



Value at Risk

P. Sercu,  
*International  
Finance: Theory into  
Practice*

Overview

# Chapter 14

## Value at Risk: Quantifying Overall Net Market Risk

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# Overview

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### Risk Budgeting—a Factor-based, Linear Approach

- Why work with factors not assets
- Linking Bonds to Interest-rate Factors
- Stock-market Risk
- Currency Forwards
- Options
- Swaps
- The Risk Budget

### The Linear/Normal VaR Model: Potential Flaws & Corrections

- A Zero-Drift (“Martingale”) Process
- A Constant-Variance Process
- Constant Correlations Between Factors.
- Linearizations in the Mapping  $f \rightarrow dV$
- Choice of the factors
- Normality of  $dV$
- Assets can be Liquidated in one Day

### Backtesting, Bootstrapping, Monte Carlo, and Stress Testing

- Backtesting
- Bootstrapping
- Monte Carlo Simulation
- Stress Testing

### Closing remarks

- CDOs/CDSs are hard to price
- Moral Hazard type risks



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# What are we after? (in this chapter)

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◇ **VaR** = maximal loss that could be suffered on the current portfolio with 99% confidence

**VaR: losses worse than VaR should occur only one day in 100**

- ▷ basis for a bank's Capital Requirement calculations ("Market Risk Charge"), which is set at three times VaR or more
- ▷ can be computed ...
  - from a normality model if  $\sigma_p$  is known
    - with normal factors, linearly related to returns (Riskmetrics)
  - from a reconstructed history of portfolio returns (backtesting)
  - from a simulated series of possible future events with ...
    - resampled actual returns (bootstrapping), or
    - normal or non-normal computer-generated factors, mapped into returns (Monte Carlo)



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- ◇ **Suppose we know  $\text{std}(\tilde{r}_p)$** : VaR follows easily:

## Example

- data:
  - current value 100m,
  - expected return 10%, stdev 30%, both *p.a.*
  - all assets are liquid
    - ⇒ horizon is 1 trading day, i.e. 1/260 year
- Computations
  - expected value tomorrow:
$$100m \times \left(1 + \frac{0.10}{260}\right) = 100.04m$$
  - variance is linear in time, so the 1-day std is
$$100m \times 0.30 \times \sqrt{\frac{1}{260}} = 1.9m$$
  - maximal loss below 100.04 (with 99% confidence):
$$1.9m \times 2.33 = 4.13m \text{ over one day}$$

## ◇ How to get std?

- ▷ portfolio theory: need full varcov matrix of all assets
- ▷ Need Nobs  $\gg$  N of assets, otherwise “linear dependencies”
- ▷ ⇒ *factor-covariance/normality solution*: “map” return as functions of far fewer underlying factors



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# More on sources of risk and factors)

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- ◇ **Sources of uncertainty:** (in standard software)
  - ▷ exchange risks
  - ▷ stock price risks (by country, in local currency)
  - ▷ interest rate risk (by currency and time to maturity; say 13 rates)
  - ▷ commodity price risk
  
- ◇ **A factor:** the *unexpected percentage* change in the source-of-risk variable.
  - ▷ J.P. Morgan's Riskmetrics<sup>(C)</sup> provides a covariance matrix for hundreds of these factors.
  
- ◇ **Mapping:** link between factor and asset return
  - ▷ E.g. a 5-year FC bond:  $V = V^*S$  so  $\Delta V \approx V \times \left( \frac{\Delta V^*}{V^*} + \frac{\Delta S}{S} \right)$   
with  $\frac{\Delta V^*}{V^*}$  a function of the foreign 5-year interest rate



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# Portfolio return as a function of asset returns

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## Portfolio return:

$$\begin{aligned}
 dV_p &= \sum_{i=1}^N n_{i,t} (P_{i,t+1} - P_{i,t}) \\
 &= \sum_{i=1}^N \underbrace{n_{i,t} P_{i,t}}_{\text{initial capital invested in } i} \underbrace{\frac{P_{i,t+1} - P_{i,t}}{P_{i,t}}}_{\text{return on } i} .
 \end{aligned}$$

## Example

asset	(a) $n_{i,t}$	(b) $P_{i,t}$	(c)=(a) × (b) initial capital	(d) $P_{i,t+1}$	(e)=(a) × (d) final capital	(e) return	(c) × (e) cap ×
Nikkei	1,000	13,000	13,000,000	13,650	13,650,000	0.05	650,000
Copper	600	30,000	18,000,000	660	19,800,000	0.10	1,800,000
Total			31,000,000		33,450,000		2,450,000



