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**Ariel Rubinstein: Lecture Notes in Microeconomic Theory**

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## Choice

### Choice Functions

Until now we have avoided any reference to behavior. We have talked about preferences as a summary of the decision maker's mental attitude toward a set of alternatives. But economics is about behavior, and therefore we now move on to modeling an agent's choice. The term "agent's behavior" contains not only the specification of the agent's actual choices made when he confronts certain choice problems, it also contains a full description of his behavior in all scenarios we imagine he might confront.

Consider a *grand set*  $X$  of possible alternatives. We view a choice problem as a nonempty subset of  $X$ , and we refer to a choice from  $A \subseteq X$  as specifying one of  $A$ 's members. We think about behavior as a hypothetical response to a questionnaire that contains many questions of the following type:

**Q(A):** Assume you have to choose from a set of alternatives  $A$ . Which alternative would you choose?\_\_\_\_\_

A legal response to this questionnaire requires responding to *all* questions by indicating *a unique* element in  $A$  for every question  $Q(A)$ .

In some contexts, not all questions are meaningful. Therefore we allow that the questionnaire consist of a subset of questions, one for each element of a set  $D$  of subsets of  $X$ . We will refer to a pair  $(X, D)$  as a *context*.

### **Example:**

Imagine that we are interested in a student's behavior regarding his selection from the set of universities to which he has been admitted.

Let  $X = \{x_1, \dots, x_N\}$  be the set of all universities in the scope of the student's imagination. A choice problem  $A$  is interpreted as the set of universities to which he has been admitted. The fact that the student was admitted to some subset of universities does not imply his admission outcome for other universities. Therefore,  $D$  contains the  $2^N - 1$  nonempty subsets of  $X$ . But if, for example, the universities are listed according to difficulty in being admitted ( $x_1$  being the most difficult) and if the fact that the student is admitted to  $x_k$  means that he is admitted to all less prestigious universities, that is, to all  $x_l$  with  $l > k$ , then  $D$  will consist of the  $N$  sets  $A_1, \dots, A_N$  where  $A_k = \{x_k, \dots, x_N\}$ .

Given a context  $(X, D)$ , a *choice function*  $C$  assigns to each set  $A \in D$  a unique element of  $A$  with the interpretation that  $C(A)$  is the chosen element from the set  $A$ .

Our understanding is that a decision maker behaving in accordance with the function  $C$  will choose  $C(A)$  if he has to make a choice from a set  $A$ . This does not mean that we can actually observe the choice function. At most we might observe some particular choices made by the decision maker in some instances. Thus, a choice function is a description of hypothetical behavior.

### Rational Choice Functions

It is typically assumed in economics that choice is an outcome of "rational deliberation". Namely, the decision maker has in mind a preference relation  $\succsim$  on the set  $X$  and, given any choice problem  $A$  in  $D$ , he chooses an element in  $A$  which is  $\succsim$  optimal. Assuming that it is well defined, we define the *induced choice function*  $C_{\succsim}$  as the function that assigns to every nonempty set  $A \in D$  the  $\succsim$ -best element of  $A$ . Note that the preference relation is fixed, that is, it is independent of the choice set being considered.

### Dutch Book Arguments

Some of the justifications for this assumption are normative, that is, they reflect a perception that people should be rational in this sense and, if they are not, they should convert to reasoning of this type. One interesting class of arguments that aimed at supporting this approach is referred to in the literature as "Dutch book arguments."

The claim is that an economic agent who behaves according to a choice function that is not induced from maximization of a preference relation will not survive.

