Fore!

3.2

How many golf balls would it take to circle the Earth at the equator?*
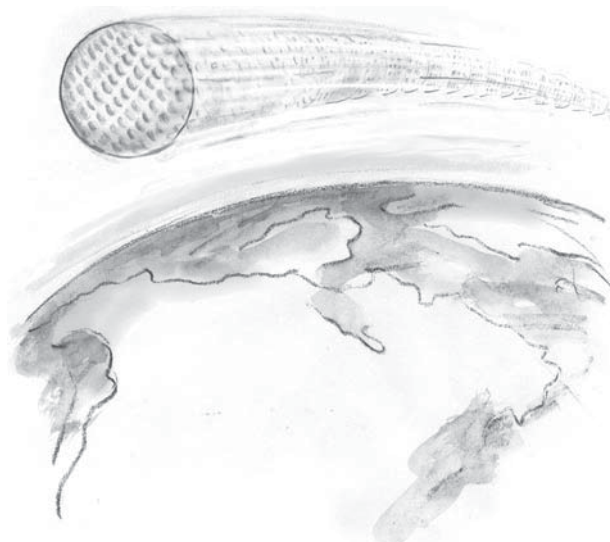
* Thanks to Tom Isenhour for this question [6]. Can we have that raise now, Tom?

HINT: What's the diameter of a golf ball? Remember that 1 inch is 2.5 cm.

HINT: What is the circumference of the Earth?

HINT: If you remember the radius then the circumference is $c = 2\pi R$. (If you don't remember the radius, c is still $2\pi R$ but the formula is a lot less useful.)

HINT: There is a 3-hour time difference between LA and NY. There are 24 time zones total.

HINT: The Earth's circumference is eight times the distance from NY to LA. You can fly that distance in 6 hours.

✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳

How many golf balls would it take to circle the Earth at the equator?*

ANSWER: To answer this question we need the diameter of a golf ball and the circumference of the Earth. Let's start with the easier part. A golf ball has a diameter of a bit less than 2 inches or about 4 cm.

There are several ways to estimate the circumference of the Earth. For example, there is a three-hour time difference between New York and Los Angeles and there are 24 times zones covering the Earth. Therefore, the circumference of the Earth is about eight times the distance from NY to LA. If you don't remember that the distance between them is 3000 miles, then you can estimate it from the fact that it takes about six hours to fly from NY to LA and a modern jet flies at about 500 mph. Thus, the circumference is about $c = 8 \times 3000 \, \text{mi} = 2.4 \times 10^4 \, \text{mi}$.

Alternatively, we know that passenger jets fly slower than the Earth rotates (since you always arrive after you leave [in local time]) and that some military jets can fly faster than the Earth rotates. Since passenger jets fly at about 500 mph and military jets can fly up to 2000 mph, we can estimate the Earth's rotation at 1000 mph. Since the Earth rotates completely in 24 hours, its circumference must be $c = 24 \times 1000 \, \text{mph} = 2.4 \times 10^4 \, \text{mi}$.

Of course, if you remembered that the circumference is 25,000 miles (or 40,000 km) or that the radius of the Earth is 4000 miles (6400 km) and the circumference is $c = 2\pi R$, then you didn't need to estimate it.

Now the arithmetic is straightforward. First we need to convert the circumference of the Earth from ters to centimeters. The number of golf balls is

$$N = 4 \times 10^4 \, \text{km} \times \frac{10^3 \, \text{m}}{1 \, \text{km}} \times \frac{10^2 \, \text{cm}}{1 \, \text{m}} \times \frac{1 \, \text{golf ball}}{4 \, \text{cm}}$$

$$10^9 \, \text{golf balls}$$

The Pacific Ocean is a very large water hazard. It would be extremely irritating to lose a billion golf balls in the water! We'd better use the special kind that floats.

