

Metaphysical Implications of Quantum Mechanical Dynamics

SERIES PAGE TITLE

Series Author Name

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi.

Series book title1, *Author of series book1*

Series book title2, *Author of series book2*

Series book title3, *Author of series book3*

Metaphysical Implications of Quantum Mechanical Dynamics

Applications of Magic in Engineering, Physics, and Neuroscience

A. N. Author

Second Author

Author Three

Author Four

PRINCETON UNIVERSITY PRESS
PRINCETON AND OXFORD

© 2023 Princeton University Press

Requests for permission to reproduce material from this work should be sent to Permissions, Princeton University Press

Published by Princeton University Press,
41 William Street, Princeton, New Jersey 08540

In the United Kingdom: Princeton University Press,
6 Oxford Street, Woodstock, Oxfordshire OX20 1TR

press.princeton.edu

All Rights Reserved

British Library Cataloging-in-Publication Data is available

Printed on acid-free paper.

Typesetting and Printing information: Typeset in \LaTeX Using the Princeton University Press Macros

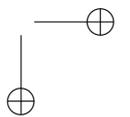
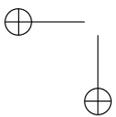
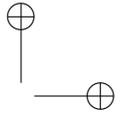
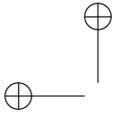
To our families, with gratitude,
—JB and MJ

Epigraph text rendait une étrange musique
Comme l'eau courante et le vent,
Ou le grain qu'un vanneur d'un mouvement rythmique
Agite et tourne son van.

Charles Baudelaire, *Une charogne* (Les Fleurs du Mai)

Brief Contents

LIST OF ALGORITHMS	<i>xv</i>
NOTATION	<i>xxiii</i>
PART I FIRST PART TITLE	
1 The First Chapter	3
PART II SECOND PART TITLE	
2 The Second Chapter	13
APPENDICES	
A Derivation of Expression for the Melnikov Function, Being a Complete Explication of Mysteries of the World	21
B	23
LIST OF ABBREVIATIONS	25
LIST OF DEFINITIONS	27
BOOK END NOTES	33
LIST OF CONTRIBUTORS	37
INDEX	39

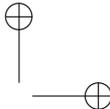


Contents

List of Figures	xi
List of Tables	xiii
List of Algorithms	xv
Foreword	xvii
Preface	xix
Acknowledgments	xxi
Introduction	xxii
Notation	xxiii
PART I First Part Title	
1 Short version of the First Chapter	3
1.1 The First Section	3
1.2 Listing Environments	4
1.3 A bevy of Theorem Environments	6
1.4 More Theorem Examples	7
1.5 Math and Tables	7
PART II Second Part Title	
2 The Second Chapter	13
2.1 Longtable sample	14
2.2 Rotated Table	15
2.3 First	19

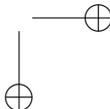
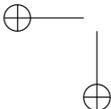
Appendices

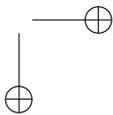
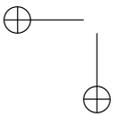
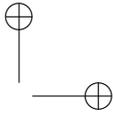
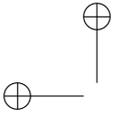
Appendix A Derivation of Expression	21
A.1 A Section Head in the Appendix	21
Appendix B	23
B.1 A Section Head in the Appendix	23
List of Abbreviations	25
List of Definitions	27
Glossary	29
Notes	33
Bibliography	35
List of Contributors	37
Index	39



List of Figures

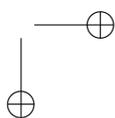
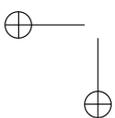
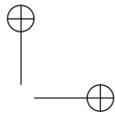
1.1	This is a short caption.	5
1.2	This is an example of a long caption that can accomodate two or more lines correctly.	5
A.1	Test figure caption in Appendix	22