The following is a “sad” story about a monkey in a forest with three trees,  $a$ ,  $b$ , and  $c$ . The monkey is about to pick a tree to sleep in. It has in mind a binary relation  $\succsim$  that reflects the comparison he makes mentally between any two trees such that  $a \succ b$ ,  $b \succ c$ , and  $c \succ a$ . Assume that whenever he is on tree  $a$  he sees only tree  $b$ , whenever he is on tree  $b$  he sees only tree  $c$ , and whenever he is on tree  $c$  he observes only tree  $a$ . The monkey’s choice function is  $C(\{a, b\}) = b$ ,  $C(\{b, c\}) = c$ ,  $C(\{a, c\}) = a$ . The monkey will perpetually jump from tree to tree to tree—not a good mode of behavior in the “cruel” environment of nature.

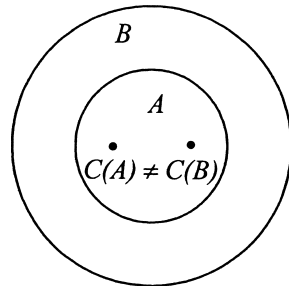
A similar “story,” more appropriate to human beings, is called the “money pump” argument. Assume that a decision maker behaves like the monkey regarding three alternatives  $a$ ,  $b$ , and  $c$ . Assume that (for all  $x$  and  $y$ ) the choice  $C(x, y) = y$  is strong enough that while he is “holding” the option to receive the alternative  $x$ , he is ready to pay 1¢ for the ability to make the choice from  $\{x, y\}$ . In this case, he can be “pumped” for his money by giving him  $a$  and offering him to replace what he holds with  $b$ ,  $c$ , and again  $a$  until his pockets are emptied, or until the decision maker learns his lesson and changes his behavior.

I bring this “Dutch book argument” here not as a necessarily convincing argument for rationality but just as an interesting argument. The above argument could be easily criticized. Its appeal requires, in particular, that we be convinced that the environment in which the economic agent operates would offer the agent the above sequence of choice problems.

### Rationalizing

Economists were often criticized for making the assumption that decision makers maximize a preference relation. The most common response to this criticism is that we don’t really need this assumption. All we need to assume is that the decision maker’s behavior can be described *as if* he were maximizing some preference relation.

Let us state this “economic defense” more precisely. We will say that a choice function  $C$  can be rationalized if there is a preference relation  $\succsim$  on  $X$  so that  $C = C_{\succsim}$  (that is,  $C(A) = C_{\succsim}(A)$  for any  $A$  in the domain of  $C$ ).



**Figure 3.1**  
Violation of condition \*.

We will now identify a condition under which a choice function can indeed be presented as if derived from some preference relation (i.e., can be rationalized).

**Condition \*:**

We say that  $C$  satisfies condition \* if for any two problems  $A, B \in D$ , if  $A \subset B$  and  $C(B) \in A$  then  $C(A) = C(B)$ . (See fig. 3.1.)

Note that if  $\succsim$  is a preference relation on  $X$ , then  $C_{\succsim}$  (defined on a set of subsets of  $X$  that have a single most preferred element) satisfies \*.

Alternatively, consider the “second-best procedure” in which the decision maker has in mind an ordering  $\succsim$  of  $X$  and for any given choice problem set  $A$  chooses the element from  $A$ , which is the  $\succsim$ -maximal from the nonoptimal alternatives. The second-best procedure does not satisfy condition \*: If  $A$  contains all the elements in  $B$  besides the  $\succsim$ -maximal, then  $C(B) \in A \subset B$  but  $C(A) \neq C(B)$ .

We will now show that condition \* is a sufficient condition for a choice function to be formulated *as if* the decision maker is maximizing some preference relation.

**Proposition:**

Assume that  $C$  is a choice function with a domain containing at least all subsets of  $X$  of size no greater than 3. If  $C$  satisfies \*, then there is a preference  $\succsim$  on  $X$  so that  $C = C_{\succsim}$ .

**Proof:**

Define  $\succsim$  by  $x \succsim y$  if  $x = C(\{x, y\})$ .

Let us first verify that the relation  $\succsim$  is a preference relation.

