro economics:** budget allocation problem has as efficiency condition that

$$\frac{\text{marginal utility}_j}{\text{marginal cost}_j} = \text{same across all goods } j$$

- ◇ **Mean-Variance:** the “good” side is not utility but expected return; and the bad side is variance. So the efficiency condition is

$$\frac{j\text{'s contribution to } E(\tilde{r}_p)}{j\text{'s contribution to } \text{var}(\tilde{r}_p)} = \text{same across all assets } j$$

$$\frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_p)} = \lambda, \text{ for all risky assets } j=1, \dots, N$$

(In)Efficiency and Risk Aversion: examples



The Cost of
International Capital

P. Sercu,
International
Finance: Theory into
Practice

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution
How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

	$E(\tilde{r}_j - r)$	(co)variances	
Asset 1	0.092	$\text{cov}(\tilde{r}_1, \tilde{r}_1) = 0.04$	$\text{cov}(\tilde{r}_1, \tilde{r}_2) = 0.05$
Asset 2	0.148	$\text{cov}(\tilde{r}_2, \tilde{r}_1) = 0.05$	$\text{cov}(\tilde{r}_2, \tilde{r}_2) = 0.09$

portfolio with weights $x_1 = .40, x_2 = .40, x_0 = .20$ is not efficient:

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.092 \times 0.40 + 0.05 \times 0.40 = 0.036$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.05 \times 0.40 + 0.09 \times 0.40 = 0.056,$$

$$\frac{0.092}{0.092} = 2.555 \neq \frac{0.056}{0.056} = 2.643$$

portfolio with weights $x_1 = .40, x_2 = .60, x_0 = .00$ is efficient:

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.04 \times 0.40 + 0.05 \times 0.60 = 0.046$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.05 \times 0.40 + 0.09 \times 0.60 = 0.074,$$

$$\frac{0.046}{0.046} = 2 = \frac{0.074}{0.074}$$

portfolio with weights $x_1 = .20, x_2 = .30, x_0 = .50$ is efficient:

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.04 \times 0.20 + 0.05 \times 0.30 = 0.023$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.05 \times 0.20 + 0.09 \times 0.30 = 0.037,$$

$$\frac{0.023}{0.023} = 4 = \frac{0.037}{0.037}$$

(In)Efficiency and Risk Aversion: examples



The Cost of
International Capital

P. Sercu,
International
Finance: Theory into
Practice

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution
How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

	$E(\tilde{r}_j - r)$	(co)variances	
Asset 1	0.092	$\text{cov}(\tilde{r}_1, \tilde{r}_1) = 0.04$	$\text{cov}(\tilde{r}_1, \tilde{r}_2) = 0.05$
Asset 2	0.148	$\text{cov}(\tilde{r}_2, \tilde{r}_1) = 0.05$	$\text{cov}(\tilde{r}_2, \tilde{r}_2) = 0.09$

portfolio with weights $x_1 = .40, x_2 = .40, x_0 = .20$ is not efficient:

$$\begin{aligned} \text{cov}(\tilde{r}_1, \tilde{r}_p) &= 0.092 \times 0.40 + 0.05 \times 0.40 = 0.036 \\ \text{cov}(\tilde{r}_2, \tilde{r}_p) &= 0.05 \times 0.40 + 0.09 \times 0.40 = 0.056, \\ \frac{0.036}{0.056} &= 2.555 \neq \frac{0.092}{0.148} = 2.643 \end{aligned}$$

portfolio with weights $x_1 = .40, x_2 = .60, x_0 = .00$ is efficient:

$$\begin{aligned} \text{cov}(\tilde{r}_1, \tilde{r}_p) &= 0.092 \times 0.40 + 0.05 \times 0.60 = 0.046 \\ \text{cov}(\tilde{r}_2, \tilde{r}_p) &= 0.05 \times 0.40 + 0.09 \times 0.60 = 0.074, \\ \frac{0.046}{0.074} &= 2 = \frac{0.092}{0.148} \end{aligned}$$

portfolio with weights $x_1 = .20, x_2 = .30, x_0 = .50$ is efficient:

$$\begin{aligned} \text{cov}(\tilde{r}_1, \tilde{r}_p) &= 0.092 \times 0.20 + 0.05 \times 0.30 = 0.023 \\ \text{cov}(\tilde{r}_2, \tilde{r}_p) &= 0.05 \times 0.20 + 0.09 \times 0.30 = 0.037, \\ \frac{0.023}{0.037} &= 4 = \frac{0.092}{0.148} \end{aligned}$$

(In)Efficiency and Risk Aversion: examples



The Cost of
International Capital

P. Sercu,
International
Finance: Theory into
Practice

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution
How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

	$E(\tilde{r}_j - r)$	(co)variances	
Asset 1	0.092	$\text{cov}(\tilde{r}_1, \tilde{r}_1) = 0.04$	$\text{cov}(\tilde{r}_1, \tilde{r}_2) = 0.05$
Asset 2	0.148	$\text{cov}(\tilde{r}_2, \tilde{r}_1) = 0.05$	$\text{cov}(\tilde{r}_2, \tilde{r}_2) = 0.09$

portfolio with weights $x_1 = .40, x_2 = .40, x_0 = .20$ is not efficient:

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.036 \times 0.40 + 0.05 \times 0.40 = 0.036$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.05 \times 0.40 + 0.09 \times 0.40 = 0.056,$$

$$\frac{0.036}{0.092} = 2.555 \neq \frac{0.056}{0.148} = 2.643$$

portfolio with weights $x_1 = .40, x_2 = .60, x_0 = .00$ is efficient:

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.04 \times 0.40 + 0.05 \times 0.60 = 0.046$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.05 \times 0.40 + 0.09 \times 0.60 = 0.074,$$

$$\frac{0.046}{0.092} = 2 = \frac{0.074}{0.148}$$

portfolio with weights $x_1 = .20, x_2 = .30, x_0 = .50$ is efficient:

$$\text{cov}(\tilde{r}_1, \tilde{r}_p) = 0.04 \times 0.20 + 0.05 \times 0.30 = 0.023$$

$$\text{cov}(\tilde{r}_2, \tilde{r}_p) = 0.05 \times 0.20 + 0.09 \times 0.30 = 0.037,$$

$$\frac{0.023}{0.092} = 4 = \frac{0.037}{0.148}$$



The Market Portfolio as the Benchmark

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ Homogeneous expectations and opportunities:

- ▷ Peter's tangency portfolio is the same as Paul's and Mary's
- ▷ So everybody holds a combination of *the* tangency portfolio and riskfree assets
- ▷ So the aggregate portfolio of all investors is a combination of *the* tangency portfolio and riskfree assets, with a weight that reflects the average risk-aversion
- ▷ In a closed economy, this aggregate portfolio must be the portfolio of all existing shares ("the market portfolio").