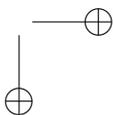
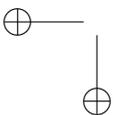
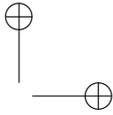




List of Tables

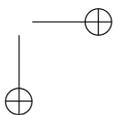
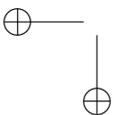
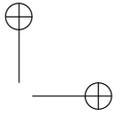
1.1	An example of a generic table.	8
2.1	A sample long table.	14
2.2	Effects of two types of scaling proposed by Dennard and co-workers.	16
A.1	Test table caption in Appendix	22





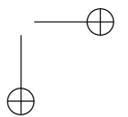
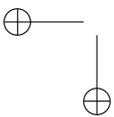
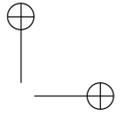
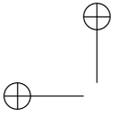
List of Algorithms

2.1	1 Short caption for algorithm	17
A.1	Here is algorithm caption.	21



Foreword

Here is the foreword. Here is the foreword. Here is the foreword. Here is the foreword.
Here is the foreword.



Preface

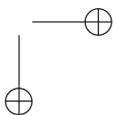
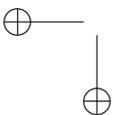
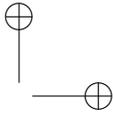
Here is the preface. Here is the preface.

Extract is used for a quotation that is only one paragraph long. Extract is used for a quotation that is only one paragraph long.

—Source of text in Extract

Here is the continuing preface. Here is the continuing preface.

The Author
Date Details



Acknowledgments

Acknowledgments found here. Acknowledgments found here. Acknowledgments found here. Acknowledgments found here. Acknowledgments found here.

Introduction

Introduction here. Iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

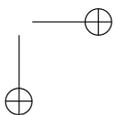
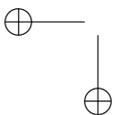
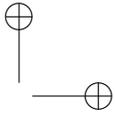
Notation

$g_{\mu\nu}(x^\lambda) = g_{\nu\mu}(x^\lambda)$ symmetric tensor

$g_{\mu\nu} \equiv \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ Minkowski spacetime

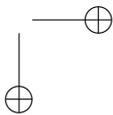
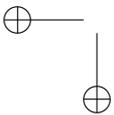
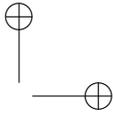
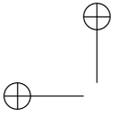
$s^2 = \Delta r^2 - c^2 \Delta t^2$ Spacetime interval

$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$ Einstein field equations



Part I

First Part
Title



Chapter One

The First Chapter

Applications of Magic in Engineering, Physics, and Neuroscience

1.1 THE FIRST SECTION THE FIRST SECTION THE FIRST SECTION

Lorem ipsum Lorem dolor sit ¹ amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. ²

1.1.1 The First Subsection

This is the subsection. This is the subsection.

1.1.1.1 A SubSubSection

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet.

GEOMETRICAL ASSUMPTIONS: AN EXAMPLE OF A D-HEAD

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit.

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi

$$V_T(x) \leq \left(\frac{\beta}{2} + \frac{M^2 \alpha}{2} e^{-2\omega T} \right) |x|^2 \leq \left(\frac{\beta}{\alpha} + M^2 e^{-2\omega T} \right) G(x).$$

A DIALOGUE

From the NY Times article of February 11, 2016, *Gravitational Waves Detected, Confirming Einstein's Theory*:

Francis Córdova It's been decades, through a lot of different technological innovations, [and the foundation's advisory board had] really scratched their heads on this one.

Janna Levin I was astounded!

Robert Garisto [the editor of Physical Review Letters] I got goose bumps while reading the LIGO paper.

1.2 LISTING ENVIRONMENTS

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros.

An example of an unnumbered list. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

Lobortis nisl ut aliquip ex ea commodo consequat.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait. End of unnumbered list.