# Asset return as a function of factors

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Individual asset  $i$  as a function of factors  $f_j, j = 1, \dots, M$ :

$$\begin{aligned}
 \widetilde{dV}_i &= \sum_{j=1}^M \frac{\partial V_i}{\partial X_j} \widetilde{dX}_j + \frac{\partial V_i}{\partial t} dt + \text{higher-order terms} \\
 &= \sum_{j=1}^M \underbrace{V_i}_{\text{inv.}} \times \underbrace{\frac{\partial V_i}{\partial X_j} \frac{X_j}{V_i}}_{\text{elasticity w.r.t. } X_j} \times \underbrace{\frac{\widetilde{dX}_j}{X_j}}_{\text{factor}} + \underbrace{\frac{\partial V_i}{\partial t} dt}_{\text{(not risky)}} + \dots, \\
 &\quad \underbrace{\hspace{10em}}_{\text{pseudo-inv in } j \text{ via } i, E_{i,j}} \\
 &= \sum_{j=1}^M E_{i,j} \widetilde{f}_j + \dots
 \end{aligned}$$

Portfolio then behaves as series of pseudo-investments  $E_{p,j}$  in factors  $j$ :

$$\widetilde{dV}_p = \sum_{i=1}^N n_i \widetilde{dV}_i + \dots = \sum_{j=1}^M \underbrace{\sum_{i=1}^N n_i E_{i,j}}_{=E_{p,j}} \widetilde{f}_j + \dots = \sum_{j=1}^M E_{p,j} \widetilde{f}_j + \dots$$



# Asset return as a function of factors

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# Linking Bonds to Interest-rate factors – 1

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- ◇ **Step 1: Strip the bonds** decompose every individual bond or loan into a replicating package of promissory notes.

## Example

A 3-year 6% bond paying out 1m with first coupon date in 8 months boils down to

- ▷ one promissory note (PN) ad 60,000, 8 month
- ▷ one PN ad 60,000, 20 month
- ▷ one PN ad 1060,000, 32 month



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## ◇ Step 2: track $dV_t$ to $dR/R$

$$\begin{aligned}V_t &= V_T \times (1 + R_{t,T})^{-(T-t)}, \\dV_t &= \underbrace{-(T-t)}_{\text{Modif Dur}} \times V_T \times (1 + R_{t,T})^{-(T-t)-1} dR_{t,T}, \\&= \underbrace{-\frac{T-t}{1 + R_{t,T}}}_{\text{elasticity}} \times R_{t,T} \times \underbrace{\frac{V_T}{(1 + R_{t,T})^{T-t}}}_{= V_t} \frac{dR_{t,T}}{R_{t,T}}\end{aligned}$$

Example: 8-mo PN;  $V_T = 100K$ ,  $R_{8mo} = 0.03$  p.a.

▷ Value  $V_t = 100,000 / (1.03)^{2/3} = 98048.70$

▷ Duration =  $(2/3) / 1.03 = 0.6472$

▷ Elasticity =  $-0.6472 \times 0.03 = -0.01941$

▷  $dV = \underbrace{98048.70 \times (-0.01941)}_{-1903.86} \times \frac{dR_{8mo}}{R_{8mo}}$

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- ◇ **Step 3: Link  $dR_{t,T}/R_{t,T}$  to official factors:** The 8-mo rate is not in the riskmetrics database. So link it, via (non?)linear interpolation, to those that are.

Example: 8-mo rate linked to 6- and 9-mo ones

- ▷ Simplest solution: geometric interpolation:  $R_{8mo} = x R_{6mo} + (1-x) R_{9mo}$ 
  - find  $x$  if  $R_{8mo}$  is observed,
  - set  $x = 1/3$  if  $R_{8mo}$  must be calculated.
- ▷ Implication:  $\frac{dR_{8mo}}{R_{8mo}} = x \frac{dR_{6mo}}{R_{6mo}} + (1-x) \frac{dR_{9mo}}{R_{9mo}}$
- ▷ Value impact in previous example

$$\begin{aligned}dV &= -1903.86 \times \left( x \frac{dR_{6mo}}{R_{6mo}} + (1-x) \frac{dR_{9mo}}{R_{9mo}} \right) \\ &= \underbrace{-1903.86 \times x}_{E \text{ to 6mo rate}} \frac{dR_{6mo}}{R_{6mo}} + \underbrace{-1903.86 \times (1-x)}_{E \text{ to 9mo rate}} \frac{dR_{9mo}}{R_{9mo}}\end{aligned}$$

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- ▷ Implication:  $\frac{dR_{8mo}}{R_{8mo}} = x \frac{dR_{6mo}}{R_{6mo}} + (1-x) \frac{dR_{9mo}}{R_{9mo}}$
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$$\begin{aligned}dV &= -1903.86 \times \left( x \frac{dR_{6mo}}{R_{6mo}} + (1-x) \frac{dR_{9mo}}{R_{9mo}} \right) \\ &= \underbrace{-1903.86 \times x}_{E \text{ to 6mo rate}} \frac{dR_{6mo}}{R_{6mo}} + \underbrace{-1903.86 \times (1-x)}_{E \text{ to 9mo rate}} \frac{dR_{9mo}}{R_{9mo}}\end{aligned}$$



# Linking Bonds to Interest-rate Factors – 3

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- ◇ **Step 3: Link  $dR_{t,T}/R_{t,T}$  to official factors:** The 8-mo rate is not in the riskmetrics database. So link it, via (non?)linear interpolation, to those that are.