◇ **Bottom line:** the market portfolio must be efficient too; so

$$\forall j: \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)} = \lambda_m,$$
$$\Rightarrow E(\tilde{r}_j - r) = \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m). \quad (1)$$



The Market Portfolio as the Benchmark

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

◇ Homogeneous expectations and opportunities:

- ▷ Peter's tangency portfolio is the same as Paul's and Mary's
- ▷ So everybody holds a combination of *the* tangency portfolio and riskfree assets
- ▷ So the aggregate portfolio of all investors is a combination of *the* tangency portfolio and riskfree assets, with a weight that reflects the average risk-aversion
- ▷ In a closed economy, this aggregate portfolio must be the portfolio of all existing shares ("the market portfolio").

◇ Bottom line: the market portfolio must be efficient too; so

$$\forall j: \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)} = \lambda_m,$$
$$\Rightarrow E(\tilde{r}_j - r) = \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m). \quad (1)$$



The Market Portfolio as the Benchmark

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

◇ Homogeneous expectations and opportunities:

- ▷ Peter's tangency portfolio is the same as Paul's and Mary's
- ▷ So everybody holds a combination of *the* tangency portfolio and riskfree assets
- ▷ So the aggregate portfolio of all investors is a combination of *the* tangency portfolio and riskfree assets, with a weight that reflects the average risk-aversion
- ▷ In a closed economy, this aggregate portfolio must be the portfolio of all existing shares ("the market portfolio").

◇ **Bottom line:** the market portfolio must be efficient too; so

$$\begin{aligned}\forall j: \quad \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)} &= \lambda_m, \\ \Rightarrow E(\tilde{r}_j - r) &= \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m). \quad (1)\end{aligned}$$



The Market Portfolio as the Benchmark

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ Homogeneous expectations and opportunities:

- ▷ Peter's tangency portfolio is the same as Paul's and Mary's
- ▷ So everybody holds a combination of *the* tangency portfolio and riskfree assets
- ▷ So the aggregate portfolio of all investors is a combination of *the* tangency portfolio and riskfree assets, with a weight that reflects the average risk-aversion
- ▷ In a closed economy, this aggregate portfolio must be the portfolio of all existing shares (“the market portfolio”).

◇ Bottom line: the market portfolio must be efficient too; so

$$\begin{aligned}\forall j: \quad \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)} &= \lambda_m, \\ \Rightarrow E(\tilde{r}_j - r) &= \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m). \quad (1)\end{aligned}$$



The Market Portfolio as the Benchmark

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ Homogeneous expectations and opportunities:

- ▷ Peter's tangency portfolio is the same as Paul's and Mary's
- ▷ So everybody holds a combination of *the* tangency portfolio and riskfree assets
- ▷ So the aggregate portfolio of all investors is a combination of *the* tangency portfolio and riskfree assets, with a weight that reflects the average risk-aversion
- ▷ In a closed economy, this aggregate portfolio must be the portfolio of all existing shares (“the market portfolio”).

◇ Bottom line: the market portfolio must be efficient too; so

$$\begin{aligned}\forall j : \quad \frac{E(\tilde{r}_j - r)}{\text{COV}(\tilde{r}_j, \tilde{r}_m)} &= \lambda_m, \\ \Rightarrow E(\tilde{r}_j - r) &= \lambda_m \text{COV}(\tilde{r}_j, \tilde{r}_m). \quad (1)\end{aligned}$$



Identifying λ_m —the CAPM

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Piece of cake. If $\forall j$: $\lambda_m = \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)}$, then for any portfolio p (with weights x_j):

$$\lambda_m = \frac{\sum_{j=1}^n x_j E(\tilde{r}_j - r)}{\sum_{j=1}^n x_j \text{cov}(\tilde{r}_j, \tilde{r}_m)} = \frac{E(\tilde{r}_p - r)}{\text{cov}(\tilde{r}_p, \tilde{r}_m)}$$

Now pick the market portfolio as our p . Then

$$\lambda_m = \frac{E(\tilde{r}_m - r)}{\text{cov}(\tilde{r}_m, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)}.$$

Substitute:

$$\begin{aligned} E(\tilde{r}_j - r) &= \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)} \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \beta_j E(\tilde{r}_m - r), \end{aligned}$$

with β_j as in $\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j$, the “market model”. $\beta_j \tilde{r}_m$ is the undiversifiable risk or market risk, ϵ_j the firm-specific or idiosyncratic or diversifiable risk.



Identifying λ_m —the CAPM

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Piece of cake. If $\forall j$: $\lambda_m = \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)}$, then for any portfolio p (with weights x_j):

$$\lambda_m = \frac{\sum_{j=1}^n x_j E(\tilde{r}_j - r)}{\sum_{j=1}^n x_j \text{cov}(\tilde{r}_j, \tilde{r}_m)} = \frac{E(\tilde{r}_p - r)}{\text{cov}(\tilde{r}_p, \tilde{r}_m)}$$

Now pick the market portfolio as our p . Then

$$\lambda_m = \frac{E(\tilde{r}_m - r)}{\text{cov}(\tilde{r}_m, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)}.$$

Substitute:

$$\begin{aligned} E(\tilde{r}_j - r) &= \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)} \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \beta_j E(\tilde{r}_m - r), \end{aligned}$$

with β_j as in $\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j$, the “market model”. $\beta_j \tilde{r}_m$ is the **undiversifiable risk** or **market risk**, ϵ_j the firm-specific or idiosyncratic or **diversifiable risk**.



Identifying λ_m —the CAPM

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Piece of cake. If $\forall j$: $\lambda_m = \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)}$, then for any portfolio p (with weights x_j):

$$\lambda_m = \frac{\sum_{j=1}^n x_j E(\tilde{r}_j - r)}{\sum_{j=1}^n x_j \text{cov}(\tilde{r}_j, \tilde{r}_m)} = \frac{E(\tilde{r}_p - r)}{\text{cov}(\tilde{r}_p, \tilde{r}_m)}$$

Now pick the market portfolio as our p . Then

$$\lambda_m = \frac{E(\tilde{r}_m - r)}{\text{cov}(\tilde{r}_m, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)}.$$

Substitute:

$$\begin{aligned} E(\tilde{r}_j - r) &= \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)} \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \beta_j E(\tilde{r}_m - r), \end{aligned}$$

with β_j as in $\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j$, the “market model”. $\beta_j \tilde{r}_m$ is the **undiversifiable risk** or **market risk**, ϵ_j the firm-specific or idiosyncratic or **diversifiable risk**.