- *Completeness*: Follows from the fact that  $C(\{x, y\})$  is always well defined.
- *Transitivity*: If  $x \succsim y$  and  $y \succsim z$ , then  $C(\{x, y\}) = x$  and  $C(\{y, z\}) = y$ . If  $C(\{x, z\}) \neq x$  then  $C(\{x, z\}) = z$ . By  $*$  and  $C(\{x, z\}) = z$ ,  $C(\{x, y, z\}) \neq x$ . By  $*$  and  $C(\{x, y\}) = x$ ,  $C(\{x, y, z\}) \neq y$ , and by  $*$  and  $C(\{y, z\}) = y$ ,  $C(\{x, y, z\}) \neq z$ . A contradiction to  $C(\{x, y, z\}) \in \{x, y, z\}$ .

We still have to show that  $C(B) = C_{\succsim}(B)$ . Assume that  $C(B) = x$  and  $C_{\succsim}(B) \neq x$ . That is, there is  $y \in B$  so that  $y \succ x$ . By definition of  $\succsim$ , this means  $C(\{x, y\}) = y$ , contradicting  $*$ .

**What Is an Alternative**

Some of the cases where rationality is violated can be attributed to the incorrect specification of the space of alternatives. Consider the following example taken from Luce and Raiffa (1957): A diner in a restaurant chooses *chicken* from the menu  $\{steak\ tartare, chicken\}$  but chooses *steak tartare* from the menu  $\{steak\ tartare, chicken, frog\ legs\}$ . At first glance it seems that he is not “rational” (since his choice conflicts with  $*$ ). Assume that the motivation for the choice is that the existence of *frog legs* is an indication of the quality of the chef. If the dish *frog legs* is on the menu, the cook must then be a real expert, and the decision maker is happy ordering *steak tartare*, which requires expertise to make. If the menu lacks *frog legs*, the decision maker does not want to take the risk of choosing *steak tartare*.

Rationality is “restored” if we make the distinction between “*steak tartare* served in a restaurant where *frog legs* are also on the menu (and the cook must then be a real chef)” and “*steak tartare* in a restaurant where *frog legs* are not served (and the cook is likely a novice).” Such a distinction makes sense since the *steak tartare* is not the same in the two choice sets.

The lesson from the above discussion is that we should be careful in specifying the term “alternative.” Note, however, that defining any alternative in terms of its physical description and the choice

set from which it is to be chosen would empty the rationality hypothesis of its meaning.

### Choice Functions as “Internal Equilibria”

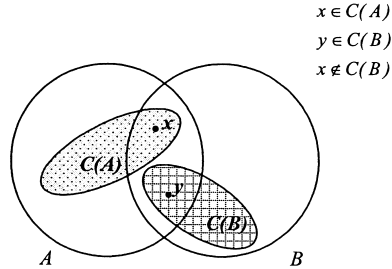
The choice function definition we have been using requires that a single element be assigned to each choice problem. If the decision maker follows the rational-man procedure using a preference relation with indifferences, the previously defined induced choice function  $C_{\succsim}(A)$  might be undefined because for some choice problems there would be more than one optimal element. This is one of the reasons that in some cases we use the alternative following concept to model behavior.

A *choice function*  $C$  is required to assign to every nonempty  $A \subseteq X$  a nonempty *subset* of  $A$ , that is,  $C(A) \subseteq A$ . According to our interpretation of a choice problem, a decision maker has to select a unique element from every choice set. Thus,  $C(A)$  cannot be interpreted as the choice made by the decision maker when he has to make a choice from  $A$ . The revised interpretation of  $C(A)$  is the *set* of all elements in  $A$  that are satisfactory in the sense that the decision maker has no desire to move away from any of them. In other words, a choice function reflects an “internal equilibrium”: If the decision maker facing  $A$  considers an alternative outside  $C(A)$ , he will not continue searching for another alternative. If he happens to consider an alternative inside  $C(A)$ , he will take it.