This also provides an interesting peg on which to hang the concept of "parts per billion" (ppb). If the air contains so many ppb of some potentially toxic substance, that is about the number of, say, red golf balls in the otherwise white ones surrounding the Earth. You could walk along the equator for months before finding your first red golf ball.

Tons of Trash

3.8

How much domestic trash
is collected each year in
the US (in m^3 or tons)?
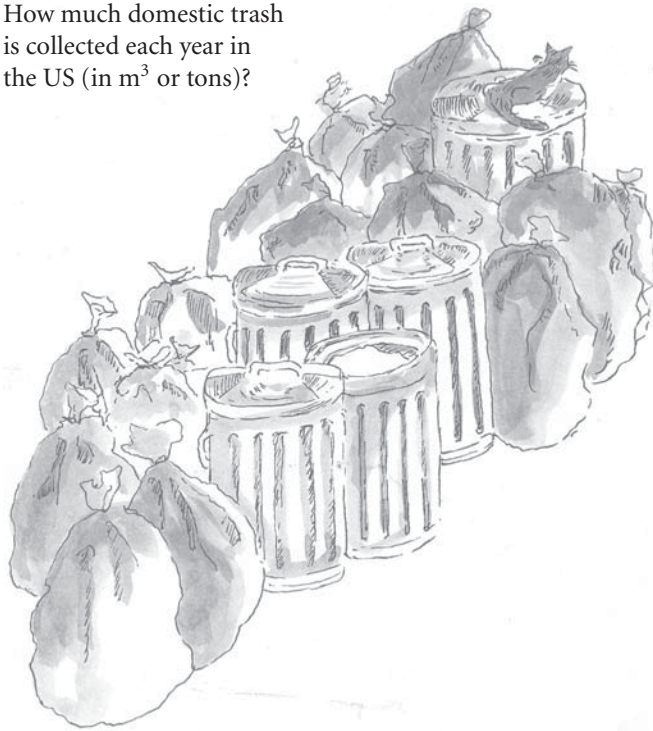
HINT: How much trash do you throw out each week?

HINT: A kitchen garbage bag is 13 gal (50 L) but can be
compacted.

HINT: Estimate how many homes there are in the US.

✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳ ✳

Tons of Trash

How much domestic trash is collected each year in the US (in m^3 or tons)?

ANSWER: Before the era of recycling (and when the children lived at home), we used to empty our 13-gallon trash can in the kitchen about every other day. Now, it's only about once a week, though the recycling bin now gets emptied once a week (with newspapers, boxes and other cardboard containers, bottles, plastic containers, cans, etc.). On second thought, it's easier to lump them all together for this problem. Since considering recycling changes the answer by less than a factor of two for our household, we will ignore it for this problem.

OK, if we empty the trash three or four times a week, that's about 50 gallons of trash for four people. Now a gallon is about 4 liters, and a liter is $10^{-3}\,m^3$, so our 50 gallons per week is 200 liters or $0.2\,m^3$. In one year (50 weeks), four people produce $50 \times 0.2 = 10\,m^3$ of garbage. That is about 300 cubic feet. Yick!

It's worse than that. There are 3×10^8 of us, or about 10^8 households, so we produce $10^9\,m^3$ of uncompacted garbage.

Now let's try to figure out the mass of that garbage. There are two things to consider here: first, trash is mostly not liquid (ooh, you threw the soup away!), and therefore there's a lot of air space in the trash bag, and second, related to it, the density of the trash is much less than that of water. Let's estimate the density. That full 13-gal (50-L) trash bag probably only weighs between 10 and 20 lb (5 and 10 kg). Thus, its density is between 0.1 and 0.2 kg/L (or 0.1 to 0.2 tons/m^3 or between 10 and 20% that of water).

Let's take an average density of 0.2 tons/m^3. Thus, in one year, my family produced $m = 10\,m^3 \times 0.2\,tons/m^3 = 2$ tons of garbage (in US units, that is, um, 2 tons).* Note that, since the average density is so low, compacting the trash in garbage trucks should reduce the volume by a factor of about three (more than one and less than five).