## Example: 8-mo rate linked to 6- and 9-mo ones

- ▷ Simplest solution: geometric interpolation:  $R_{8mo} = R_{6mo}^x R_{9mo}^{1-x}$ 
  - find  $x$  if  $R_{8mo}$  is observed,
  - set  $x = 1/3$  if  $R_{8mo}$  must be calculated.
- ▷ Implication:  $\frac{dR_{8mo}}{R_{8mo}} = x \frac{dR_{6mo}}{R_{6mo}} + (1-x) \frac{dR_{9mo}}{R_{9mo}}$
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- ◇ **General:** Top-down approach: the investment in country-A stocks is assumed to be a position in the country's market index,  $M$ . So elasticity (or beta) " $\approx$ " 1: no correction to mkt values in HC "is" needed

- ◇ **Domestic Stocks**

$$\begin{aligned}dV &= V \times \frac{dV}{V} \text{ and } \frac{dV}{V} \approx \frac{dM}{M} \\ &\approx V \times \frac{dM}{M}\end{aligned}$$

- ◇ **Foreign Stocks:** exposed to both  $M^*$  and  $S$ :

$$\begin{aligned}V &= V^* \times S \Rightarrow \frac{dV}{V} = \frac{dV^*}{V^*} + \frac{dS}{S} \approx \frac{dM^*}{M^*} + \frac{dS}{S} \\ \frac{dV}{V} &= V \times \frac{dV}{V} \approx V \times \frac{dM^*}{M^*} + V \times \frac{dS}{S}\end{aligned}$$



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Decompose into three first-pass factors: a forward purchase of ISK 100m at 0.0125 for delivery at time  $T$  boils down to a portfolio of two PNS:

- ◇ **Asset:** your claim on the bank = the bank's PN *ad* 100m Kronar, whose market value depends on
  - ▷ the ISK risk-free rate for  $T$ , and
  - ▷ the exchange rate
- ◇ **Liability:** you wrote a PN *ad* HC 1.25m, whose value depends on
  - ▷ the HC risk-free rate for date  $T$
- ◇ **Bottom line: five exposures**
  - ▷ two "adjacent" domestic interest rates
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◇ **Replication:** in the short run, a currency option behaves like a portfolio of

▷ forex PN

▷ domestic PN

$$C = \underbrace{\frac{S_t}{(1 + R_{t,T}^*)^{T-t}}}_{\text{HC price FC PN}} \underbrace{N(d_1)}_{\substack{\text{\# of FC} \\ \text{PN's you} \\ \text{hold}}} - \underbrace{\frac{X}{(1 + R_{t,T})^{T-t}}}_{\text{price HC PN face X}} \underbrace{N(d_2)}_{\substack{\text{\# of HC} \\ \text{PN's you} \\ \text{hold}}}$$

Similar to forward purchase except for  $N(d_i) \neq 1$ .

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- ◇ **Replication:** a swap is like an exchange of two bonds that differ in terms of
  - ▷ interest rate: fixed, floating
  - ▷ currency of denomination
  - ▷ both
- ◇ **FRN bond:** behaves like a PN expiring at first coupon date
- ◇ **Fixed-rate bond:** behaves like a portfolio of PNS
- ◇ **Possibly Xrisk**



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- ◇ **Portfolio's sensitivity to factors** is obtained by summing:

$$\begin{aligned}
 dV_p &= E_{1,1}f_1 + E_{1,2}f_2 + \dots + E_{1,M}f_M \\
 &+ E_{2,1}f_1 + E_{2,2}f_2 + \dots + E_{2,M}f_M \\
 &+ E_{3,1}f_1 + E_{3,2}f_2 + \dots + E_{3,M}f_M \\
 &\dots \\
 &+ E_{n,1}f_1 + E_{n,2}f_2 + \dots + E_{n,M}f_M \\
 &= \left[ \sum_{i=1}^n E_{i,1} \right] f_1 + \left[ \sum_{i=1}^n E_{i,2} \right] f_2 + \dots + \left[ \sum_{i=1}^n E_{i,M} \right] f_M \\
 &= E_{p,1}f_1 + E_{p,2}f_2 + \dots + E_{p,M}f_M
 \end{aligned}$$

- ◇ **Portfolio variance** follows the usual formula—except that we use factors not returns, and elasticity-corrected amounts not amounts:

$$\text{var}(dV_p) = \sum_{j=1}^M E_{j,i} \sum_{i=1}^M E_{i,j} \text{cov}(f_i, f_j)$$



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# The Risk Budget — example

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## Example: Three positions:

- (i) domestic stock worth HC 150;
  - (ii) foreign stock worth HC 200; and
  - (iii) a 10-year forward sale worth, in PV, HC 100 each leg.
- Let  $R_{10} = 5\%$  and  $R_{10}^* = 4\%$ .