Identifying λ_m —the CAPM

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution
How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

Piece of cake. If $\forall j$: $\lambda_m = \frac{E(\tilde{r}_j - r)}{\text{cov}(\tilde{r}_j, \tilde{r}_m)}$, then for any portfolio p (with weights x_j):

$$\lambda_m = \frac{\sum_{j=1}^n x_j E(\tilde{r}_j - r)}{\sum_{j=1}^n x_j \text{cov}(\tilde{r}_j, \tilde{r}_m)} = \frac{E(\tilde{r}_p - r)}{\text{cov}(\tilde{r}_p, \tilde{r}_m)}$$

Now pick the market portfolio as our p . Then

$$\lambda_m = \frac{E(\tilde{r}_m - r)}{\text{cov}(\tilde{r}_m, \tilde{r}_m)} = \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)}.$$

Substitute:

$$\begin{aligned} E(\tilde{r}_j - r) &= \lambda_m \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \frac{E(\tilde{r}_m - r)}{\text{var}(\tilde{r}_m)} \text{cov}(\tilde{r}_j, \tilde{r}_m), \\ &= \beta_j E(\tilde{r}_m - r), \end{aligned}$$

with β_j as in $\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j$, the “market model”. $\beta_j \tilde{r}_m$ is the **undiversifiable risk** or **market risk**, ϵ_j the firm-specific or idiosyncratic or **diversifiable risk**.



The CAPM for dummies: just using words

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

- ◇ The beta is a measure of the asset's relative risk—that is, the asset's market covariance risk $\text{cov}(\tilde{r}_j, \tilde{r}_m)$, rescaled by the portfolio's total risk, $\text{var}(\tilde{r}_m)$.

Beta can be estimated from the market-model regression.

- ◇ A risky asset with beta equal to zero should have an expected return that is equal to the risk-free rate, even if the asset's return is uncertain.

The reason is that the marginal contribution to the total market risk is zero.

- ◇ If an asset's beta or relative risk is non-zero, the asset's expected return should contain a risk premium. The additional return that can be expected per unit of beta is the market's expected excess return above the risk-free rate.



The CAPM for dummies: just using words

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

- ◇ The beta is a measure of the asset's relative risk—that is, the asset's market covariance risk $\text{cov}(\tilde{r}_j, \tilde{r}_m)$, rescaled by the portfolio's total risk, $\text{var}(\tilde{r}_m)$.

Beta can be estimated from the market-model regression.

- ◇ A risky asset with beta equal to zero should have an expected return that is equal to the risk-free rate, even if the asset's return is uncertain.

The reason is that the marginal contribution to the total market risk is zero.

- ◇ If an asset's beta or relative risk is non-zero, the asset's expected return should contain a risk premium. The additional return that can be expected per unit of beta is the market's expected excess return above the risk-free rate.



The CAPM for dummies: just using words

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

- ◇ The beta is a measure of the asset's relative risk—that is, the asset's market covariance risk $\text{cov}(\tilde{r}_j, \tilde{r}_m)$, rescaled by the portfolio's total risk, $\text{var}(\tilde{r}_m)$.

Beta can be estimated from the market-model regression.

- ◇ A risky asset with beta equal to zero should have an expected return that is equal to the risk-free rate, even if the asset's return is uncertain.

The reason is that the marginal contribution to the total market risk is zero.

- ◇ If an asset's beta or relative risk is non-zero, the asset's expected return should contain a risk premium. The additional return that can be expected per unit of beta is the market's expected excess return above the risk-free rate.



The CAPM for yuppies: APT interpretation

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ You can always avoid a stock's idiosyncratic risk:

- ▷ form j 's "shadow portfolio" \hat{j} : invest a weight β_j in m , and $(1 - \beta_j)$ riskfree
- ▷ now compare the returns on j and on \hat{j} :

$$\begin{aligned}\tilde{r}_j &= \alpha_j + \beta_j \tilde{r}_m + \epsilon_j, \\ \tilde{r}_{\hat{j}} &= (1 - \beta_j) r + \beta_j \tilde{r}_m.\end{aligned}$$

- ◇ You cannot seriously expect to be rewarded for a risk that you can easily avoid. So j and \hat{j} have the same expected return:

$$\begin{aligned}E(\tilde{r}_j) &\stackrel{\text{eq.}}{=} E(\tilde{r}_{\hat{j}}), \\ &= (1 - \beta_j) r + \beta_j E(\tilde{r}_m), \\ &= r + \beta_j E(\tilde{r}_m - r).\end{aligned}$$

CAPM: diversifiable risk gets no reward.



The CAPM for yuppies: APT interpretation

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ You can always avoid a stock's idiosyncratic risk:

- ▷ form j 's "shadow portfolio" \hat{j} : invest a weight β_j in m , and $(1 - \beta_j)$ riskfree
- ▷ now compare the returns on j and on \hat{j} :

$$\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j,$$

$$\tilde{r}_{\hat{j}} = (1 - \beta_j) r + \beta_j \tilde{r}_m.$$

- ◇ You cannot seriously expect to be rewarded for a risk that you can easily avoid. So j and \hat{j} have the same expected return:

$$\begin{aligned} E(\tilde{r}_j) &\stackrel{\text{eq.}}{=} E(\tilde{r}_{\hat{j}}), \\ &= (1 - \beta_j) r + \beta_j E(\tilde{r}_m), \\ &= r + \beta_j E(\tilde{r}_m - r). \end{aligned}$$

CAPM: diversifiable risk gets no reward.



The CAPM for yuppies: APT interpretation

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ You can always avoid a stock's idiosyncratic risk:

- ▷ form j 's "shadow portfolio" \hat{j} : invest a weight β_j in m , and $(1 - \beta_j)$ riskfree
- ▷ now compare the returns on j and on \hat{j} :

$$\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j,$$

$$\tilde{r}_{\hat{j}} = (1 - \beta_j) r + \beta_j \tilde{r}_m.$$

- ## ◇ You cannot seriously expect to be rewarded for a risk that you can easily avoid.
- So j and \hat{j} have the same expected return:

$$\begin{aligned} E(\tilde{r}_j) &\stackrel{\text{eq.}}{=} E(\tilde{r}_{\hat{j}}), \\ &= (1 - \beta_j) r + \beta_j E(\tilde{r}_m), \\ &= r + \beta_j E(\tilde{r}_m - r). \end{aligned}$$

CAPM: diversifiable risk gets no reward.



The CAPM for yuppies: APT interpretation

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ You can always avoid a stock's idiosyncratic risk:

- ▷ form j 's "shadow portfolio" \hat{j} : invest a weight β_j in m , and $(1 - \beta_j)$ riskfree
- ▷ now compare the returns on j and on \hat{j} :

$$\tilde{r}_j = \alpha_j + \beta_j \tilde{r}_m + \epsilon_j,$$

$$\tilde{r}_{\hat{j}} = (1 - \beta_j) r + \beta_j \tilde{r}_m.$$

- ## ◇ You cannot seriously expect to be rewarded for a risk that you can easily avoid.
- So j and \hat{j} have the same expected return:

$$\begin{aligned} E(\tilde{r}_j) &\stackrel{\text{eq.}}{=} E(\tilde{r}_{\hat{j}}), \\ &= (1 - \beta_j) r + \beta_j E(\tilde{r}_m), \\ &= r + \beta_j E(\tilde{r}_m - r). \end{aligned}$$

CAPM: diversifiable risk gets no reward.



How and when to use the single-country CAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

◇ Finding beta?