But enough about us, let's look at the whole country. With a population of 3×10^8, we produce a total mass and compacted volume of trash of about

$$M = 10^8\,\text{households} \times \frac{2\,\text{tons/year}}{\text{household}}$$
$$= 2 \times 10^8\,\text{tons of trash/year}$$

$$V = 10^8\,\text{households} \times \frac{1}{3} \times \frac{10\,m^3/\text{year}}{\text{household}}$$
$$= 3 \times 10^8\,m^3\,\text{of trash/year}$$

Now let's compare to reality. According to the US Environmental Protection Agency [9], in 2005 the US generated 245 million (2.45×10^8) tons of municipal solid waste (including recycling).
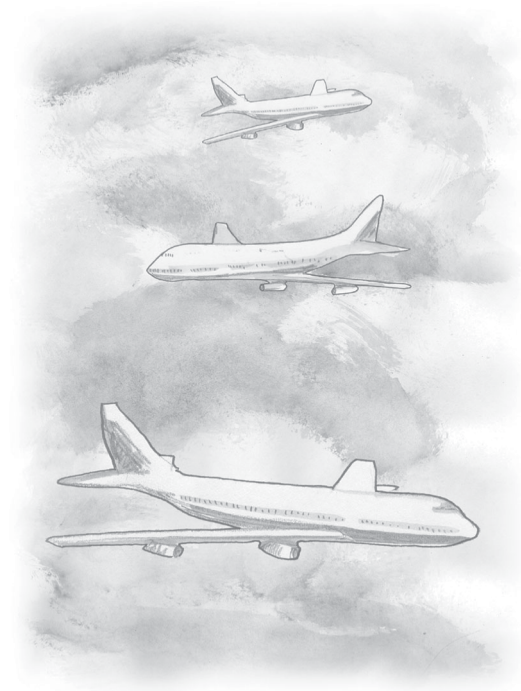
Now we need to figure out what to do with it all. But that's the subject of the next question.

* There are lots of different tons: metric, short, long, ... Since they differ by only 10%, we will use them interchangeably.

Juggling People

3.10

On average, how many people are airborne over the US at any given moment?

HINT: Don't choose 3:00 AM, choose sometime during the day.

HINT: Think about the fraction of time you spend flying, e.g., the number of hours or days you fly per, year compared the number of hours or days in a year.

HINT: The fraction of time people spend flying is equal to the fraction of people flying at any given time.

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On average, how many people are airborne over the US at any given moment?

ANSWER: There are two basic ideas here. First, the fraction of time the average person spends flying equals the average fraction of people that are airborne at any instant. This means that if you spend 10% of your time flying, then on average 10% of the population is airborne at any given time.* Note that this only works if there are enough people to average things out.† Second, we can use our own experience to estimate the fraction of time an average person spends in the air, or shopping or sleeping or (you name it). In other words,

$$\frac{\text{number flying now}}{\text{US population}} = \frac{\text{time spent flying}}{1\text{yr}}$$

Back in chapter 1 we estimated that the average American takes between two and four flights per year. The typical flight will take between one and six hours (not counting time spent parking, waiting in lines, consuming the delectable airport comestibles, . . .) so we will estimate three flights per year at three hours per flight, or nine hours per year in flight. Now we insert the numbers we know:

$$\frac{\text{number flying now}}{3 \times 10^8 \text{people}} = \frac{9\,\text{hr}}{400\,\text{days} \times 25\,\text{hr/day}}$$