	stock	stock*	Xrate	$R_{10}$	$R_{10}^*$
	<b>elasticities</b>				
home stock	1	0	0	0	0
foreign stock	0	1	1	0	0
forward sale	0	0	-1	-0.476	0.385
	<b>pseudo capital</b>				
home stock	150	0	0	0	0
foreign stock	0	200	200	0	0
forward sale	0	0	-100	-47.6	38.5
portfolio	150	200	100	-47.6	38.6

$$dV_p \approx 150 \bar{r}_{\text{stock}} + 200 \bar{r}_{\text{stock}^*} + 100 \frac{dS}{S} - 47.8 \frac{dR_{10}}{R_{10}} + 38.6 \frac{dR_{10}^*}{R_{10}^*}.$$



# VaR: the arrows & the boxes



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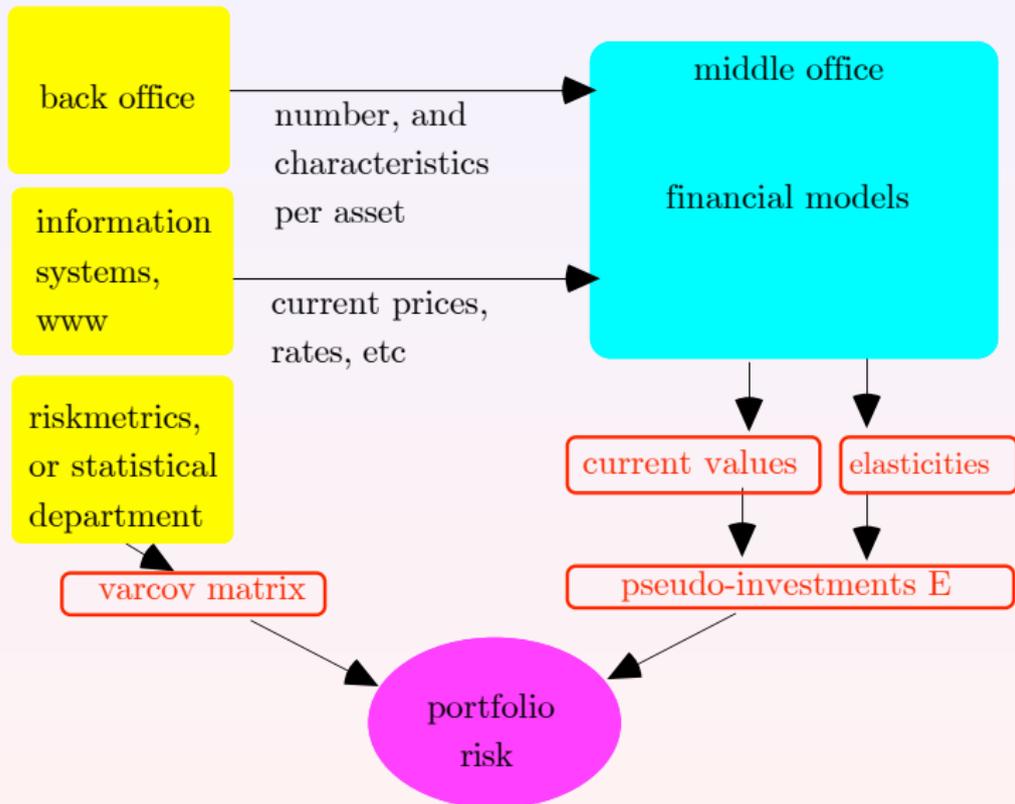
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- Zero Drift Processes
- A Constant-Variance Process
- Constant Correlations Between Factors
- Linearizations in the Mapping  $f \rightarrow dV$
- Choice of the factors
- Normality of  $dV$
- Assets can be Liquidated in one Day

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## Risk Budgeting—a Factor-based, Linear Approach

- Why work with factors not assets
- Linking Bonds to Interest-rate Factors
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## The Linear/Normal VaR Model: Potential Flaws & Corrections

- A Zero-Drift (“Martingale”) Process
- A Constant-Variance Process
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## Backtesting, Bootstrapping, Monte Carlo, and Stress Testing

- Backtesting
- Bootstrapping
- Monte Carlo Simulation
- Stress Testing

## Closing remarks

- CDOs/CDSs are hard to price
- Moral Hazard type risks



# Potential Flaws & Corrections — Overview

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1. Intertemporal independence of changes in the levels of prices or interest rates
2. Constant distribution of percentage changes in the levels of prices or interest rates
3. Constant linear relationships between the changes in the levels of prices or interest rates
4. Linearizations of links between underlying variables and between asset prices and factors
5. Choice of factors: in some respects too many, in other respects too few factors
6. Normality of the portfolio value
7. Liquidity.



# Zero-Drift (“Martingale”) Processes

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- ◇ **Assumption 1:**  $f_i$  fully unpredictable, apart from (tiny) drift

## ◇ Counterexamples

- ▷ mean reversion in  $R$ ; in  $S$ ; in goods prices; even in stocks

$$dR = \kappa(\mu_R - R) + \dots + \epsilon$$

- ▷ error-correction-typelinks between vars, e.g. various  $R$ s

$$dR_1 = \lambda(R_2 - R_1 - \nu_{1,2}) + \dots + \epsilon$$

## ◇ Evaluation

- ▷ not a serious problem at one-day or -week horizon



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# Constant-Variance Processes

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◇ **Assumption 2:**  $\text{var}_t(f_{i,t+1})$  is constant so that variance is easily estimated, and risk increases linearly in length of horizon  $T - t$

## ◇ Counterexamples

- ▷ managed Xrates: changes are a mixture of
  - intra-band-changes (whose distributions depend on the position in the band), and
  - “jumps” (re-alignments) whose chances are time-varying
- ▷ stock and bond prices are also subject to jump risk (e.g. crashes) with time-varying probabilities.
- ▷ asset markets experience waves of nervousness/sleepiness: variance has many possible levels instead of just two??