Lorem ipsum dolor sit amet, Lorem consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

- *An example of a bullet list.* Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.
 - Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.
1. *An example of a numbered list.* Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.



Figure 1.1: This is a short caption.



Figure 1.2: This is an example of a long caption that can accomodate two or more lines correctly.

- a) Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.
- b) Lobortis nisl ut aliquip ex ea commodo consequat.
2. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat, Lobortis nisl ut aliquip ex ea commodo consequat.
3. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi.

1.3 A BEVY OF THEOREM ENVIRONMENTS

Here is the normal page width. Here is the normal page width.

Theorem 1.1. *This is theorem.*

Lemma 1.2. *This is a lemma.*

Theorem 1.3 (The Optional Argument). *Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.*

Proof. This is a proof ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. ■

Corollary 1.4. *Corollary modifies Theorem, and is numbered using the most recent theorem number as well as the corollary number.*

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

Conjecture 1.5. *We conject the following.*

Proposition 1.6. *Here is a new proposition.*

MAKING A NEW THEOREM ENVIRONMENT WITH AMSTHM

Here are the steps needed to make a new theorem command:

Setting theorem style: The definition will be made using the theorem style current when the newtheorem is made; use either

1. `\theoremstyle{plain}` for italic text; or
2. `\theoremstyle{definition}` for roman text.

Setting the theorem counter: The theorem counter we choose will be used on all the following theorem type environments.

(Notice that the theoremstyle given is surrounded with curly brackets, so that the style given does not confuse the style commands in the PUP-Book.cls file)

1. Chapter.Section.Thmnumber:


```
{\theoremstyle{plain} \newtheorem{theorem}{Theorem}[section]}
```
2. Only theorem number:


```
{\theoremstyle{plain} \newtheorem{theorem}{Theorem}}
```

3. Chapter number.Theorem number
(default and PUP preferred theorem numbering style)

```
{\theoremstyle{plain} \newtheorem{theorem}{Theorem}[chapter]}
```

Using the current theorem definition to build a new kind of theorem:

```
\newtheorem{suppose}[theorem]{Supposition}
```

Using the new theorem environment:

```
\begin{suppose}
Here is a supposition.
\end{suppose}
```

Supposition 1.7. *Here is a supposition.*

1.4 MORE THEOREM EXAMPLES

Assumption 1.8. Here is the assumption.

Here is the normal page width. Here is the normal page width.

Definition 1.9. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

Definition 1.10 (Title of Definition). Here is definition with a title.

Example 1.11. This is an **example**. An integrable dynamical system with two centers and a saddle point: the standard Duffing-Holmes oscillator.

Remark 1.12. This is remarkable.

Remark 1.13 (Optional title of remark). Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Exercise 1.14. This is an exercise

1.5 MATH AND TABLES

The pairs (S, C) with

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Table 1.1: An example of a generic table.

	Spanned Head		
	Column 1	Column 2	Column 3
Row 1	(x_1, y_1)	(x_2, y_1)	(x_3, y_1)
Row 2	(x_1, y_2)	(x_2, y_2)	(x_3, y_2)
Row 30	(x_1, y_3)	(x_2, y_3)	(x_3, y_3)

Table footnote Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi

and

$$S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

are examples of SNS-matrix pairs.

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exercitation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Let $S = [s_{ij}]$ ($1 \leq i, j \leq n$) be a $(0, 1, -1)$ -matrix of order n . Then S is a *sign-nonsingular matrix* (SNS-matrix) provided that each real matrix with the same sign pattern as S is nonsingular. There has been considerable recent interest in constructing and characterizing SNS-matrices. There has also been interest in strong forms of sign-nonsingularity. In this paper we give a new generalization of SNS-matrices and investigate some of their basic properties.