- ▷ single-asset betas are noisy; many prefer industry betas. See e.g. http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/Betas.htm
- ▷ intervaling effect: monthly-return beta estimates are higher than daily-return ones
- ▷ correct for thin trading: Scholes-Williams-Fowler-Rorke; Dimson;
- ▷ correct for leverage: CoCa assumed full-equity financing

◇ Finding the market risk premium?

- ▷ Take long-term estimates: 20 yrs is not enough
- ▷ Correct past data for inflation?
- ▷ Issue of survivorship bias
- ▷ Just assume 4 or 5%??

◇ When to use the model?

- ▷ When local mkt is segmented from RoW
- ▷ Or *assuming* that RoW is not important?



How and when to use the single-country CAPM

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

From Asset Returns to Portfolio
Return

The tangency solution

How the weights affect mean
and variance

How to make a portfolio
efficient

The Market Portfolio as the
Benchmark

The International
CAPM

Wrapping up

◇ Finding beta?

- ▷ single-asset betas are noisy; many prefer industry betas. See e.g. http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/Betas.htm
- ▷ intervaling effect: monthly-return beta estimates are higher than daily-return ones
- ▷ correct for thin trading: Scholes-Williams-Fowler-Rorke; Dimson;
- ▷ correct for leverage: CoCa assumed full-equity financing

◇ Finding the market risk premium?

- ▷ Take long-term estimates: 20 yrs is not enough
- ▷ Correct past data for inflation?
- ▷ Issue of survivorship bias
- ▷ Just assume 4 or 5%??

◇ When to use the model?

- ▷ When local mkt is segmented from RoW
- ▷ Or *assuming* that RoW is not important?



How and when to use the single-country CAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Wrapping up

◇ Finding beta?

- ▷ single-asset betas are noisy; many prefer industry betas. See e.g. http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/Betas.htm
- ▷ intervaling effect: monthly-return beta estimates are higher than daily-return ones
- ▷ correct for thin trading: Scholes-Williams-Fowler-Rorke; Dimson;
- ▷ correct for leverage: CoCa assumed full-equity financing

◇ Finding the market risk premium?

- ▷ Take long-term estimates: 20 yrs is not enough
- ▷ Correct past data for inflation?
- ▷ Issue of survivorship bias
- ▷ Just assume 4 or 5%??

◇ When to use the model?

- ▷ When local mkt is segmented from RoW
- ▷ Or *assuming* that RoW is not important?



Outline

The Cost of International Capital

P. Sercu,
International

Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

Translate first, then discount—or vv?

Two procedures

When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return

The tangency solution

How the weights affect mean and variance

How to make a portfolio efficient

The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?

Why Xrisk pops up in the InCAPM

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up



Why do we need an InCAPM?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ Standard country-by-country CAPM?

- ▷ Summing until the national level—e.g. Canada—the country's aggregate portfolio should be efficient in CAD terms,
- ▷ ... but it can no longer be equated to the stocks issued by local companies:
 - Canadian investors hold many foreign stocks
 - Many Canadian stocks are held abroad

◇ Aggregating to the world level solves this problem, but

- ▷ we might run into problems with “homogeneous opportunities”
- ▷ we do run into problems with “homogeneous expectations”
 - Canadian T-bill is risky to Americans, not to Canadians
 - US T-bill: vv—and these are just the most obvious examples

⇒ We need a CAPM that takes into account the fact that investors “think” in different currencies.



Why do we need an InCAPM?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ Standard country-by-country CAPM?

- ▷ Summing until the national level—e.g. Canada—the country's aggregate portfolio should be efficient in CAD terms,
- ▷ ... but it can no longer be equated to the stocks issued by local companies:
 - Canadian investors hold many foreign stocks
 - Many Canadian stocks are held abroad

◇ Aggregating to the world level solves this problem, but

- ▷ we might run into problems with “homogeneous opportunities”
 - ▷ we do run into problems with “homogeneous expectations”
 - Canadian T-bill is risky to Americans, not to Canadians
 - US T-bill: vv—and these are just the most obvious examples
- ⇒ We need a CAPM that takes into account the fact that investors “think” in different currencies.



Why do we need an InCAPM?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ **Standard country-by-country CAPM?**

- ▷ Summing until the national level—e.g. Canada—the country's aggregate portfolio should be efficient in CAD terms,
- ▷ ... but it can no longer be equated to the stocks issued by local companies:
 - Canadian investors hold many foreign stocks
 - Many Canadian stocks are held abroad

◇ **Aggregating to the world level solves this problem, but**

- ▷ we might run into problems with “homogeneous opportunities”
- ▷ we do run into problems with “homogeneous expectations”
 - Canadian T-bill is risky to Americans, not to Canadians
 - US T-bill: vv—and these are just the most obvious examples

⇒ We need a CAPM that takes into account the fact that investors “think” in different currencies.



Why do we need an InCAPM?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ **Standard country-by-country CAPM?**

- ▷ Summing until the national level—e.g. Canada—the country's aggregate portfolio should be efficient in CAD terms,
- ▷ ... but it can no longer be equated to the stocks issued by local companies:
 - Canadian investors hold many foreign stocks
 - Many Canadian stocks are held abroad

◇ **Aggregating to the world level solves this problem, but**

- ▷ we might run into problems with “homogeneous opportunities”
- ▷ we do run into problems with “homogeneous expectations”
 - Canadian T-bill is risky to Americans, not to Canadians
 - US T-bill: vv—and these are just the most obvious examples

⇒ **We need a CAPM that takes into account the fact that investors “think” in different currencies.**



Why Xrisk pops up in the InCAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ Obvious down-to-earth reasons:

- ▷ We need to translate FC returns into the investor's HC
- ▷ The FC T-bill even has \tilde{s} ($=\Delta S/S$) as its sole source of risk

◇ An analytical problem:

- ▷ Demand equations (or efficiency conditions) are in different currencies—say CAD and USD
- ▷ ... so how can we aggregate and find the link between aggregate demand and the world market portfolio w ?
- ▷ \Rightarrow we need to translate the US investor's efficiency condition (which is in USD) into CAD—or *vice versa*—before we can aggregate



Why Xrisk pops up in the InCAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ Obvious down-to-earth reasons:

- ▷ We need to translate FC returns into the investor's HC
- ▷ The FC T-bill even has \tilde{s} ($=\Delta S/S$) as its sole source of risk

◇ An analytical problem:

- ▷ Demand equations (or efficiency conditions) are in different currencies—say CAD and USD
- ▷ ... so how can we aggregate and find the link between aggregate demand and the world market portfolio w ?
- ▷ \Rightarrow we need to translate the US investor's efficiency condition (which is in USD) into CAD—or *vice versa*—before we can aggregate



Why Xrisk pops up in the InCAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ Obvious down-to-earth reasons:

- ▷ We need to translate FC returns into the investor's HC
- ▷ The FC T-bill even has \tilde{s} ($=\Delta S/S$) as its sole source of risk

◇ An analytical problem:

- ▷ Demand equations (or efficiency conditions) are in different currencies—say CAD and USD
- ▷ ... so how can we aggregate and find the link between aggregate demand and the world market portfolio w ?
- ▷ \Rightarrow we need to translate the US investor's efficiency condition (which is in USD) into CAD—or *vice versa*—before we can aggregate

How to translate returns and their moments?