# Constant-Variance Processes

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# The GARCH correction

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**Correction Step 1 option 1: GARCH models:** the risk we expect for today depends on ...

- [AR:] how nervous we felt yesterday morning  $v$  average day:
- [MA:] to what extent we were surprised last night  $v$  i/t morning

$$\begin{aligned}
 \text{var}_t &= \overline{\text{var}} + \underbrace{\chi [\text{var}_{t-1} - \overline{\text{var}}]}_{\text{morning}} + \underbrace{\psi [\epsilon_{t-1}^2 - \text{var}_{t-1}]}_{\text{evening}} \\
 &= (1 - \chi) \overline{\text{var}} + (\chi - \psi) \text{var}_{t-1} + \psi \epsilon_{t-1}^2 \\
 &= (1 - \phi - \psi) \overline{\text{var}} + \underbrace{\phi \text{var}_{t-1}}_{\text{AR}} + \underbrace{\psi \epsilon_{t-1}^2}_{\text{MA}}, \quad \phi = \chi - \psi.
 \end{aligned}$$

Generalisation to GARCH(p,q):

$$\text{var}_t = \overline{\text{var}} + \sum_{i=1}^p \phi_i \text{var}_{t-i} + \sum_{j=1}^q \psi_j \epsilon_{t-j}^2$$



# The GARCH correction

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# RiskMetrics' Choice: GARCH(0,1)

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- ▶ standard statistics: constant weight, 1/Nobs
- ▶ GARCH(0,1): exponentially decaying weight,  
 $\psi(1 - \psi)^{n-i}$ ,  $i = 1, \dots, n$ :

$$\begin{aligned}
 \text{var}_t &= (1 - \psi) \text{var}_{t-1} + \psi \epsilon_{t-1}^2 \\
 &= (1 - \psi) \{ (1 - \psi) \text{var}_{t-2} + \psi \epsilon_{t-2}^2 \} + \psi \epsilon_{t-1}^2 \\
 &= (1 - \psi) \{ (1 - \psi) [ (1 - \psi) \text{var}_{t-3} + \psi \epsilon_{t-3}^2 ] + \psi \epsilon_{t-2}^2 \} + \psi \epsilon_{t-1}^2 \\
 &= \dots \\
 &= \psi [ \epsilon_{t-1}^2 + (1 - \psi) \epsilon_{t-2}^2 + (1 - \psi)^2 \epsilon_{t-3}^2 + \dots + (1 - \psi)^{n-1} \epsilon_{t-n}^2 ] \\
 &\quad + \underbrace{(1 - \psi)^n}_{\rightarrow 0} \text{var}_{t-n},
 \end{aligned}$$

RiskMetrics:  $0.06 \cdot 0.94^{n-i}$  daily,  $0.03 \cdot 0.97^{n-i}$  monthly:



# RiskMetrics' Choice: the Movie

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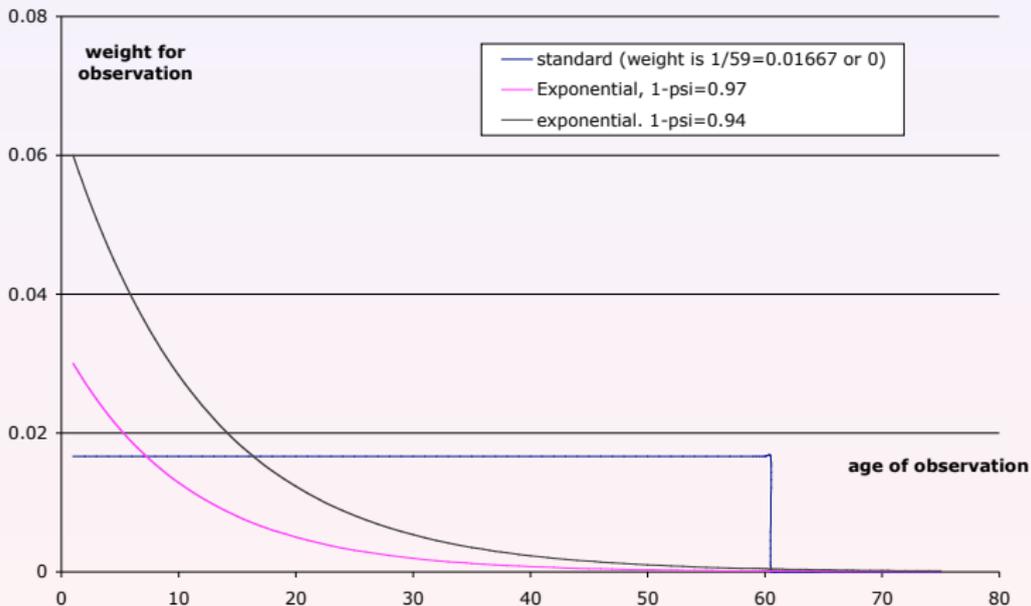
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weights in a exponential-decay model