$$\int_a^b \left(\sum_i E_i B_{i,k,x}(t) \right) \left(\sum_j F_j B_{j,l,y}(t) \right) dt, \quad (1.1)$$

$$\int_a^b f(t) \left(\sum_i E_i B_{i,k,x}(t) \right) dt, \quad (1.2)$$

where $B_{i,k,x}$ is the i th B-spline of order k defined over the knots $x_i, x_{i+1}, \dots, x_{i+k}$. We will consider B-splines normalized so that their integral is one. The splines may be of different orders and defined on different knot sequences x and y . Often the limits of integration will be the entire real line, $-\infty$ to $+\infty$.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril

$$\frac{x+1}{y-1} \quad (1.3)$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi³ enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. This implies that for every $\alpha > \beta$ there exists $\bar{T} > 0$ such that for $T \geq \bar{T}$ we have $V_T(x) \leq \rho_T G(x)$ with $\rho_T < 1$, and thus the theorem applies for $T \geq \bar{T}$. Duis autem vel eum facilisi.⁴

EXERCISES

- 1.1 Here is the first exercise, in the first set of exercises.
 1.2 Here is the second exercise.

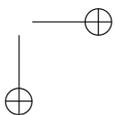
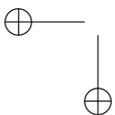
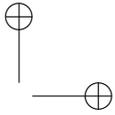
Another set of exercises:

EXERCISES

- 1.3 Here is the first exercise in the second set of exercises. Here is the first exercise.
 (a) Here is a sub-exercise. Here is the first exercise.
 (b) Here is another sub-exercise. Here is the first exercise.
 1.4 The final exercise in this set.

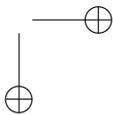
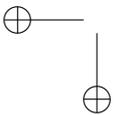
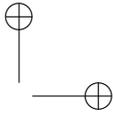
SELECTED ANSWERS TO EXERCISES

- 1.1 Here is the answer for exercise listed.
 1.3 Here is the answer for the first exercise in the second exercise set.
 (a) Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.
 (b) Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.



Part II

Second Part Title



Chapter Two

The Second Chapter

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum

—Author of Chapter Epigraph

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. ¹ consequat.

$$\begin{cases} \frac{d}{dt}x(t) = f(x(t), u(t)) \text{ for } t > 0, \\ x(0) = x_0, \quad u(t) \in U. \end{cases} \quad (2.1)$$

EXERCISES

2.1 Here is the first exercise in the second chapter.

2.2 Here is the second exercise.

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse suscipit lobortis nisl ut aliquip ex ea commodo consequat.

$$\min \int_{T_i}^{T_i+T} f^0(x(t), u(t)) dt + G(x(T_i + T)) \quad (2.2)$$

molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan, suscipit lobortis nisl ut aliquip ex ea commodo consequat, et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait suscipit lobortis nisl ut aliquip ex ea commodo consequat, nulla facilisi.

Table 2.1 – *Continued from previous page*

<i>First column</i>	<i>Second column</i>	<i>Third column</i>
And	So	On

2.2 ROTATED TABLE

On the next page you will see a rotated table. Tables should only be rotated if they are larger than 30pc. In other words, you wouldn't need to rotate a table unless it was too wide to fit in the normal text width.

Table 2.2: Effects of two types of $\alpha\beta \sum_B^A$ scaling proposed by Dennard and co-workers^{a,b}

Parameter	κ Scaling	κ, λ Scaling
Dimension	κ^{-1}	λ^{-1}
Voltage	κ^{-1}	κ^{-1}
Current	κ^{-1}	λ/κ^2
Dopant Concentration	κ	λ^2/κ

^aRefs. 19 and 20.^b $\kappa, \lambda > 1$.

```

1: if  $i \geq \text{maxval}$  then
2:    $i \leftarrow 0$ 
3: else
4:   if  $i + k \leq \text{maxval}$  then
5:      $i \leftarrow i + k$ 
6:   end if
7: end if

```

Algorithm 2.1: Here is an algorithm caption. Here is an algorithm caption.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. ²

Box 2.1 / Here is a Boxed Text Title

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

$$f(x, u) \cdot G_x(x) + f^0(x, u) \geq \delta \quad (2.3)$$

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi.