We now define the induced choice function (assuming it is never empty) as  $C_{\succsim}(A) = \{x \in A \mid x \succsim y \text{ for all } y \in A\}$ . Condition \* is now replaced by the condition that if  $x$  is revealed to be at least as good as  $y$  in one choice problem,  $y$  will never be “chosen” without  $x$  when  $x$  is feasible:

#### ***The Weak Axiom of Revealed Preference (WA):***

We say that  $C$  satisfies WA if whenever  $x, y \in A \cap B$ ,  $x \in C(A)$  and  $y \in C(B)$ , it is also true that  $x \in C(B)$  (fig. 3.2). In other words, if  $y$  is “chosen” while  $x$  is available, then it will never be the case that  $x$  is “chosen” without  $y$  when both are available.



**Figure 3.2**  
Violation of the weak axiom.

**Proposition:**

Assume that  $C$  is a choice function with a domain that includes at least all subsets of size not greater than 3. Assume that  $C$  satisfies WA. Then, there is a preference  $\succsim$  so that  $C = C_{\succsim}$ .

**Proof:**

Define  $x \succsim y$  if  $x \in C(\{x, y\})$ . We will now show that the relation is a preference:

- *Completeness:* Follows from  $C(\{x, y\}) \neq \emptyset$ .
- *Transitivity:* If  $x \succsim y$  and  $y \succsim z$  then  $x \in C(\{x, y\})$  and  $y \in C(\{y, z\})$ . If  $x \notin C(\{x, z\})$ , then  $C(\{x, z\}) = \{z\}$ . By WA,  $x \notin C(\{x, y, z\})$  (by WA  $x$  cannot be revealed to be as good as  $z$  because  $z$  was chosen without  $x$  from  $\{x, z\}$ ). Similarly,  $y \notin C(\{x, y, z\})$  (by WA,  $y$  cannot be chosen without  $x$  while  $x \in C(\{x, y\})$ ). And also,  $z \notin C(\{x, y, z\})$  (by WA,  $z$  cannot be chosen without  $y$  while  $y \in C(\{y, z\})$ ). This contradicts the nonemptiness of  $C(\{x, y, z\})$ .

It remains to be shown that  $C(B) = C_{\succsim}(B)$ .

Assume that  $x \in C(B)$  and  $x \notin C_{\succsim}(B)$ . That is, there is  $y \in B$  so that  $y$  is strictly better than  $x$ , or in other words,  $C(\{x, y\}) = \{y\}$ , thus contradicting WA.

Assume that  $x \in C_{\succsim}(B)$  and  $x \notin C(B)$ . Let  $y \in C(B)$ . By WA  $x \notin C(\{x, y\})$  and thus  $C(\{x, y\}) = \{y\}$ , and therefore  $y \succ x$ , contradicting  $x \in C_{\succsim}(B)$ .

### The Satisficing Procedure

The fact that we can present any choice function satisfying condition  $*$  (or WA) as an outcome of the optimization of some preference relation is a key argument for the view that the scope of microeconomic models is much wider than that of the models in which agents carry out explicit optimization. But have we indeed expanded the scope of economic models beyond the circumstances in which decision makers carry out explicit optimization?

Consider the following “decision scheme,” named *satisficing* by Herbert Simon. Let  $v : X \rightarrow \Re$  be a valuation of the elements in  $X$ , and let  $v^* \in \Re$  be a threshold of satisfaction. Let  $O$  be an ordering of the alternatives in  $X$ . Given a set  $A$ , the decision maker arranges the elements of this set in a list  $L(A, O)$  according to the ordering  $O$ . He then chooses the first element in  $L(A, O)$  that has a  $v$ -value at least as large as  $v^*$ . If there is no such element in  $A$ , the decision maker chooses the last element in  $L(A, O)$ .