We rearrange this to get

$$\text{number flying now} = 3 \times 10^8 \text{people} \times \frac{9\,\text{hr}}{10^4\,\text{hr}}$$

$$= 3 \times 10^5 \text{people}$$

That means there are about three hundred thousand people airborne over the US at this moment. I hope they all land safely.
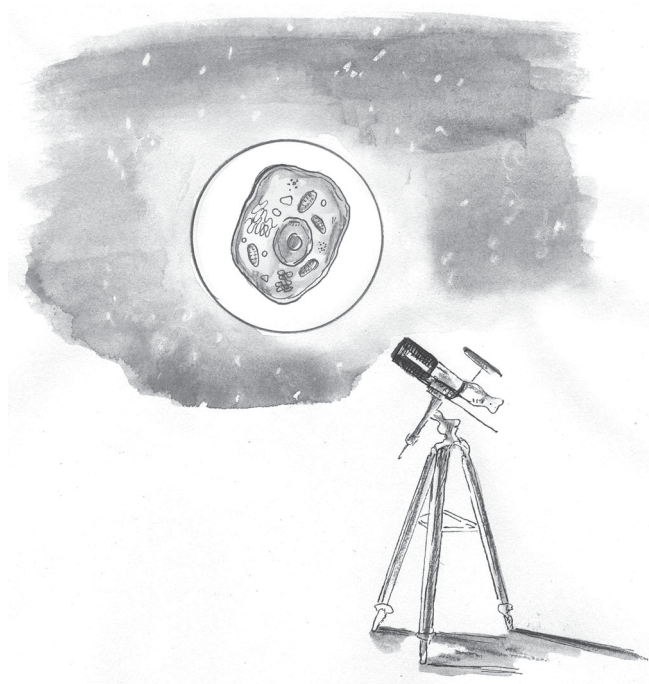
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* This does **not** mean that if you spend 10% of your time flying, that 10% of the average person (that's about one leg) is airborne.
† One other person, or even ten others wouldn't suffice. There have to be enough people so that at every moment some are in the air. This is not a problem for this question.

More Numerous Than the Stars in the Sky

How many cells are there in the human body?

❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋

HINT: How might you estimate your volume?

HINT: Volume = mass/density. What is your mass? The density of water is 1 kg/L or 10^3 kg/m^3

HINT: Alternatively, estimate your volume as a rectangular box.

HINT: A really big cell is about the smallest object you can see.

How many cells are there in the human body?

ANSWER: Sorry to ask such a personal question, since we hardly know one another, but what's your volume? It's on your driver's licence, along with your surface area ... oops, we were being futuristic. Since you didn't answer the question, let's figure it out. Let's estimate your mass, Mr. Jones, at a nice even 100 kg (or approximately 200 lb; ladies, you can modify this to suit your own figures). Since we can safely assume that you float,* your average density is rather close to that of water or about 1 kg/L or 10^3 kg/m^3. Thus, 100 kg of water occupies 100 kg $\times$ $(1\,\text{m}^3/10^3\,\text{kg})$ = 0.1 m^3. So your volume, Sir, is about 0.1 m^3.

We can do this another way. Let's approximate our body as a box of length l, width w, and height h, so our volume is $V = l \times w \times h$. What choices shall we make for these quantities? Length is easy, $l \approx 6$ ft. Now what about w and h? John's cross section is decidedly not rectangular, but since we're not going to say what shape it is,† we'll pick an average width, $w = 1$ ft (remember to average over head, neck, torso, legs, and feet). As for h, the front–back dimension, it will be about, say, 6 in. (he's not barrel-chested). Therefore, his volume is $6 \times 1 \times \frac{1}{2} = 3$ ft^3. Now 1 m^3 is about $3 \times 3 \times 3 = 27$ ft^3, so 3 ft^3 is about 0.1 m^3.

Now let's figure out the size of a cell using our own eyes. We cannot generally see individual cells with the unaided eye. If you look at a ruler, the lines on the ruler are a fraction of a millimeter (10^{-3} m) wide. I can quite easily see something one-tenth of a millimeter (10^{-4} m) in size. I cannot see cells, so they must be smaller than that. The inventor of the microscope used his first crude microscope (with a magnification between 10 and 100) to see cells. Thus, a typical cell must be 10 to 100 times smaller

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* That's easy for us to say—we're standing at the side of the pool.
† *Note*: We are dealing with *round* numbers.

than 10^{-4} m or between 10^{-5} and 10^{-6} m (that's 1 to 10 μm [micrometer]) in size.