By introducing the product topology on $R^{m \times m} \times R^{n \times n}$ with the induced inner product

$$\langle (A_1, B_1), (A_2, B_2) \rangle := \langle A_1, A_2 \rangle + \langle B_1, B_2 \rangle, \quad (2.4)$$

we calculate the Fréchet derivative of F as follows:

$$\begin{aligned}
F'(U, V)(H, K) &= \langle R(U, V), H \Sigma V^T + U \Sigma K^T - \\
&\quad P(H \Sigma V^T + U \Sigma K^T) \rangle \\
&= \langle R(U, V), H \Sigma V^T + U \Sigma K^T \rangle \\
&= \langle R(U, V) V \Sigma^T, H \rangle + \langle \Sigma^T U^T R(U, V), K^T \rangle.
\end{aligned} \quad (2.5)$$

In the middle line of (2.5) we have used the fact that the range of R is always perpendicular to the range of P .

$$\nabla F(U, V) = (R(U, V)V\Sigma^T, R(U, V)^T U\Sigma) \in R^{m \times m} \times R^{n \times n}. \quad (2.6)$$

Because of the product topology, we know

$$\mathcal{T}_{(U, V)}(\mathcal{O}(m) \times \mathcal{O}(n)) = \mathcal{T}_U \mathcal{O}(m) \times \mathcal{T}_V \mathcal{O}(n), \quad (2.7)$$

where $\mathcal{T}_{(U, V)}(\mathcal{O}(m) \times \mathcal{O}(n))$ stands for the tangent space to the manifold $\mathcal{O}(m) \times \mathcal{O}(n)$ at $(U, V) \in \mathcal{O}(m) \times \mathcal{O}(n)$ and so on. The projection of $\nabla F(U, V)$ onto $\mathcal{T}_{(U, V)}(\mathcal{O}(m) \times \mathcal{O}(n))$, therefore, is the product of the projection of the first component of $\nabla F(U, V)$ onto $\mathcal{T}_U \mathcal{O}(m)$ and the projection of the second component of $\nabla F(U, V)$ onto $\mathcal{T}_V \mathcal{O}(n)$. In particular, we claim that the projection $g(U, V)$ of the gradient $\nabla F(U, V)$ onto $\mathcal{T}_{(U, V)}(\mathcal{O}(m) \times \mathcal{O}(n))$ is given by the pair of matrices:

$$g(U, V) = \left(\frac{R(U, V)V\Sigma^T U^T - U\Sigma V^T R(U, V)^T}{2} U, \right. \\ \left. \frac{R(U, V)^T U\Sigma V^T - V\Sigma^T U^T R(U, V)}{2} V \right). \quad (2.8)$$

Thus, the vector field

$$\frac{d(U, V)}{dt} = -g(U, V) \quad (2.9)$$

defines a steepest descent flow on the manifold $\mathcal{O}(m) \times \mathcal{O}(n)$ for the objective function $F(U, V)$. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

$$\begin{cases} \frac{d}{dt}x(t) = f(x(t), u(t)), & t > 0, \\ x(0) = x_0, \end{cases} \quad (2.10)$$

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Box 2.2 / Frank Wilczek on Einstein and Gravitation

Einstein's general relativity, as a theory of gravitation, is so tight conceptually that it allows only two free parameters: Newton's constant and the cosmological term. It has passed every test that physicists and astronomers have devised. Yet there are reasons to remain dissatisfied.

2.3 FIRST

First, the strength of gravity is grossly disproportionate to the strength of other forces. If we believe in the unity of nature's operating system, how can that be?

2.3.1 Second

Second, the measured value of the mass density of space devoid of matter—the cosmological term, often called dark energy—is incommensurate with reasonable expectations. Why is it much smaller than theory suggests, yet not zero?

