Review Problems

The following is a collection of questions I have given in exams during the last few years.

Problem 1 (Princeton 2002)

Consider a consumer with a preference relation in a world with two goods, X (an aggregated consumption good) and M ("membership in a club," for example), which can be consumed or not. In other words, the consumption of X can be any nonnegative real number, while the consumption of M must be either 0 or 1.

Assume that consumer preferences are strictly monotonic, continuous, and satisfy the following property:

> *Property E*: For every x there is y such that $(y, 0) \succ (x, 1)$ (that is, there is always some amount of money that can compensate for the loss of membership).

1. Show that any consumer's preference relation can be represented by a utility function of the type
$$u(x, m) = \begin{cases} x & \textit{if} \quad m = 0 \\ x + g(x) & \textit{if} \quad m = 1 \end{cases} .$$
2. (Less easy) Show that the consumer's preference relation can also be represented by a utility function of the type
$$u(x, m) = \begin{cases} f(x) & \textit{if} \quad m = 0 \\ f(x) + v & \textit{if} \quad m = 1 \end{cases} .$$
3. Explain why continuity and strong monotonicity (without property E) are not sufficient for (1).
4. Calculate the consumer's demand function.
5. Taking the utility function to be of the form described in (1), derive the consumer's indirect utility function. For the case where the function g is differentiable, verify the Roy equality with respect to commodity M.

Problem 2 (Princeton 2001)

A consumer has to make his decision *before* he is informed whether a certain event, which is expected with probability α, happened or not. He assigns a vNM utility $v(x)$ to the consumption of the bundle x in case the event occurs, and a vNM utility $w(x)$ to the consumption of x should the event not occur. The consumer maximizes his expected utility. Both v and w satisfy the standard assumptions about the consumer. Assume also that v and w are concave.

1. Show that the consumer's preference relation is convex.
2. Find a connection between the consumer's indirect utility function and the indirect utility functions derived from v and w.
3. A new commodity appears on the market: "A discrete piece of information that tells the consumer whether the event occurred or not." The commodity can be purchased prior to the consumption decision. Use the indirect utility functions to characterize the demand function for the new commodity.

Problem 3 (Princeton 2001)

1. Define a formal concept for "$\succsim_1$ is closer to $\succsim_0$ than $\succsim_2$."
2. Apply your definition to the class of preference relations represented by $U_1 = tU_2 + (1-t)U_0$, where the function U_i represents $\succsim_i$ $(i = 0, 1, 2)$.
3. Consider the above definition in the consumer context. Denote by $x_k^i(p, w)$ the demand function of $\succsim_i$ for good k. Is it true that if $\succsim_1$ is closer to $\succsim_0$ than $\succsim_2$, then $|x_k^1(p, w) - x_k^0(p, w)| \leq |x_k^2(p, w) - x_k^0(p, w)|$ for any commodity k and for every price vector p and wealth level w?

Problem 4 (Princeton 1997)

A decision maker forms preferences over the set X of all possible distributions of a population over two categories (like living in two locations). An element in X is a vector (x_1, x_2) where $x_i \geq 0$ and $x_1 + x_2 = 1$. The decision maker has two considerations in mind:

- He thinks that if $x \succsim y$, then for any z, the mixture of $\alpha \in [0, 1]$ of x with $(1-\alpha)$ of z should be at least as good as the mixture of α of y with $(1-\alpha)$ of z.

- He is indifferent between a distribution that is fully concentrated in location 1 and one which is fully concentrated in location 2.

1. Show that the only preference relation that is consistent with the two principles is the degenerate indifference relation ($x \sim y$ for any $x, y \in X$).
2. The decision maker claims that you are wrong as his preference relation is represented by a utility function $|x_1 - 1/2|$. Why is he wrong?

Problem 5 (Princeton 2000. Based on Fishburn and Rubinstein 1982.)

Let $X = \Re^+ \times \{0, 1, 2, \ldots\}$, where (x, t) is interpreted as receiving \$$x$ at time t. A preference relation on X has the following properties:

- There is indifference between receiving \$0 at time 0 and receiving \$0 at any other time.
- For any positive amount of money, it is better to receive it as soon as possible.
- Money is desirable.
- The preference between (x, t) and $(y, t+1)$ is independent of t.
- Continuity.

1. Define formally the continuity assumption for this context.
2. Show that the preference relation has a utility representation.
3. Verify that the preference relation represented by the utility function $u(x)\delta^t$ (with $\delta < 1$ and u continuous and increasing) satisfies the above properties.
4. Formulize a concept "one preference relation is more impatient than another."
5. Discuss the claim that preferences represented by $u_1(x)\delta_1^t$ are more impatient than preferences represented by $u_2(x)\delta_2^t$ if and only if $\delta_1 < \delta_2$.

Problem 6 (Tel Aviv 2003)

Consider the following consumer problem. There are two goods, 1 and 2. The consumer has a certain endowment. Before the consumer are two "exchange functions": he can exchange x units of good 1 for $f(x)$ units of good 2, or he can exchange y units of good 2 for $g(y)$ units of good 1. Assume the consumer can only make one exchange.