Here's another approach. We can see cells with an ordinary light microscope.* This means that the cells must be larger than the wavelength of visible light or we couldn't see them. The wavelength of visible light ranges from blue light at about $0.4\,\mu$m to red light at about $0.7\,\mu$m (400 to 700 nm). Thus, cells must be larger than 1 μm. However, while we can see the major features inside a cell, we cannot see that much detail, so that typical cells must be much smaller than about 100 μm.

A typical human cell with a diameter of 10 μm $(10 \times 10^{-6} \text{ m} = 1 \times 10^{-5} \text{ m})$ will have a volume

$$V_{\text{cell}} \approx \text{diameter}^3 = (10^{-5}\,\text{m})^3 = 10^{-15}\,\text{m}^3$$

Now the number of cells in our body is just the ratio of the volumes:

$$N_{\text{cells}} = \frac{V_{\text{body}}}{V_{\text{cell}}} = \frac{10^{-1}\,\text{m}^3}{10^{-15}\,\text{m}^3} = 10^{14}$$

This means that you, Sir, and I each have about 100 trillion cells in our bodies. Goodness me! That's about a thousand times as many stars as reside in our galaxy. Start counting . . . and Mr. Lucas? It's time for the first "Cell Wars" trilogy . . .
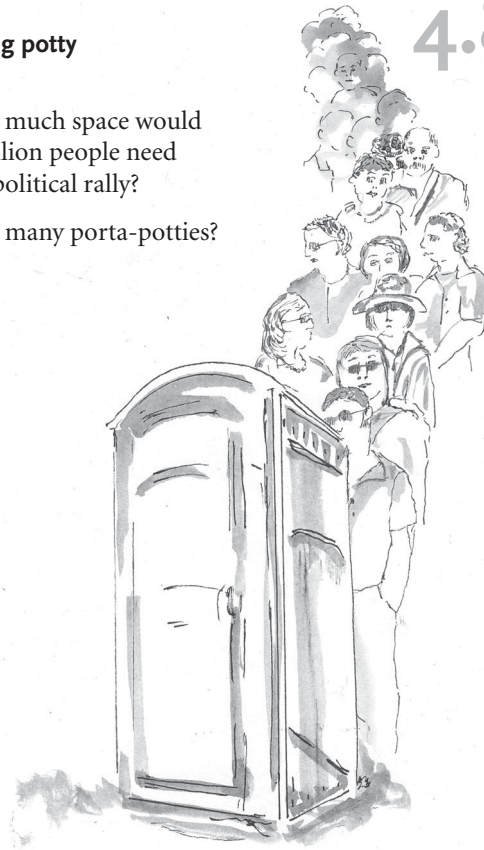
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* A light microscope uses light rather than, say, electrons to "see" with. It may still be rather heavy.

Going potty

4.8

How much space would
a million people need
at a political rally?

How many porta-potties?

* * * * * * * * * * * * * * *

HINT: How much elbow room do you have at a political rally?
How many people are crammed into a typical square meter?

HINT: The fraction of people in the porta-potty equals the
fraction of time each them spends in that indelicate activity.

HINT: What fraction of the time do you spend in the loo?

Return to book description

Going potty

4.8

How much space would a million people need at a political rally? How many porta-potties?

ANSWER: In a political rally, people are packed reasonably tightly together. There is still space to move around, so we're not packed as tightly as in the very first question. Let's estimate that there's about two feet between people. Since each person is about one foot in size, each person occupies a space that is about 3 ft by 3 ft or $1\,m^2$. That means that 10^6 people occupy $10^6\,m^2$ or $1\,km^2$. That is about the size of the National Mall in Washington, DC or Central Park in New York. Now that you have all those people there, you need to provide facilities for inputs and outputs, that is to say refreshments and porta-potties.