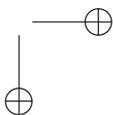
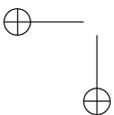
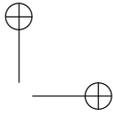
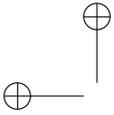
2.3.1.1 Third

Third, the equations that follow from straightforward quantization of general relativity break down in extreme conditions. What are the consequences? Those issues are important agenda items for the next 100 years of physics. In the boxes, I've indicated a promising way to approach the question of the weakness of gravity. Here I'll offer a few comments on the other issues. ... (Frank Wilczek, *Physics Today*, April 2016, scitation.aip.org/content/aip/magazine/physicstoday/article/69/4/10.1063/PT.3.3137)

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi.

This is shaded. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi.

This is not shaded. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.



Appendix A

Derivation of Expression for the Melnikov Function, Being a Complete Explication of Mysteries of the World

Here is an optional subtitle

A.1 A SECTION HEAD IN THE APPENDIX

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. ¹ Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

$$\begin{cases} \inf \int_0^\infty (\ell(x(t)) + \frac{1}{2}|u(t)|^2) dt, \\ \dot{x}(t) = a(x(t)) + B(x(t))u(t), \quad x(0) = x_0, \end{cases} \quad (\text{A.1})$$

- 1: **if** $i \geq \text{maxval}$ **then**
- 2: $i \leftarrow 0$
- 3: **else**
- 4: **if** $i + k \leq \text{maxval}$ **then**
- 5: $i \leftarrow i + k$
- 6: **end if**
- 7: **end if**

Algorithm A.1: Here is algorithm caption.



Figure A.1: Test figure caption in Appendix

Table A.1: Test table caption in Appendix

Here is a little tiny table
Centering over 4 columns

Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh

$$Ax \cdot W_x(x) - \frac{1}{2} |b^T W_x(x)|^2 + \ell(x) - Ax \cdot U_x + \frac{1}{2} |b^T U_x|^2 = 0.$$

euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam. quis nostrud exerci tation ullamcorper suscipit

Appendix B

B.1 A SECTION HEAD IN THE APPENDIX

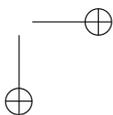
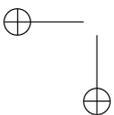
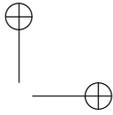
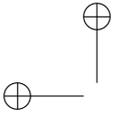
Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. ¹

```
my_packages c("tidyverse", "broom", "coefplot", "cowplot",
             "here", "interplot", "margins", "maps", "mapproj",
             "mapdata", "MASS", "quantreg", "rlang", "scales",
             "survey", "srvyr", "viridis", "viridisLite", "devtools")
```

Showing normal text in gray box. dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea .

```
Here is a
      code listing
```

```
Here is a
      code listing
      without gray background
```



List of Abbreviations

WHO World Health Organization, a global presence that puts countries at the centre of our work. From our onstanding Geneva headquarters to our 6 regional offices, 150 country offices and other offices around the world, WHO plays an essential role improving local health systems and coordinating the global response to health threats. Discover how we work to support the efforts of governments and partners to ensure everyone, everywhere has an equal chance at a safe and healthy life. (from WHO website)

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

WTO World Trade Organization

- GD** Group Discussion
- WHO** World Health Organization
- WTO** World Trade Organization
- GD** Group Discussion
- WHO** World Health Organization
- WTO** World Trade Organization
- GD** Group Discussion
- WHO** World Health Organization
- WTO** World Trade Organization
- GD** Group Discussion
- WHO** World Health Organization
- WTO** World Trade Organization
- GD** Group Discussion

List of Definitions

Poisson distribution

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

The Poisson distribution can also be used for the number of events in other specified interval types such as distance, area or volume.