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ Exact formula:

$$1 + r_j = \frac{\tilde{V}_{j,t+1}}{V_{j,t}} = \frac{\tilde{V}_{j,t+1}^* \tilde{S}_{t+1}}{V_{j,t}^* S_t} = (1 + \tilde{r}_j^*)(1 + \tilde{s}),$$
$$\Rightarrow 1 + r_j^* = \frac{1 + \tilde{r}_j}{1 + \tilde{s}}.$$

◇ Quadratic approximation

$$r_j^* = \tilde{r}_j - \tilde{s} - \tilde{r}_j \tilde{s} + \tilde{s}^2$$

Example

input data:		true r^*	linear approx	quadr approx
r	s	$(r - s)/(1 + s)$	$r - s$	$(r - s)(1 - s)$
0.1	0.000	0.1000	0.1000	0.1000
0.1	0.025	0.0732	0.0750	0.0731
0.1	0.050	0.0476	0.0500	0.0475
0.1	0.075	0.0233	0.0250	0.0231
0.1	0.100	0.0000	0.0000	0.0000
0.1	0.125	-0.0222	-0.0250	-0.0219
0.1	0.150	-0.0435	-0.0500	-0.0425
0.1	0.175	-0.0638	-0.0750	-0.0619
0.1	0.200	-0.0833	-0.1000	-0.0800

How to translate returns and their moments?



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ Exact formula:

$$1 + r_j = \frac{\tilde{V}_{j,t+1}}{V_{j,t}} = \frac{\tilde{V}_{j,t+1}^* \tilde{S}_{t+1}}{V_{j,t}^* S_t} = (1 + \tilde{r}_j^*)(1 + \tilde{s}),$$
$$\Rightarrow 1 + r_j^* = \frac{1 + \tilde{r}_j}{1 + \tilde{s}}.$$

◇ Quadratic approximation

$$r_j^* = \tilde{r}_j - \tilde{s} - \tilde{r}_j \tilde{s} + \tilde{s}^2$$

Example

input data:		true r^*	linear approx	quadr approx
r	s	$(r - s)/(1 + s)$	$r - s$	$(r - s)(1 - s)$
0.1	0.000	0.1000	0.1000	0.1000
0.1	0.025	0.0732	0.0750	0.0731
0.1	0.050	0.0476	0.0500	0.0475
0.1	0.075	0.0233	0.0250	0.0231
0.1	0.100	0.0000	0.0000	0.0000
0.1	0.125	-0.0222	-0.0250	-0.0219
0.1	0.150	-0.0435	-0.0500	-0.0425
0.1	0.175	-0.0638	-0.0750	-0.0619
0.1	0.200	-0.0833	-0.1000	-0.0800



How to translate returns and their moments?

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ Quadratic approximation

$$r_j^* = \tilde{r}_j - \tilde{s} - \tilde{r}_j \tilde{s} + \tilde{s}^2$$

◇ Approximation to mean & variance:

$$\begin{aligned} E(r_p^*) &= E(\tilde{r}_p) - E(\tilde{s}) - E(\tilde{r}_p \tilde{s}) + E(\tilde{s}^2) \\ &= E(\tilde{r}_p) - E(\tilde{s}) - [E(\tilde{r}_p)E(\tilde{s}) + \text{cov}(\tilde{r}_p, \tilde{s})] + [E(\tilde{s})^2 + \text{var}(\tilde{s})], \end{aligned}$$

$$\xrightarrow{d} E(\tilde{r}_p) - E(\tilde{s}) - \text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).$$

$$\text{var}(\tilde{r}_p^*) \xrightarrow{d} \text{var}(\tilde{r}_p - \tilde{s}) = \text{var}(\tilde{r}_p) - 2\text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).$$

Example: do Americans-in-Paris like USD exposure?

	Example 1: Positive covariance		Example 2: Negative covariance		Comment
W_{US} (in CAD)	12,000	16,000	12,000	16,000	same distribution for W_{US} ... and same distribution for S ; but the large-cov case has ...
S (CAD/USD)	1.00	1.50	1.50	1.00	
W_{US}^* (in USD)	12,000	10,667	8,000	16,000	– a lower mean W_{US}^* ... – and a lower stdev W_{US}^*
$E(W_{US}^*)$	11,333		12,000		
stdev(W_{US}^*)	667		4,000		



How to translate returns and their moments?

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ Quadratic approximation

$$r_j^* = \tilde{r}_j - \tilde{s} - \tilde{r}_j \tilde{s} + \tilde{s}^2$$

◇ Approximation to mean & variance:

$$\begin{aligned} E(r_p^*) &= E(\tilde{r}_p) - E(\tilde{s}) - E(\tilde{r}_p \tilde{s}) + E(\tilde{s}^2) \\ &= E(\tilde{r}_p) - E(\tilde{s}) - [E(\tilde{r}_p)E(\tilde{s}) + \text{cov}(\tilde{r}_p, \tilde{s})] + [E(\tilde{s})^2 + \text{var}(\tilde{s})], \end{aligned}$$

$$\xrightarrow{d} E(\tilde{r}_p) - E(\tilde{s}) - \text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).$$

$$\text{var}(\tilde{r}_p^*) \xrightarrow{d} \text{var}(\tilde{r}_p - \tilde{s}) = \text{var}(\tilde{r}_p) - 2\text{cov}(\tilde{r}_p, \tilde{s}) + \text{var}(\tilde{s}).$$

Example: do Americans-in-Paris like USD exposure?

	Example 1: Positive covariance		Example 2: Negative covariance		Comment
W_{US} (in CAD)	12,000	16,000	12,000	16,000	same distribution for W_{US} ... and same distribution for S ; but the large-cov case has ...
S (CAD/USD)	1.00	1.50	1.50	1.00	
W_{US}^* (in USD)	12,000	10,667	8,000	16,000	— a lower mean W_{US}^* ... — and a lower stdev W_{US}^*
$E(W_{US}^*)$	11,333		12,000		
$\text{stdev}(W_{US}^*)$	667		4,000		

Do assets have a financial nationality?



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

Consider

$$\tilde{r}_j = \delta_j + \gamma \tilde{s}_{\text{CAD/USD}} + \epsilon_j,$$

with γ = relative exposure (or elasticity of V wrt S).

Relative exposures of various assets

asset type	gamma	effect of rising USD
CAD TBill	0	no effect
USD TBill	1	one-to-one effect
Canadian importer	< 0	CAD price should fall
Canadian exporter	> 0	CAD price should rise
US exporter	< 1	USD price should fall
US importer	> 1	USD price should rise

Q: is gold, or oil, a dollar investment “because it is quoted in USD”?



Do assets have a financial nationality?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

Consider

$$\tilde{r}_j = \delta_j + \gamma \tilde{s}_{\text{CAD/USD}} + \epsilon_j,$$

with γ = relative exposure (or elasticity of V wrt S).