Again, as with the flighty Americans or the nose picking, the fraction of the time I spend in some activity (except for bungee-jumping: none!) is equal to the fraction of people doing it right now. So: how long do you spend in the bathroom during the daytime? Ignore time spent grooming or powdering your nose. One minute is probably too short and 100 minutes is far too long ("Are you going to be in there ALL day?"). So let's settle on 10 minutes, shall we? Also, let's just take a 15-hr day because most people don't go to political rallies while they are sleeping. Let's convert 15 hours to $15 \times 60 = 10^3$ minutes (so it has the same units as our bathroom interval). Thus, the fraction of daytime spent in the bathroom (and hence the fraction of people in the bathroom at any instant) is 10 minutes out of 10^3 minutes or 1%.

This means that we need one potty for every 100 people, or a total of 10^4 potties for 10^6 people. Wow, that's a huge logistical effort! Note that if there is one potty for every 100 people, given that I spend 1/100 of my time there, then all the potties will be 100% occupied during the entire rally. Since the demand for potties is generally not uniform, there will be serious queuing problems (that is British for "really long lines"). So a few more might be advantageous . . .

Let's get one thing straight!

4.9

How long is all the DNA
in your body?
How long is the DNA
of all humanity?

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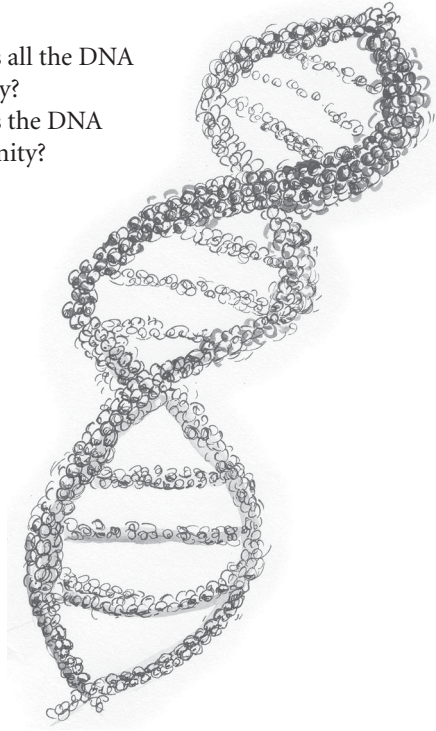
HINT: How much DNA is in one cell?

HINT: DNA is composed of long strings of "base pairs," each composed of about 1000 atoms.

HINT: One atom is about 10^{-10} m in size.

HINT: Each base pair is approximately a cube of side length 10^{-9} m.

HINT: The nucleus of a cell is filled with DNA and is about 1/10 the size of the cell.

HINT: We just estimated the number of cells several questions ago.

How long is all the DNA in your body?
How long is the DNA of all humanity?

ANSWER: This one is a little complicated. We need to calculate the size of the building blocks of DNA and the total volume of DNA in the cell. Then we can use that to figure out how long the total DNA is.

The nucleus of a cell is filled primarily with long strands of DNA called chromosomes.* Each chromosome is composed of long strings of base pairs (the familiar letters $ATGC$ from a long forgotten biology course). Each base pair is a very complicated molecule itself. This means that it must contain a lot of atoms, certainly more than 100 and less than 10^4, so we'll estimate 10^3 atoms per base pair. We will treat each base pair as a cube, 10 atoms on a side. Since all atoms are about 10^{-10} m, our base pairs are 10^{-9} m in length. The volume of a base pair is then the length cubed:

$$V_{bp} = (10^{-9}\,\text{m})^3 = 10^{-27}\,\text{m}^3$$

We estimated earlier that a cell is about 10^{-5} m in size. The nucleus is about 1/10 of that, or 10^{-6} m. Thus, the volume of the nucleus is

$$V_n = (10^{-6}\,\text{m})^3 = 10^{-18}\,\text{m}^3$$

This means that each nucleus can contain a number of base pairs

$$N_{bp} = \frac{V_n}{V_{bp}} = \frac{10^{-18}\,\text{m}^3}{10^{-27}\,\text{m}^3} = 10^9$$

That is one billion base pairs. That's a LOT of information.