Let us show that the choice function induced by this procedure satisfies condition  $*$ . Assume that  $a$  is chosen from  $B$  and is also a member of  $A \subset B$ . The list  $L(A, O)$  is obtained from  $L(B, O)$  by eliminating all elements in  $B - A$ . If  $v(a) \geq v^*$  then  $a$  is the first satisfactory element in  $L(B, O)$ , and is also the first satisfactory element in  $L(A, O)$ . Thus  $a$  is chosen from  $A$ . If all elements in  $B$  are unsatisfactory, then  $a$  must be the last element in  $L(B, O)$ . Since  $A$  is a subset of  $B$ , all elements in  $A$  are unsatisfactory and  $a$  is the last element in  $L(A, O)$ . Thus,  $a$  is chosen from  $A$ .

Note, however, that even a “small” variation in this scheme leads to a variation of the procedure such that it no longer satisfies  $*$ . For example:

*Satisficing using two orderings:* Let  $X$  be a population of university graduates who are potential candidates for a job. Given a set of actual candidates, count their number. If the number is smaller than 5, order them alphabetically. If the number of candidates is above 5, order them by their social security number. Whatever ordering is used, choose the first candidate whose undergraduate average is above 85. If there are none, choose the last student on the list.

Condition  $*$  is not satisfied. It may be that  $a$  is the first candidate with a satisfactory grade in a long list of students ordered by their social security numbers. Still,  $a$  might not be the first candidate with a satisfactory grade on a list of only three of the candidates appearing on the original list when they are ordered alphabetically.

The satisficing procedure, though it is stated in a way that seems unrelated to the maximization of a preference relation or utility function, can be described as if the decision maker maximizes a preference relation. I know of no other examples of interesting general schemes for choice procedures that satisfy \* other than the “rational man” and the satisficing procedures. However, later on, when we discuss consumer theory, we will come across several other appealing examples of demand functions that can be rationalized though they appear to be unrelated to the maximization of a preference relation.

### **Psychological Motives Not Included within the Framework**

The more modern attack on the standard approach to modeling economic agents comes from psychologists, notably from Amos Tversky and Daniel Kahneman. They have provided us with beautiful examples demonstrating not only that rationality is often violated, but that there are systematic reasons for the violation resulting from certain elements within our decision procedures. Here are a few examples of this kind that I find particularly relevant.

#### ***Framing***

The following experiment (conducted by Tversky and Kahneman 1986) demonstrates that the way in which alternatives are framed may affect decision makers’ choices. Subjects were asked to imagine being confronted by the following choice problem:

An outbreak of disease is expected to cause 600 deaths in the US. Two mutually exclusive programs are expected to yield the following results:

- a. 400 people will die.
- b. With probability  $1/3$ , 0 people will die and with probability  $2/3$ , 600 people will die.

In the original experiment, a different group of subjects was given the same background information and asked to choose from the following alternatives:

- c. 200 people will be saved.
- d. With probability  $1/3$ , all 600 will be saved and with probability  $2/3$ , none will be saved.

While only 22% of the first group chose  $a$ , 72% of the second group chose  $c$ . My experience offering both questions to 170 graduate students in New York, Princeton, and Tel Aviv is similar even though they were the same students who responded to the two questions: 31% of the students chose  $a$  and 53% chose  $c$ .

These are “problematic” results since, by any reasonable criterion  $a$  and  $c$  are identical alternatives, as are  $b$  and  $d$ . Thus, the choice from  $\{a, b\}$  should be consistent with the choice from  $\{c, d\}$ . The results expose the sensitivity of choice to the framing of the alternatives. What is more basic to rational decision making than taking the same choice when only the manner in which the problems are stated is different?