Relative exposures of various assets

asset type	gamma	effect of rising USD
CAD TBill	0	no effect
USD TBill	1	one-to-one effect
Canadian importer	< 0	CAD price should fall
Canadian exporter	> 0	CAD price should rise
US exporter	< 1	USD price should fall
US importer	> 1	USD price should rise

Q: is gold, or oil, a dollar investment “because it is quoted in USD”?



Do assets have a financial nationality?

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

Consider

$$\tilde{r}_j = \delta_j + \gamma \tilde{s}_{\text{CAD/USD}} + \epsilon_j,$$

with γ = relative exposure (or elasticity of V wrt S).

Relative exposures of various assets

asset type	gamma	effect of rising USD
CAD TBill	0	no effect
USD TBill	1	one-to-one effect
Canadian importer	< 0	CAD price should fall
Canadian exporter	> 0	CAD price should rise
US exporter	< 1	USD price should fall
US importer	> 1	USD price should rise

Q: is gold, or oil, a dollar investment “because it is quoted in USD”?

Do assets have a financial nationality?



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

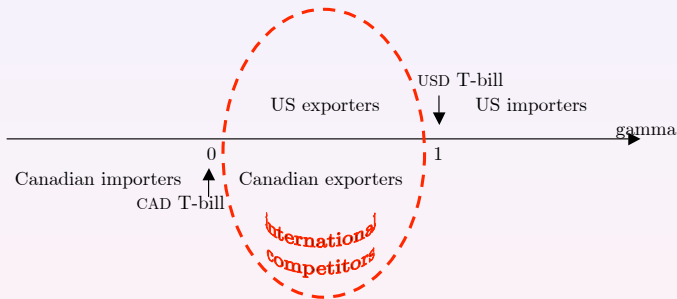
Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ The picture:



◇ Summary

- ▷ exposures are not either 0 (CDN) or 1 (US), but spread around these values
- ▷ lots of overlap in the middle: internationally competing firms have little nationality

Do assets have a financial nationality?



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

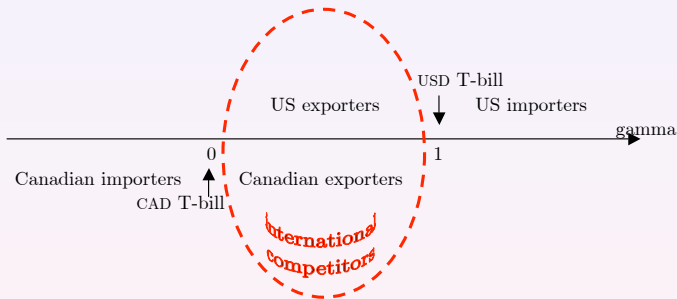
Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ The picture:



◇ Summary

- ▷ exposures are not either 0 (CDN) or 1 (US), but spread around these values
- ▷ lots of overlap in the middle: internationally competing firms have little nationality

Aggregating the Efficiency Conditions



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ The original conditions—a Babylonian chaos:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j^* - r^*) = \lambda \text{cov}(\tilde{r}_j^*, \tilde{r}_{p^*}).$$

◇ Peace and Harmony Restored:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1 - \lambda) \text{cov}(\tilde{r}_j, s).$$

◇ Aggregate (after weighting by invested wealths):

$$\text{world: } E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, s),$$

Proof:

$$\begin{aligned} W_{ca} E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p) \\ W_{us} E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{us} \tilde{r}_{p^*}) + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \hline (W_{ca} + W_{us}) E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p + W_{us} \tilde{r}_{p^*}) + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \Rightarrow E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \underbrace{\frac{W_{us}}{W_{ca} + W_{us}} (1 - \lambda)}_{\kappa} \text{cov}(\tilde{r}_j, s). \end{aligned}$$

Aggregating the Efficiency Conditions



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ The original conditions—a Babylonian chaos:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j^* - r^*) = \lambda \text{cov}(\tilde{r}_j^*, \tilde{r}_{p^*}).$$

◇ Peace and Harmony Restored:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1 - \lambda) \text{cov}(\tilde{r}_j, s).$$

◇ Aggregate (after weighting by invested wealths):

$$\text{world: } E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, s),$$

Proof:

$$\begin{aligned} W_{ca} E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p) \\ W_{us} E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{us} \tilde{r}_{p^*}) + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \hline (W_{ca} + W_{us}) E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p + W_{us} \tilde{r}_{p^*}) + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \Rightarrow E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \underbrace{\frac{W_{us}}{W_{ca} + W_{us}} (1 - \lambda)}_{\kappa} \text{cov}(\tilde{r}_j, s). \end{aligned}$$

Aggregating the Efficiency Conditions



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ The original conditions—a Babylonian chaos:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j^* - r^*) = \lambda \text{cov}(\tilde{r}_j^*, \tilde{r}_{p^*}).$$

◇ Peace and Harmony Restored:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1 - \lambda) \text{cov}(\tilde{r}_j, s).$$

◇ Aggregate (after weighting by invested wealths):

$$\text{world: } E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, s),$$

Proof:

$$\begin{aligned} W_{ca} E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p) \\ W_{us} E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{us} \tilde{r}_{p^*}) + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \hline (W_{ca} + W_{us}) E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p + W_{us} \tilde{r}_{p^*}) + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \Rightarrow E(\tilde{r}_j - r) &= \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \underbrace{\frac{W_{us}}{W_{ca} + W_{us}} (1 - \lambda)}_{\kappa} \text{cov}(\tilde{r}_j, s). \end{aligned}$$

Aggregating the Efficiency Conditions



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ The original conditions—a Babylonian chaos:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j^* - r^*) = \lambda \text{cov}(\tilde{r}_j^*, \tilde{r}_{p^*}).$$

◇ Peace and Harmony Restored:

$$\text{CDN's } p : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_p),$$

$$\text{US 's } p^* : \quad E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_{p^*}) + (1 - \lambda) \text{cov}(\tilde{r}_j, s).$$

◇ Aggregate (after weighting by invested wealths):

$$\text{world: } E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, s),$$

Proof:

$$\begin{aligned} \frac{W_{ca} E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p)}{W_{us} E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, W_{us} \tilde{r}_{p^*})} & \quad + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \hline (W_{ca} + W_{us}) E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, W_{ca} \tilde{r}_p + W_{us} \tilde{r}_{p^*}) & \quad + W_{us} (1 - \lambda) \text{cov}(\tilde{r}_j, s) \\ \Rightarrow E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) & \quad + \underbrace{\frac{W_{us}}{W_{ca} + W_{us}} (1 - \lambda) \text{cov}(\tilde{r}_j, s)}_{\text{"}\kappa\text{"}}. \end{aligned}$$



A 2-country InCAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ **Need to identify two prices of risk: λ , κ**