Now let's straighten out all that DNA. There are 10^9 base pairs at 10^{-9} m each so the total length of DNA in the cell is about 1 m. According to biology textbooks, the length is between 1 and 3 m [12, 13].

Now we estimated previously that there are 10^{14} cells in our bodies. At 1 m of DNA each, that stretches out to 10^{14} m. That is 1000 times the distance from the Earth to the Sun, ten times the distance from here to Pluto, or about 1% of a light year.

If we stretched out the DNA of everyone on the planet, the combined DNA strand would extend about 10^8 light years—about 50 times farther than the next galaxy! But who would be around to appreciate the comparison?

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* This is a physicist's view of a cell and is accurate to within a factor of two or three. If you want more precision, call a friendly biologist.

Horse Exhaust

How much waste is generated (per kilometer) by horse-drawn carriages and by automobiles? Give your answer in kg/km.

❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋ ❋

HINT: Horse exhaust is either solid or liquid and car exhaust is gaseous.

HINT: Assume that all the input (food and fuel) becomes output (waste).

HINT: How far does a horse-drawn carriage travel in one day? Horses are not that much faster than people.

HINT: How much does the horse eat and drink? A horse is about 10 times as massive as a person.

HINT: How far does a car travel on a gallon of gasoline?

How much waste is generated (per kilometer) by horse-drawn carriages and by automobiles? Give your answer in kg/km.

ANSWER: We'll need to first estimate how far a horse can pull a carriage in one day and then estimate how much food and water it consumes in the process. Note that we are assuming that the horse does not gain or lose weight and therefore it converts its food and drink into an equal mass of horse exhaust.

How far can an average horse pull an average carriage in one day? Since we are making the horse work for about eight hours, it is certainly not going to gallop, canter, or trot for most of it. Let's start with the average walking speed of a horse. A horse will certainly walk faster than a person (3–4 mph or about 5–6 kph or about 2 m/s) but not that much faster. This gives us a range of about 5–10 mph. In 8 hours, it can travel 40–80 miles, so we'll choose 60 miles (or 100 km).*

How can we figure out the food consumption? We can try to directly estimate the food consumed or we can start with a human and scale up. A horse will certainly eat more than 1 quart (or liter) of grain and less than 100 quarts. We'll take the geometrical mean and estimate 10 L. That has a mass of about 10 kg (20 lb)† and will therefore produce 10 kg of waste. Similarly, the horse will drink more than 1 L and less than 100 L so we will estimate that it drinks about 10 L (also 10 kg).

Now let's start with a human and scale up. We eat 2–3 lb (1–1.5 kg) of food and drink 1–2 L (or quarts) of liquid per day. A horse is about ten times our size (in volume or mass) and thus probably consumes about ten times more than we do. This gives about the same consumption as the previous estimate.

Thus, the horse will produce about 10 kg each of liquid and solid waste in the course of traveling 60 miles. The total waste will be about 0.3 kg/mi.

The automobile burns about one gallon every 20 miles. A gallon is about 4 L and thus has a mass of about 4 kg. Thus, the car produces 4 kg of waste in 20 miles or 0.2 kg/mi.

A car produces about the same amount of waste per mile as a horse.* The problem is that while the car's exhaust blows away into the air (and is almost entirely carbon dioxide and water), the horse's exhaust sticks around (literally). Imagine New York City if it had millions of horses instead of millions of cars. Better yet, don't.

---

* That factor of 1.6 is almost irrelevant. Our estimates are just not that exact.

† The density of anything organic is reasonably close to water. The density of water is 1 g/cm^3 or 1 kg/L or 1 ton/m^3.

* Both produce plant food! The car produces carbon dioxide and the horse produces fertilizer.