Calculus derivative

In mathematics, the derivative of a function of a real variable measures the sensitivity to change of the function value with respect to a change in its argument. Derivatives are a fundamental tool of calculus.

Third derivative

In calculus, a branch of mathematics, the third derivative is the rate at which the second derivative, or the rate of change of the rate of change, is changing.

The third derivative of a function $y = f(x)$ can be denoted by

$$\frac{d^3y}{dx^3}, \quad f'''(x), \quad \text{or} \quad \frac{d^3}{dx^3}[f(x)].$$

Other notations can be used, but the above are the most common.

Poisson distribution

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

The Poisson distribution can also be used for the number of events in other specified interval types such as distance, area or volume.

Calculus derivative In mathematics, the derivative of a function of a real variable measures the sensitivity to change of the function value with respect to a change in its argument. Derivatives are a fundamental tool of calculus.

Third derivative In calculus, a branch of mathematics, the third derivative is the rate at which the second derivative, or the rate of change of the rate of change, is changing.

The third derivative of a function $y = f(x)$ can be denoted by

$$\frac{d^3y}{dx^3}, \quad f'''(x), \quad \text{or} \quad \frac{d^3}{dx^3}[f(x)].$$

Other notations can be used, but the above are the most common.

Poisson distribution In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

The Poisson distribution can also be used for the number of events in other specified interval types such as distance, area or volume.

Calculus derivative In mathematics, the derivative of a function of a real variable measures the sensitivity to change of the function value with respect to a change in its argument. Derivatives are a fundamental tool of calculus.

Third derivative In calculus, a branch of mathematics, the third derivative is the rate at which the second derivative, or the rate of change of the rate of change, is changing.

The third derivative of a function $y = f(x)$ can be denoted by

$$\frac{d^3y}{dx^3}, \quad f'''(x), \quad \text{or} \quad \frac{d^3}{dx^3}[f(x)].$$

Other notations can be used, but the above are the most common.

Glossary

NormGibbs	Draw a sample from a posterior distribution of data with an unknown mean and variance using Gibbs sampling.
pNull	Test a one sided hypothesis from a numerically specified posterior CDF or from a sample from the posterior
sintegral	A numerical integration using Simpson's rule
long term here	Sample of a term that might break over lines.
NormGibbs	Draw a sample from a posterior distribution of data with an unknown mean and variance using Gibbs sampling.
pNull	Test a one sided hypothesis from a numerically specified posterior CDF or from a sample from the posterior
sintegral	A numerical integration using Simpson's rule
long term here	Sample of a term that might break over lines.
NormGibbs	Draw a sample from a posterior distribution of data with an unknown mean and variance using Gibbs sampling.
pNull	Test a one sided hypothesis from a numerically specified posterior CDF or from a sample from the posterior
sintegral	A numerical integration using Simpson's rule
long term here	Sample of a term that might break over lines.
NormGibbs	Draw a sample from a posterior distribution of data with an unknown mean and variance using Gibbs sampling.
pNull	Test a one sided hypothesis from a numerically specified posterior CDF or from a sample from the posterior
sintegral	A numerical integration using Simpson's rule
long term here	Sample of a term that might break over lines.
NormGibbs	Draw a sample from a posterior distribution of data with an unknown mean and variance using Gibbs sampling.

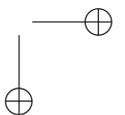
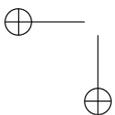
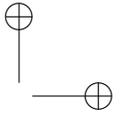
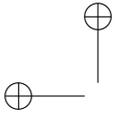
- pNull** Test a one sided hypothesis from a numerically specified posterior CDF or from a sample from the posterior
- sintegral** A numerical integration using Simpson's rule
- long term
here** Sample of a term that might break over lines.

Making Answers to Exercises

Answers to exercises all through the book may be given at the end of the book; or at the end of the chapter, or both.