### *Simplifying the Choice Problem and the Use of Similarities*

The following experiment was also conducted by Tversky and Kahneman. One group of subjects was presented with the following choice:

Choose one of the two roulette games  $a$  or  $b$ . Your prize is the one corresponding to the outcome of the chosen roulette game as specified in the following tables:

	Color	White	Red	Green	Yellow
(a)	Chance %	90	6	1	3
	Prize \$	0	45	30	-15

	Color	White	Red	Green	Yellow
(b)	Chance %	90	7	1	2
	Prize \$	0	45	-10	-15

A different group of subjects was presented the same background information and asked to choose between:

	Color	White	Red	Green	Blue	Yellow
(c)	Chance %	90	6	1	1	2
	Prize \$	0	45	30	-15	-15

and

	Color	White	Red	Green	Blue	Yellow
(d)	Chance %	90	6	1	1	2
	Prize \$	0	45	45	-10	-15

In the original experiment, 58% of the subjects in the first group chose  $a$ , while nobody in the second group chose  $c$ . I presented the two problems, one after the other, to 170 graduate students in New York, Princeton, and Tel Aviv: 43% chose  $a$  and 10% chose  $c$ . Interestingly, the median response time among the students who answered  $a$  was 60 seconds, whereas the median response time of the students who answered  $b$  was 91 seconds.

The results demonstrate a common procedure people practice when confronted with a complicated choice problem. We often transfer the complicated problem into a simpler one by “canceling” similar elements. While  $d$  clearly dominates  $c$ , the comparison between  $a$  and  $b$  is not as easy. Many subjects “cancel” the probabilities of Yellow and Red and are left with comparing the prizes of Green, a process that leads them to choose  $a$ .

Incidentally, several times in the past, when I presented these choice problems in class, I have had students (some of the best students, in fact) who chose  $c$ . They explained that they identified the second problem with the first and used the procedural rule: “I chose  $a$  from  $\{a, b\}$ . The alternatives  $c$  and  $d$  are identical to the alternatives  $a$  and  $b$ , respectively. It is only natural then, that I choose  $c$  from  $\{c, d\}$ .” This observation brings to our attention a hidden facet of the rational-man model. The model does not allow a decision maker to employ a rule such as: “In the past I chose  $x$  from  $B$ . The choice problems  $A$  and  $B$  are similar. Therefore, I shall choose  $x$  from  $A$ .”

### ***Reason-Based Choice***

Making choices sometimes involves finding reasons to pick one alternative over the others. When the deliberation involves the use of reasons strongly associated with the problem at hand (“internal reasons”), we often find it difficult to reconcile the choice with the rational man paradigm.

Imagine, for example, a European student who would choose *Princeton* if allowed to choose from  $\{\textit{Princeton}, \textit{LSE}\}$  and would choose *LSE* if he had to choose from  $\{\textit{Princeton}, \textit{Chicago}, \textit{LSE}\}$ . His explanation is that he prefers an American university so long as he does not have to choose between American schools—a choice he deems harder. Having to choose from  $\{\textit{Princeton}, \textit{Chicago}, \textit{LSE}\}$ , he finds it difficult deciding between *Princeton* and *Chicago* and therefore chooses not to cross the Atlantic. His choice does not satisfy \*, not because of a careless specification of the alternatives (as in the restaurant’s menu example discussed previously), but because his reasoning involves an attempt to avoid the difficulty of making a decision.

Another example follows Huber, Payne, and Puto (1982):

Let  $a = (a_1, a_2)$  be “a holiday package of  $a_1$  days in Paris and  $a_2$  days in London.” Choose one of the four vectors  $a = (7, 4)$ ,  $b = (4, 7)$ ,  $c = (6, 3)$ , and  $d = (3, 6)$ .

All subjects in the experiment agreed that a day in Paris and a day in London are desirable goods. Some of the subjects were requested to choose between the three alternatives  $a$ ,  $b$ , and  $c$ ; others had to choose between  $a$ ,  $b$ , and  $d$ . The subjects exhibited a clear tendency toward choosing  $a$  out of the set  $\{a, b, c\}$  and choosing  $b$  out of the set  $\{a, b, d\}$ .

A related experiment is reported by Tversky and Shafir (1992): Subjects reviewed a list of twelve lotteries, including:

- (x) 65% chance to win \$15.
- (y) 30% chance to win \$35.
- (z) 65% chance to win \$14.