# Other Corrections for Non-constant Variance

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### ◇ **Correction Step 1 option 2: ISD** instead of GARCH

- ▷ smirks: as many ISDs are there are  $T - t$ 's and  $X/S$ 's
- ▷ Deep ITM/OTM: sensitive to bid-ask

### ◇ **Correction Step 2: Covariances**

- ▷ GARCH-like models for covs: require many parameters or heavy restrictions
- ▷ CCC: constant conditional correlation:  
$$\text{COV}_t(f_i, f_j) = \sigma_{i,t} \sigma_{j,t} \rho(f_i, f_j)$$

### ◇ **Correction Step 3: add risk as a factor**

- ▷ variance is needed not just for  $\text{varcov}(f_i, f_j)$  but also as a pricing factor in derivatives
- ▷ options: vega
- ▷ bonds: convexity



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# T-Garch(1,1) plots (MY, RS, AR); GARCH(1,1) (AR)

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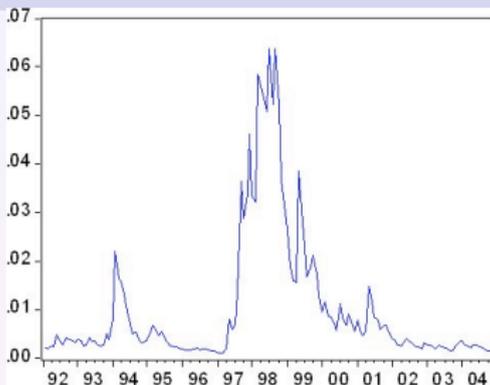
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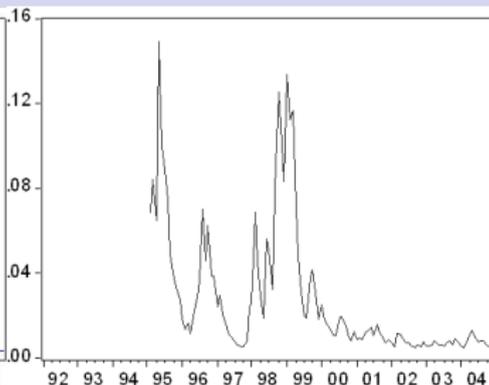
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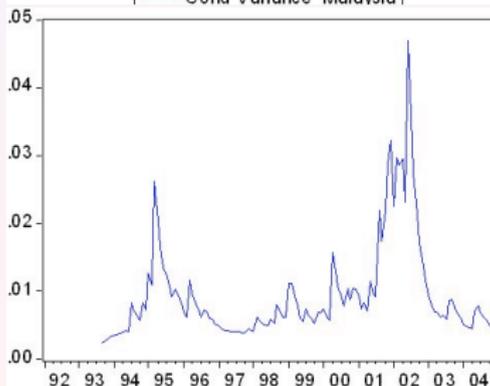
Closing remarks



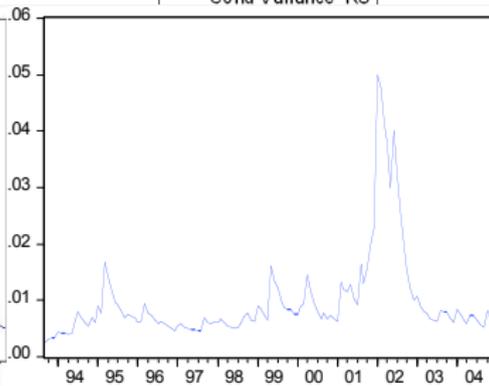
— Cond Variance Malaysia



— Cond Variance RS



— Cond Variance Argentina



— Conditional variance



# Constant Correlations Between Factors.

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◇ **Assumption 3:** constant correlations (i.e. linear links, up to noise) between factors

## ◇ Counterexamples/Corrections

- ▷ **Threshold effect:** in case of large changes, especially downward ones, correlations between stock returns turn out to be much larger than usual
  - ▷ asymmetric T-GARCH models
  - ▷ threshold correlations
- ▷ **"Intervaling" effect:** over longer periods (e.g. weekly returns), correlations turn out to be higher than for daily returns.
  - ▷ Scholes and Williams (1977) or Dimson (1979): take into account cross-correlations



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## Closing remarks

◇ **Assumption 4** Asset prices are linear in the factors, or, more fairly, non-linearities are not very relevant.

## ◇ Counterexamples:

- ▷ bond prices are non-linear in the underlying rates
- ▷ option premia are non-linear in the underlying prices

## ◇ Evaluation:

- ▷ bad when changes are big (crashes!)
- ▷ ... or horizons are longish (illiquid assets)

## ◇ Remediation:

- ▷ stress testing/Monte Carlo (see below)
- ▷ use quadratic approximations (option's gamma; bond's convexity); expand list of factors with squared factors



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◇ **Assumption 5:** to describe portfolios we just need exchange rates, stock market indices, interest rates (up to 13 per currency), and commodity prices.