1. Show that if the exchange functions are continuous and the consumer's preference relation satisfies monotonicity and continuity, then a solution to the consumer problem exists.
2. Explain why strong convexity of the preference relation is not sufficient to guarantee a unique solution if the functions f and g are increasing and convex.
3. What does the statement "the function f is increasing and convex" mean?
4. Suppose both functions f and g are differentiable and concave and that the product of their derivatives at point 0 is 1. Suppose also that the preference relation is strongly convex. Show that under these conditions, the agent will not find two different exchanges, one exchanging good 1 for good 2, and one exchanging good 2 for good 1, optimal.
5. Now assume $f(x) = ax$ and $g(y) = by$. Explain this assumption. Find a condition that will ensure it is not profitable for the consumer to make more than one exchange.

Problem 7 (Tel Aviv 1999)

Consider a consumer in a world with K goods and preferences satisfying the standard assumptions regarding the consumer. At the start of trade, the consumer is endowed with a bundle of goods e and he chooses the best bundle from the budget set $B(p, e) = \{x|px = pe\}$. The consumer's preference over bundles of goods can be represented by a utility function u. Define $V(p, e) = max\{u(x)|px = pe\}$.

1. Explain the meaning of the function V and show that $V(tp, e) = V(p, e)$ where t is any positive number.
2. Show that for every bundle e, the set of vectors p, such that $V(p, e)$ is less than or equal to $V(p^*, e)$, is convex.
3. Fix all prices but p_i, and all quantities in the initial bundle but w_i. Show that the slope of the indifference curve of V in the two-dimensional space where the parameters on the axes are p_i, and w_i is $(x_i(p, w) - w_i)/p_i$ where $x(p, w)$ is the solution to the consumer's problem $B(p, w)$.

Problem 8 (Tel Aviv 1998)

A consumer with wealth $w = 10$ "must" obtain a book from one of three stores. Denote the prices at each store as p_1, p_2, p_3. All prices are below w in the relevant range. The consumer has devised a strategy: he compares the prices at the first two stores and obtains the book from the first store if its price is not greater than the price at the second store. If $p_1 > p_2$, he compares the prices of the second and third stores and obtains the book from the second store if its price is not greater than the price at the third store. He uses the remainder of his wealth to purchase other goods.

1. What is this consumer's "demand function"?
2. Does this consumer satisfy "rational man" assumptions?
3. Consider the function $v(p_1, p_2, p_3) = w - p_{i^*}$, where i^* is the store from which the consumer purchases the book if the prices are (p_1, p_2, p_3). What does this function represent?
4. Explain why $v(\cdot)$ is not monotonically decreasing in p_i. Compare with the indirect utility function of the classic consumer model.

Problem 9 (Tel Aviv 1999)

Tversky and Kahneman (1986) report the following experiment: each participant receives a questionnaire asking him to make two choices, one from $\{a, b\}$ and the second from $\{c, d\}$:

a. A sure profit of \$240.
b. A lottery between a profit of \$1000 with probability 25% and 0 with probability 75%.

c. A sure loss of \$750.
d. A lottery between a loss of \$1000 with probability 75% and 0 with probability 25%.

The participant will receive the sum of the outcomes of the two lotteries he chooses. Seventy-three percent of participants chose the combination *a* and *d*. What do you make of this result?

Problem 10 (Princeton 2000)

Consider the following social choice problem: a group has n members (n is odd) who must choose from a set containing 3 elements

$\{A, B, L\}$, where A and B are prizes and L is the lottery which yields each of the prizes A and B with equal probability. Each member has a strict preference over the three alternatives that satisfies vNM assumptions. Show that there is a non-dictatorial social welfare function which satisfies the independence of irrelevant alternatives axiom (even the strict version I^*) and the Pareto axiom (*Par*). Reconcile this fact with Arrow's Impossibility Theorem.

Problem 11 (Tel Aviv 2003. Based on Gilboa and Schmeidler 1995.)

An agent must decide whether to do something, Y, or not to do it, N.

A history is a sequence of results for past events in which the agent chose Y; each result is either a success S or a failure F. For example, (S, S, F, F, S) is a history with five events in which the action was carried out. Two of them (events 3 and 4) ended in failure while the rest were successful.

The decision rule D is a function that assigns the decision Y or N to every possible history.

Consider the following properties of decision rules:

- $A1$ After every history that contains only successes, the decision rule will dictate Y, and after every history that contains only failures, the decision rule will dictate N.
- $A2$ If the decision rule dictates a certain action following some history, it will dictate the same action following any history that is derived from the first history by reordering its members. For example, $D(S, F, S, F, S) = D(S, S, F, F, S)$.
- $A3$ If $D(h) = D(h')$, then this will also be the decision following the concatenation of h and h'. (Reminder: The concatenation of $h = (F, S)$ and $h' = (S, S, F)$ is (F, S, S, S, F)).

1. For every $i = 1, 2, 3$, give an example of a decision rule that does not fulfill property Ai but does fulfill the other two properties.
2. Give an example of a decision rule that fulfills all three properties.
3. (Difficult) Characterize the decision rules that fulfill the three properties.