Tire tracks

5.6

How far does a car travel
before a one-molecule
layer of rubber is
worn off the tires?

* * * * * * * * * * * * * * * * * *
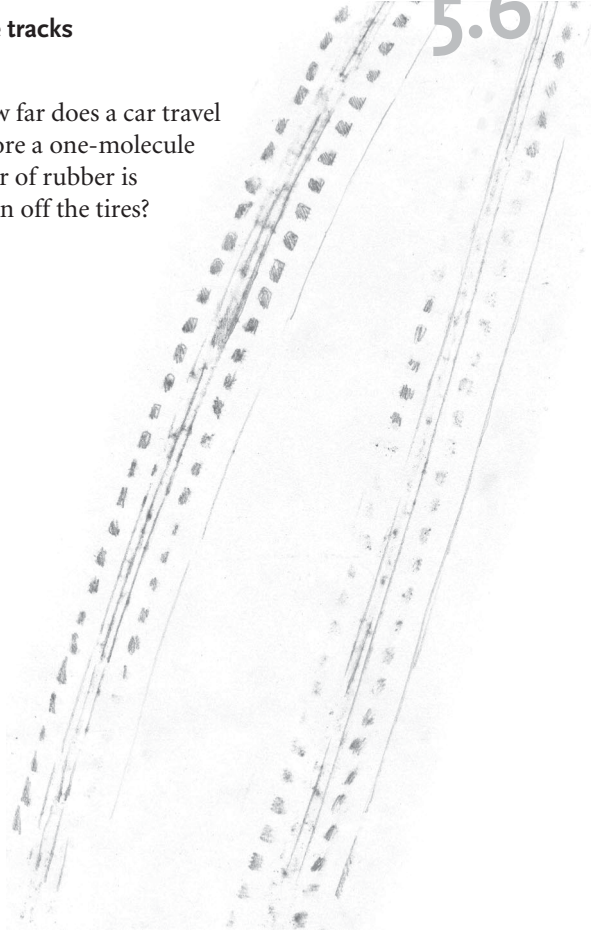
HINT: What is the lifetime of a tire (in miles)?

HINT: How thick is the tread of a new tire? How much of this is
worn away during a tire's lifetime?

HINT: A rubber molecule is only a few tenths of a nanometer
(i.e., a few $\times 10^{-10}$ m) in size.

5.6

How far does a car travel **before a one-molecule layer of rubber is** worn off the tires?

ANSWER: First we need to figure the lifetime of tires in miles. As usual, there are a few ways to do this. You can estimate the lifetime of a tire in years and assume the usual 12,000 miles per year. Tires definitely last more than 1 year and less than 10, so estimates of between 3 and 5 years are reasonable. Alternatively, you can read the tire ads, which advertise the tire lifetimes, or remember the lifetime of the last set of tires you bought. Tires typically last 30–60 thousand miles. They typically have between 1/4 and 1/2 in. (i.e., about 1 cm) of tread.

Thus, 1 cm of tread is worn off in about 4×10^4 mi. We want to know how long it takes to wear off a thickness of one molecule or 5×10^{-10} m of tread. That distance is

$$
\begin{aligned}
d &= \frac{4 \times 10^4 \, \text{mi}}{1 \, \text{cm}} \times \frac{100 \, \text{cm}}{1 \, \text{m}} \times 5 \times 10^{-10} \, \text{m} \\
&= 20 \times 10^{-4} \, \text{mi} \\
&= 2 \times 10^{-3} \, \text{mi}
\end{aligned}
$$

Now we need to make sense of this result. 10^{-3} mi is hard to figure out, but 10^{-3} km is just 1 m. Since a mile is only a little bigger than a km, we have

$$
d = 2 \times 10^{-3} \, \text{mi} = 3 \times 10^{-3} \, \text{km} = 3 \, \text{m}
$$

Three meters is about 10 feet. That is only one or two complete rotations of the tire.

Thus, you wear off a one-molecule thickness of rubber with every rotation of your tire.