▷ So we need two benchmarks

▷ We wake the world market, and the USD T-bill

◇ **Let's cheat a bit** and assume that $\text{cov}(\tilde{r}_w, \tilde{s}) = 0$

▷ general: $E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, \tilde{s})$

▷ world: $E(\tilde{r}_w - r) = \lambda \text{cov}(\tilde{r}_w, \tilde{r}_w) + \kappa 0$

$$\Rightarrow \lambda = \frac{E(\tilde{r}_w - r)}{\text{var}(\tilde{r}_w)}$$

▷ USD TBill: $E(\tilde{s}_j + r^* - r) = \lambda 0 + \kappa \text{cov}(\tilde{s}, \tilde{s})$

$$\Rightarrow \kappa = \frac{E(\tilde{s} + r^* - r)}{\text{var}(\tilde{s})}$$

$$\Rightarrow \text{general: } E(\tilde{r}_j - r) = \frac{E(\tilde{r}_w - r)}{\text{var}(\tilde{r}_w)} \text{cov}(\tilde{r}_j, \tilde{r}_w) + \frac{E(\tilde{s} + r^* - r)}{\text{var}(\tilde{s})} \text{cov}(\tilde{r}_j, \tilde{s}) \\ = \beta_j E(\tilde{r}_w - r) + \gamma_j E(\tilde{s} + r^* - r)$$

◇ **The no-cheating model:** same, except that the regression slopes are from a *multiple* regression,

$$r_j = \alpha_{j,w,s} + \beta_{j;s} r_w + \gamma_{j;w\tilde{s}} \tilde{s} + \epsilon_{j;w,s}$$



A 2-country InCAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ Need to identify two prices of risk: λ , κ

- ▷ So we need two benchmarks
- ▷ We wake the world market, and the USD T-bill

◇ Let's cheat a bit and assume that $\text{cov}(\tilde{r}_w, \tilde{s}) = 0$

▷ general:
$$E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, \tilde{s})$$

▷ world:
$$E(\tilde{r}_w - r) = \lambda \text{cov}(\tilde{r}_w, \tilde{r}_w) + \kappa 0$$

$$\Rightarrow \lambda = \frac{E(\tilde{r}_w - r)}{\text{var}(\tilde{r}_w)}$$

▷ USD TBill:
$$E(\tilde{s}_j + r^* - r) = \lambda 0 + \kappa \text{cov}(\tilde{s}, \tilde{s})$$

$$\Rightarrow \kappa = \frac{E(\tilde{s} + r^* - r)}{\text{var}(\tilde{s})}$$

$$\begin{aligned} \Rightarrow \text{general: } E(\tilde{r}_j - r) &= \frac{E(\tilde{r}_w - r)}{\text{var}(\tilde{r}_w)} \text{cov}(\tilde{r}_j, \tilde{r}_w) + \frac{E(\tilde{s} + r^* - r)}{\text{var}(\tilde{s})} \text{cov}(\tilde{r}_j, \tilde{s}) \\ &= \beta_j E(\tilde{r}_w - r) + \gamma_j E(\tilde{s} + r^* - r) \end{aligned}$$

◇ The no-cheating model: same, except that the regression slopes are from a *multiple* regression,

$$r_j = \alpha_{j,w,s} + \beta_{j,s} r_w + \gamma_{j,w} \tilde{s} + \epsilon_{j,w,s}$$



A 2-country InCAPM

The Cost of International Capital

P. Sercu,
International Finance: Theory into Practice

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ **Need to identify two prices of risk: λ , κ**

- ▷ So we need two benchmarks
- ▷ We wake the world market, and the USD T-bill

◇ **Let's cheat a bit** and assume that $\text{cov}(\tilde{r}_w, \tilde{s}) = 0$

▷ general:
$$E(\tilde{r}_j - r) = \lambda \text{cov}(\tilde{r}_j, \tilde{r}_w) + \kappa \text{cov}(\tilde{r}_j, \tilde{s})$$

▷ world:
$$E(\tilde{r}_w - r) = \lambda \text{cov}(\tilde{r}_w, \tilde{r}_w) + \kappa 0$$

$$\Rightarrow \lambda = \frac{E(\tilde{r}_w - r)}{\text{var}(\tilde{r}_w)}$$

▷ USD TBill:
$$E(\tilde{s}_j + r^* - r) = \lambda 0 + \kappa \text{cov}(\tilde{s}, \tilde{s})$$

$$\Rightarrow \kappa = \frac{E(\tilde{s} + r^* - r)}{\text{var}(\tilde{s})}$$

$$\Rightarrow \text{general: } E(\tilde{r}_j - r) = \frac{E(\tilde{r}_w - r)}{\text{var}(\tilde{r}_w)} \text{cov}(\tilde{r}_j, \tilde{r}_w) + \frac{E(\tilde{s} + r^* - r)}{\text{var}(\tilde{s})} \text{cov}(\tilde{r}_j, \tilde{s})$$

$$= \beta_j E(\tilde{r}_w - r) + \gamma_j E(\tilde{s} + r^* - r)$$

◇ **The no-cheating model:** same, except that the regression slopes are from a *multiple* regression,

$$r_j = \alpha_{j,w,s} + \beta_{j;s} r_w + \gamma_{j;w\tilde{s}} \tilde{s} + \epsilon_{j;w,s}$$



The Solnik-Sercu-Adler-Dumas model

The Cost of International Capital

P. Sercu, *International Finance: Theory into Practice*

Translate first, then discount—or vv?

The Single-Country CAPM

The International CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial nationality?

Aggregating the Efficiency Conditions

The InCAPM

Wrapping up

◇ N countries

$$E(r_j - r) = \beta_{j,..}E(r_w - r) + \gamma_{j,1;..}E(\tilde{s}_1 + r_1^* - r) + \gamma_{j,2;..}E(\tilde{s}_2 + r_2^* - r) + \dots + \gamma_{j,n;..}E(\tilde{s}_n + r_n^* - r),$$

$$r_j = \alpha_{j,w;\underline{s}} + \beta_{j,w;..}r_w + \gamma_{j,s_1;..}\tilde{s}_1 + \gamma_{j,s_2;..}\tilde{s}_2 + \dots + \gamma_{j,s_n;..}\tilde{s}_n + \epsilon_{j,w;\underline{s}}.$$

◇ Restricted versions (1): trim the list of countries

because γ s are hard to estimate and Tbill premia small

▷ small countries' exchange rates have a small risk premium

$$\kappa_l = \frac{W_l}{\sum_{\forall k} W_k} (1 - \lambda) < 1 - \lambda$$

▷ unconnected countries (no X, M links; no competitors): cov must be close to zero anyway

◇ Restricted Version (2): cut out all Xrisk terms, just keep the beta multivariate

▷ Assumes that Tbill premia are zero in the long run

▷ So produces almost a world CAPM, except for the beta



The Solnik-Sercu-Adler-Dumas model

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ **N countries**

$$\begin{aligned} E(r_j - r) &= \beta_{j,..} E(r_w - r) + \gamma_{j,1;..} E(\tilde{s}_1 + r_1^* - r) + \gamma_{j,2;..} E(\tilde{s}_2 + r_2^* - r) \\ &\quad + \dots \gamma_{j,n;..} E(\tilde{s}_n + r_n^* - r), \\ r_j &= \alpha_{j,w;\underline{s}} + \beta_{j,w;..} r_w + \gamma_{j,s_1;..} \tilde{s}_1 + \gamma_{j,s_2;..} \tilde{s}_2 + \dots \gamma_{j,s_n;..} \tilde{s}_n + \epsilon_{j,w;\underline{s}}. \end{aligned}$$