## ◇ Counterexamples:

- ▷ Counterexamples (1): Missing factors Volatility; Idiosyncratic risk (for stocks)
- ▷ Counterexamples (2): Excessive factors Do we need 13 numbers to describe the term structure and its changes? Three factors already do a great job: shifts (up-down), slope (long v short maturities), and curvature.

## ◇ Remediation:

- ▷ add volatility as a factor, e.g. ISD
- ▷ correction for underdiversification:  $\text{var}(\bar{r}_m) + \overline{\text{var}}(\bar{\epsilon})/n$
- ▷ work with a more structured, parsimonious TS model



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# Normality of $dV$

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◇ **Assumption 6** Gaussian portfolio values

◇ **Counterexamples/evaluation:**

- ▷ Could come from multivariate normality for indiv asset returns. But asset returns are “leptocurtic” not normal.
- ▷ CLT effect? assumes a large number of IID risks; so it fails if ...
  - portfolios with few assets, especially if far from normal.
  - specialized portfolios with highly correlated assets, or
  - crash scenarios, when correlations go through the roof.
- ▷ CLT is about the center of the distribution; the tail could be—and is—very non-Gaussian

Normality badly underestimates worst outcomes

◇ **Remediation:**

- ▷ stress testing/Monte carlo
- ▷ double or triple the standard VaR number



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  - specialized portfolios with highly correlated assets, or
  - crash scenarios, when correlations go through the roof.
- ▷ CLT is about the center of the distribution; the **tail** could be—and is—very non-Gaussian

Normality badly underestimates worst outcomes

◇ **Remediation:**

- ▷ stress testing/Monte carlo
- ▷ double or triple the standard VaR number



# Normality of $dV$

## Value at Risk

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*International  
Finance: Theory into  
Practice*

## Risk Budgeting

## Linear/Normal VaR: Discussion

Zero Drift Processes

A Constant-Variance Process

Constant Correlations Between  
Factors

Linearizations in the Mapping  
 $f \rightarrow dV$

Choice of the factors

Normality of  $dV$

Assets can be Liquidated in  
one Day

## Other Approaches

## Closing remarks

◇ **Assumption 6** Gaussian portfolio values

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- ▷ Could come from multivariate **normality for indiv asset returns**. But asset returns are “leptocurtic” not normal.
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- ◇ **Assumption 7** if all equity gets eaten up in 24 hours one can stop the losses and sell out immediately, without extra price pressure.

### ◇ Counterexamples:

- ▷ illiquid markets?
- ▷ liquid markets—but what if many are in the same boat as you?

**Counterexamples:** during LTCM scare, hedge funds unwinding carry trade made JPY fall by 13% in three days—even 1% in 1 hr. — ● — Greenspan gave LTCM three months to liquidate — ● — price-pressure cost of liquidating Leeson's positions was GBP 50m — ● — during the subprime crisis, even interbank money froze (!)

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- ▷ widen horizon, e.g. to 2 weeks (Basel 2) — use multiple  $\sigma$



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### ◇ Idea:

- ▷ construct a 2- or 5-year “history” of today’s portfolio
- ▷ compute daily returns
- ▷ identify one-percent worst event

### ◇ Pros

- ▷ No distributional assumptions; picks up real-world returns
- ▷ Not necessarily top-down: look at *your* stocks, or close substitutes
- ▷ No linear approximations: observed prices, or full model prices for derivatives

### ◇ Cons

- ▷ is sample representative? No big crash in 5 years—so crashes no longer “exist”?
- ▷ we get a “marginal” distribution, but we need a conditional one
- ▷ 99%  $\neq$  good enough, but 99.75 is hard to estimate
- ▷ increasing the sample size is difficult and of doubtful validity

# Backtesting



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- ◇ **Bootstrapping:** using data sampled from the sample. Extra pros/cons relative to backtesting:
  - + gives you a feel of how much a VaR estimate can depend on sample
  - + allows you to build many more artificial 10-day *sequences* than there are genuine 10day periods
    - in building 10-day scenarios one loses autocorr in returns and risks—unless one uses block bootstrapping, but then there are far fewer independent drawings
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# Monte Carlo Simulation

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## Closing remarks

- ◇ **Monte Carlo Simulation:** “sample” the factors from a theoretical distribution, then price assets. Repeat  $nK$  times. Find 1st percentile portfolio value
  - + can still chose a thick-tailed, skewed distribution with crashes instead of a normal
  - + can chose a conditional distribution that takes into account recent events
  - + can generate huge “sample”s
  - + can take into account nonlinearities in factor-generating processes and in mapping (factor  $\rightarrow$   $dV$ )
  - many modeling choices needed
  - computationally heavy



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### ◇ Idea:

- ▷ look at worst days in past  $n$  years, see how this portfolio would have done
- ▷ dream up worst days in years to come (SARS+BirdFlu pandemic killing millions?), simulate what portfolio would do if markets crashes by 50%, ISD doubles, interest rates soar, etc

### ◇ Evaluation:

- + coherent disaster scenario for factors
- + not limited to what happened in last 2 years
- + take into account nonlinearities in mapping (factor  $\rightarrow$   $dV$ )
- every realised catastrophe would have sounded unthinkable ex ante (9/11; ERM 1992; Leeson; LTCM; 2007 shriveling of interbank mkt; 2008: us-wide crash of real estate prices), so we're probably way off.