SELECTED ANSWERS TO EXERCISES

- 1.1** Here is the answer for exercise listed.
- 1.3** Here is the answer for the first exercise in the second exercise set.
 - (a)** Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.
 - (b)** Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.
- 2.1** Here is the answer to the first exercise in Chapter 2.
- 2.2** Here is the answer to the second exercise in Chapter 2.



Notes

1. Sample for footnote text enim ad minim veniam, quis nostrud exerci tation ullamcorper
 2. Here is another note from the first chapter.
 3. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi.
 4. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue dui dolore te feugait nulla facilisi.
1. Text of the sample footnote. Text of the sample footnote. Text of the sample footnote. Text of the sample footnote.
 2. Here is a sample footnote that will not become an endnote.
1. Here is a footnote or endnote in an appendix.
 1. Here is an endnote in the second appendix. Here is an endnote in the second appendix. Here is an endnote in the second appendix. Here is an endnote in the second appendix.

Sample citations

These commands are found near the top of this file. Please add these commands to your .tex file if you want to use natbib commands.

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Choose bibliography styles:
%% Annals of Math bibliography style:
    \bibliographystyle{alpha}

%% PUP preferred bibliography style:
    \usepackage[round,sort]{natbib}
    \bibliographystyle{authordate1}

```

A list of natbib commands and their results is found in the PUP-docs.pdf file, under

/endmatter/bibliography/cites.

Here are example citations:

```

\cite{einstein},
\cite[chap.~1]{einstein}, \cite{goossens93},
and \cite{knuthwebsite}.

```

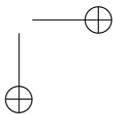
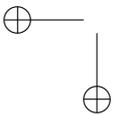
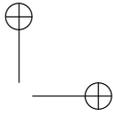
And their results: Einstein (1905), (Einstein, 1905, chap. 1), Goossens *et al.* (1993), and Knuth (2000).

Bibliography

Einstein, Albert. 1905. Zur Elektrodynamik bewegter Körper. (German) [On the electrodynamics of moving bodies]. *Annalen der Physik*, **322**(10), 891–921.

Goossens, Michel, Mittelbach, Frank, & Samarin, Alexander. 1993. *The L^AT_EX Companion*. Reading, Massachusetts: Addison-Wesley.

Knuth, Donald. 2000. *Knuth: Computers and Typesetting*.

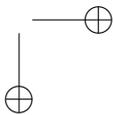
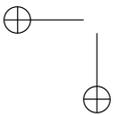
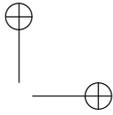


List of Contributors

Contributor name enim ad minim veniam, quis nostrud exerci tation Entry ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Contributor name Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation Entry ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

Contributor name Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation Entry ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.



Index

accumsan et iusto odio dignissim
 qui blandit praesent, 4
adipiscing, 17
aliquip ex ea commodo consequat, xxii
amet, xxii
augue
 duis, xxii
autem, 13
B-splines, 8
blandit praesent luptatum
 zzril delenit, xxii
commodo, 6, 7, 23
consectetuer, 17
consectetuer adipiscing elit, sed diam non-
 ummy, 35
different orders, 8
dolor, 17
dolore, 35
dolore te feugait, 35
duis dolore
 te feugait, 4
Entry, 35
facilisis at vero eros et accumsan et iusto
 odio dignissim, 21
feugait, 21
feugiat, 3
Lorem, 3, 4, 17
 ipsum, 3
Lorem ipsum
 dolor sit amet, 13
magna, 7, 13
minim veniam, 19
molestie, 17
nostrud, 23
nulla, 21
quis, 4
quis nostrud exerci tation
 ullamcorper, 21
SNS-matrices, 7
suscipit, 3
tation, 4
topology, 17
vel illum dolore
 eu feugiat
 nulla facilisis, 4
veniam, 8
vulputate velit esse molestie
 consequat, vel illum dolore eu feugiat
 nulla, 6
wisi, 23
zzril, 19, 23