◇ **Restricted versions (1): trim the list of countries** because γ s are hard to estimate and Tbill premia small

- ▷ small countries' exchange rates have a small risk premium

$$\kappa_l = \frac{W_l}{\sum_{\forall k} W_k} (1 - \lambda) < 1 - \lambda$$

- ▷ unconnected countries (no X, M links; no competitors): cov must be close to zero anyway

◇ **Restricted Version (2):** cut out all Xrisk terms, just keep the beta multivariate

- ▷ Assumes that Tbill premia are zero in the long run
- ▷ So produces almost a world CAPM, except for the beta



The Solnik-Sercu-Adler-Dumas model

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Why do we need an InCAPM?

Why does it contain Xrisk?

Do assets have a financial
nationality?

Aggregating the Efficiency
Conditions

The InCAPM

Wrapping up

◇ **N countries**

$$\begin{aligned} E(r_j - r) &= \beta_{j,..} E(r_w - r) + \gamma_{j,1;..} E(\tilde{s}_1 + r_1^* - r) + \gamma_{j,2;..} E(\tilde{s}_2 + r_2^* - r) \\ &\quad + \dots \gamma_{j,n;..} E(\tilde{s}_n + r_n^* - r), \\ r_j &= \alpha_{j,w;\underline{s}} + \beta_{j,w;..} r_w + \gamma_{j,s_1;..} \tilde{s}_1 + \gamma_{j,s_2;..} \tilde{s}_2 + \dots \gamma_{j,s_n;..} \tilde{s}_n + \epsilon_{j,w;\underline{s}}. \end{aligned}$$

◇ **Restricted versions (1): trim the list of countries** because γ s are hard to estimate and Tbill premia small

- ▷ small countries' exchange rates have a small risk premium

$$\kappa_l = \frac{W_l}{\sum_{\forall k} W_k} (1 - \lambda) < 1 - \lambda$$

- ▷ unconnected countries (no X, M links; no competitors): cov must be close to zero anyway

◇ **Restricted Version (2):** cut out all Xrisk terms, just keep the beta multivariate

- ▷ Assumes that Tbill premia are zero in the long run
- ▷ So produces almost a world CAPM, except for the beta



Outline

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

Translate first, then discount—or vv?

Two procedures
When to do what?

The Single-Country CAPM

From Asset Returns to Portfolio Return
The tangency solution
How the weights affect mean and variance
How to make a portfolio efficient
The Market Portfolio as the Benchmark

The International CAPM

Why do we need an InCAPM?
Why Xrisk pops up in the InCAPM
Do assets have a financial nationality?
Aggregating the Efficiency Conditions
The InCAPM

Wrapping up



Practical implications

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ When do we use what model?

- ▷ Is host a segmented market? immaterial to the foreign investor!
- ▷ Is this a domestic or a foreign investment? immaterial!
- ▷ Is home a segmented market?
 - if yes: CAPM
 - if no: InCAPM, with “world” defined as market’s total portfolio

◇ When do we use what currency?

- ▷ home and host integrated? use either HC or FC
- ▷ home and host not integrated: use HC (home)

◇ Estimated risks and returns

- ▷ Market: use long-term, lowish estimates
- ▷ beta: use your priors too, or even exclusively
 - capital goods v consumer goods; operating leverage; growth opportunities
- ▷ gamma: use your common sense, incl. setting many =0
- ▷ TBill premia: tiny and hard to estimate



Practical implications

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ When do we use what model?

- ▷ Is host a segmented market? immaterial to the foreign investor!
- ▷ Is this a domestic or a foreign investment? immaterial!
- ▷ Is home a segmented market?
 - if yes: CAPM
 - if no: InCAPM, with “world” defined as market’s total portfolio

◇ When do we use what currency?

- ▷ home and host integrated? use either HC or FC
- ▷ home and host not integrated: use HC (home)

◇ Estimated risks and returns

- ▷ Market: use long-term, lowish estimates
- ▷ beta: use your priors too, or even exclusively
 - capital goods v consumer goods; operating leverage; growth opportunities
- ▷ gamma: use your common sense, incl. setting many =0
- ▷ TBill premia: tiny and hard to estimate



Practical implications

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ When do we use what model?

- ▷ Is host a segmented market? immaterial to the foreign investor!
- ▷ Is this a domestic or a foreign investment? immaterial!
- ▷ Is home a segmented market?
 - if yes: CAPM
 - if no: InCAPM, with “world” defined as market’s total portfolio

◇ When do we use what currency?

- ▷ home and host integrated? use either HC or FC
- ▷ home and host not integrated: use HC (home)

◇ Estimated risks and returns

- ▷ Market: use long-term, lowish estimates
- ▷ beta: use your priors too, or even exclusively
 - capital goods v consumer goods; operating leverage; growth opportunities
- ▷ gamma: use your common sense, incl. setting many =0
- ▷ TBill premia: tiny and hard to estimate



Practical implications

The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ When do we use what model?

- ▷ Is host a segmented market? immaterial to the foreign investor!
- ▷ Is this a domestic or a foreign investment? immaterial!
- ▷ Is home a segmented market?
 - if yes: CAPM
 - if no: InCAPM, with “world” defined as market’s total portfolio

◇ When do we use what currency?

- ▷ home and host integrated? use either HC or FC
- ▷ home and host not integrated: use HC (home)

◇ Estimated risks and returns

- ▷ Market: use long-term, lowish estimates
- ▷ beta: use your priors too, or even exclusively
 - capital goods v consumer goods; operating leverage; growth opportunities
- ▷ gamma: use your common sense, incl. setting many =0
- ▷ TBill premia: tiny and hard to estimate

Practical implications



The Cost of
International Capital

P. Sercu,
*International
Finance: Theory into
Practice*

Translate first, then
discount—or vv?

The Single-Country
CAPM

The International
CAPM

Wrapping up

◇ When do we use what model?

- ▷ Is host a segmented market? immaterial to the foreign investor!
- ▷ Is this a domestic or a foreign investment? immaterial!
- ▷ Is home a segmented market?
 - if yes: CAPM
 - if no: InCAPM, with “world” defined as market’s total portfolio

◇ When do we use what currency?

- ▷ home and host integrated? use either HC or FC
- ▷ home and host not integrated: use HC (home)

◇ Estimated risks and returns

- ▷ Market: use long-term, lowish estimates
- ▷ beta: use your priors too, or even exclusively
 - capital goods v consumer goods; operating leverage; growth opportunities
- ▷ gamma: use your common sense, incl. setting many =0
- ▷ TBill premia: tiny and hard to estimate