Afterwards, they were presented with a pair of lotteries; some got  $x$  and  $z$  and others  $y$  and  $z$ . They had to either choose one of them or pay \$1 and receive an additional option. Significantly more subjects chose to pay the extra dollar when they had to choose between  $x$  and  $y$  than when they had to choose between  $x$  and  $z$ .

To conclude, decision makers look for reasons to prefer one alternative over the other. Typically, making decisions by using “external reasons” (which do not refer to the properties of the choice set) will not cause violations of rationality. However, applying “internal reasons” such as “I prefer the alternative  $a$  over the alternative  $b$  since  $a$  clearly dominates the other alternative  $c$  while  $b$  does not” might cause conflicts with condition \*.

### **Bibliographic Notes**

*Recommended readings:* Kreps 1990, 24–30; Mas-Colell et al. 1995, chapter 1 C,D.

An excellent book on the lecture's subject is Kreps (1988). For the sources of consistency in choice and revealed preference assumptions, see Samuelson (1948), Houthakker (1950), and Richter (1966). Simon (1955) is the source of the discussion of satisficing. For a discussion of the bounded rationality approach to choice, see Rubinstein (1998). An excellent introduction to the Dutch Books arguments is Yaari (1985). Kahneman and Tversky (2000) is a definitive textbook on the psychological criticism of the economic approach to rationality. Rabin (1998) surveys the modern economics and psychology approach.

## Problem Set 3

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### Problem 1. (*Easy*)

Discuss the compatibility of the following “procedural elements” with the “rational man” paradigm:

- a. The decision maker has in mind a ranking of all alternatives and chooses the alternative that is the worst according to this ranking.
- b. The decision maker chooses an alternative with the intention that another person will suffer the most.
- c. The decision maker asks his two children to rank the alternatives and then chooses the alternative that is the best “on average.”
- d. The decision maker has an ideal point in mind and chooses the alternative that is closest to the ideal point.
- e. The decision maker looks for the alternative that appears most often in the choice set.
- f. The decision maker always selects the first alternative that comes to his attention.
- g. The decision maker searches for someone he knows who will choose an action that is feasible for him.
- h. The decision maker orders all alternatives from left to right and selects the median.

### Problem 2. (*Moderately difficult*)

Let us say that you have to make a choice from a set  $A$ . Does it matter whether (a) you make a choice from the entire set or (b) you first partition  $A$  into the subsets  $A_1$  and  $A_2$ , then make a selection from each of the sets and finally make a choice from the elements you selected from among  $A_1$  and  $A_2$ ?

- a. Formulate a “path independence” property.
- b. Show that the rational decision maker satisfies the property.
- c. Find examples of choice procedures that do not satisfy this property.
- d. Show that if a (single-valued) choice function satisfies path independence, then it is consistent with rationality.
- e. Assume that  $C$  is a (multivalued) choice function satisfying path independence. Can it be rationalized by a preference relation?

**Problem 3.** (*Easy*)

Check whether the following two choice functions satisfy WA:

$C(A) = \{x \in A \mid \text{the number of } y \in X \text{ for which } V(x) \geq V(y) \text{ is at least } |X|/2\}$ ,  
and if the set is empty then  $C(A) = A$ .

$D(A) = \{x \in A \mid \text{the number of } y \in A \text{ for which } V(x) \geq V(y) \text{ is at least } |A|/2 \}$ .

**Problem 4.** (*Moderately difficult*)

Consider the following choice procedure. A decision maker has a strict ordering  $\succsim$  over the set  $X$  and he assigns to each  $x \in X$  a natural number  $\text{class}(x)$  interpreted as the “class” of  $x$ . Given a choice problem  $A$  he chooses the element in  $A$  that is the best among those elements in  $A$ , that belong to the “most popular” class in  $A$  (that is, the class that appears in  $A$  most often). If there is more than one most popular class, he picks the best element from the members of  $A$  that belong to a most popular class with the highest class number.