UBS' VaR looked positively huge relative to actual fluctuations, but still turned out to be below subprime losses.



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### ◇ Use many methods *and* your common sense

### ◇ Basel II:

- assume 10 days for liquidation (daily  $\times \sqrt{10}$ )
- start from MA 99% VaR numbers multiplied by  $k$ :

$$MRC_t = \text{Max} \left( k \sum_{l=1}^{60} \frac{VaR_{t-l}}{60}, VaR_{t-1} \right) + SRC_t.$$

- $k=3$ —or  $\geq 4$  if backtests do not agree with your VaR
- SRC = Specific Risk Charge
- use at least one year of daily data
- update VaR at least every quarter.



# Closing remarks

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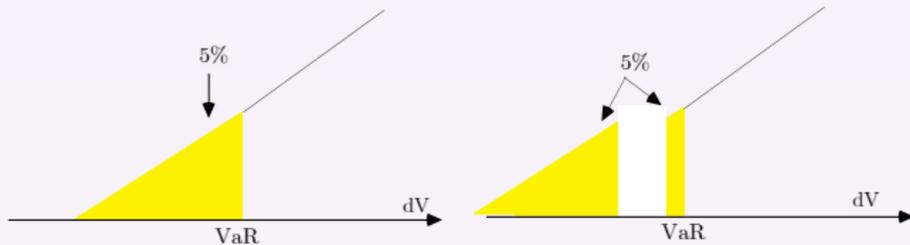
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### ◇ Risk of gaming



### ◇ Additions to VaR

- ▷ “CVAR” or (better:) expected shortfall or tail loss

$$E(\tilde{x}|x < VaR(n)) = \frac{\int_{-\infty}^{VaR(n)} xf(x) dx}{\int_{-\infty}^{VaR(n)} f(x) dx}.$$

(hard to estimate)

- ▷ Credit risks a la Basel I (CreditMetrics)—hard to estimate too



# VaR and the *Subprime* problem

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## ◇ **A hard-to-model product**

### ▷ **Default risk is much more difficult to model than market risk, ...**

- is debt a complicated American call on the assets as a whole?  
... or is default something that happens when  $V(t)$  reaches an  
¿exogenous/endogenous? default boundary?
- LT behavior of  $V(t)$  is crucial—but how does it behave? Brownian  
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- (mortgage backed securities: problem of modeling prepayment risk)

### ▷ ... and is at least as subject to “jumps” as market risk

- see the jump in CDS premia when Ford was downgraded, ¿May  
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### ▷ In addition, products are complex:

- individual loans packed together (mortgage-backed etc.) ...
- then sliced up into tranches (from “AAA” to “toxic waste”) in a way  
that is hard to verify from the outside
- often re-packaged into funds of slices
- sometimes an explicit guarantee from the sponsor (for e.g. 80%,  
Dexia), often also implicit guarantee



# VaR and the *Subprime* problem

## Value at Risk

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Practice*

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## Linear/Normal VaR: Discussion

## Other Approaches

## Closing remarks

CDOs/CDSs are hard to price

Moral Hazard type risks

## ◇ **A hard-to-model product**

### ▷ **Default risk is much more difficult to model than market risk, ...**

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- ▷ motivation of branch manager etc when bonus reflects deals made, not ex post contribution to profit  
Id: motivation of credit committees when loans are flogged rather than held:
  - 2000-06: fraction of non-investment-grade loans kept by bank fell from 90% to 60% (Europe), 60% to 20% (US)
  - e.g. up to half of customers for subprime loans lied in their applications, and obviously were not checked
- ▷ asymmetric info: bank that packages/slices/repackages the loans knows far more (winner's curse, adverse selection)
- ▷ agency problem when rating agencies are paid by the seller of the assets. IKB paid \$200m/year in fees to rating agencies & advisers
- ▷ moral hazard: the "too big to fail" free put / the Greenspan put



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### ◇ **Hiding one's riskier positions**

- ▷ parked into off-balance-sheet “Conduits”/“Structured Investment Vehicle”. 2007 Barclays estimate: \$1.4tr in conduits

### ◇ **VaR v Credit risk**

- ▷ CDOs were deemed to be tradeable securities, so they were assigned to the VaR quants
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### ◇ **It's not fun to be a Cassandra**

- ▷ Tensions between front & middle office: VaR teams felt increasingly uneasy, but risk seemed to shrink everywhere; 'cry wolf' problem
- ▷ Traders wanted their bonuses and saw VaR crowd as spoilsports



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