- a. Is the procedure consistent with the “rational man” paradigm?
- b. Can every choice function be “explained” as an outcome of such a procedure?

(Try to formalize a “property” that is satisfied by such choice procedures and is clearly not satisfied by some other choice functions.)

**Problem 5.** (*Moderately difficult*. Based on Kalai, Rubinstein, and Spiegel 2002)

Consider the following two choice procedures. Explain the procedures and try to persuade a skeptic that they “make sense.” Determine for each of them whether they are consistent with the rational-man model.

- a. The primitives of the procedure are two numerical (one-to-one) functions  $u$  and  $v$  defined on  $X$  and a number  $v^*$ . For any given choice problem  $A$ , let  $a^* \in A$  be the maximizer of  $u$  over  $A$ , and let  $b^*$  be the maximizer of  $v$  over the set  $A$ . The decision maker chooses  $a^*$  if  $v(a^*) \geq v^*$  and chooses  $b^*$  if  $v(a^*) < v^*$ .
- b. The primitives of the procedure are two numerical (one-to-one) functions  $u$  and  $v$  defined on  $X$  and a number  $u^*$ . For any given choice problem  $A$ , the decision maker chooses the element  $a^* \in A$  that maximizes  $u$  if  $u(a^*) \geq u^*$ , and  $v$  if  $u(a^*) < u^*$ .

**Problem 6.** (*Moderately difficult*)

The standard economic choice model assumes that choice is made from a *set*. Let us construct a model where the choice is assumed to be from a *list*.

Let  $X$  be a finite “grand set.” A *list* is a nonempty finite vector of elements in  $X$ . In this problem, consider a *choice function*  $C$  to be a function that assigns to each vector  $L = \langle a_1, \dots, a_k \rangle$  a single element from  $\{a_1, \dots, a_k\}$ . (Thus, for example, the list  $\langle a, b \rangle$  is distinct from  $\langle a, a, b \rangle$  and  $\langle b, a \rangle$ .) For all  $L_1, \dots, L_m$  define  $\langle L_1, \dots, L_m \rangle$  to be the list that is the concatenation of the  $m$  lists. (Note that if the length of  $L_i$  is  $k_i$ , the length of the concatenation is  $\sum_{i=1, \dots, m} k_i$ .) We say that  $L'$  *extends* the list  $L$  if there is a list  $M$  such that  $L' = \langle L, M \rangle$ .

We say that a choice function  $C$  satisfies property  $I$  if for all  $L_1, \dots, L_m$   $C(\langle L_1, \dots, L_m \rangle) = C(\langle C(L_1), \dots, C(L_m) \rangle)$ .

- a. Interpret property  $I$ . Give two (distinct) examples of choice functions that satisfy  $I$  and two examples of choice functions which do not.
- b. Define formally the following two properties of a choice function:
  - *Order Invariance*: A change in the order of the elements of the list does not alter the choice.
  - *Duplication Invariance*: Deleting an element that appears in the list elsewhere does not change the choice.
- c. Characterize the choice functions that satisfy Order Invariance, Duplication Invariance, and condition  $I$ .
- d. Assume now that in the back of the decision maker's mind is a value function  $u$  defined on the set  $X$  (such that  $u(x) \neq u(y)$  for all  $x \neq y$ ). For any choice function  $C$  define  $v_C(L) = u(C(L))$ .

We say that  $C$  *accommodates a longer list* if whenever  $L'$  extends  $L$ ,  $v_C(L') \geq v_C(L)$  and there is a list  $L'$  which extends a list  $L$  for which  $v_C(L') > v_C(L)$ .

- e. Give two interesting examples of choice functions that accommodate a longer list.
- f. Give two interesting examples of choice functions which satisfy property  $I$  but which do not accommodate a longer list.

**Problem 7. (Reading)**

Read Sen (1993). Invent two sound choice procedures and discuss their relation to the “rational man” paradigm.