Metaphysical Implications of Quantum Mechanical Dynamics

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Metaphysical Implications of Quantum Mechanical Dynamics

Applications of Magic in Engineering, Physics, and Neuroscience

A. N. Author

Second Author

Author Three

Author Four

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To our families, with gratitude,
—JB and MJ

Epigraph text rendait une étrange musique
Comme l'eau courante et le vent,
Ou le grain qu'un vanneur d'un mouvement rhytmique
Agite et tourne son van.

Charles Baudelaire, *Une charogne* (Les Fleurs du Mai)

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Here is the foreword. Here is the foreword. Here is the foreword. Here is the foreword. Here is the foreword.

Preface

Here is the preface. Here is the preface. Here is the preface. Here is the preface. Here is the preface. Here is the preface. Here is the preface. Here is the preface. Here is the preface.

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The Author
Date Details

Acknowledgments

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Introduction

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Notation

$g_{\mu\nu}(x^\lambda) = g_{\nu\mu}(x^\lambda)$	symmetric tensor
$g_{\mu\nu} \equiv \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$	Minkowski spacetime
$s^2 = \Delta r^2 - c^2 \Delta t^2$	Spacetime interval
$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$	Einstein field equations

Part I

First Part Title

Chapter One

The First Chapter

Applications of Magic in Engineering, Physics, and Neuroscience

1.1 THE FIRST SECTION THE FIRST SECTION THE FIRST SECTION

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1.1.1 The First Subsection

This is the subsection. This is the subsection. This is the subsection. This is the subsection. This is the subsection. This is the subsection.

1.1.1.1 A SubSubSection

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GEOMETRICAL ASSUMPTIONS: AN EXAMPLE OF A D-HEAD

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$$V_T(x) \leq \left(\frac{\beta}{2} + \frac{M^2\alpha}{2} e^{-2\omega T}\right)|x|^2 \leq \left(\frac{\beta}{\alpha} + M^2 e^{-2\omega T}\right) G(x).$$

A DIALOGUE

From the NY Times article of February 11, 2016, *Gravitational Waves Detected, Confirming Einstein's Theory*:

Francis Córdova It's been decades, through a lot of different technological innovations, [and the foundation's advisory board had] really scratched their heads on this one.

Janna Levin I was astounded!

Robert Garisto [the editor of Physical Review Letters] I got goose bumps while reading the LIGO paper.

1.2 LISTING ENVIRONMENTS

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An example of an unnumbered list. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

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- *An example of a bullet list.* Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.
- Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.

1. *An example of a numbered list.* Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.
   a) Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.
   b) Lobortis nisl ut aliquip ex ea commodo consequat.
2. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat, Lobortis nisl ut aliquip ex ea commodo consequat.
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Figure 1.1: This is a short caption.

Figure 1.2: This is an example of a long caption that can accomodate two or more lines correctly.

1.3 A BEVY OF THEOREM ENVIRONMENTS

Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width.

Theorem 1.1. *This is theorem.*

Lemma 1.2. *This is a lemma.*

Theorem 1.3 (The Optional Argument)**.** *Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.*

Proof. This is a proof ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. ∎

Corollary 1.4. *Corollary modifies Theorem, and is numbered using the most recent theorem number as well as the corollary number.*

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Conjecture 1.5. *We conject the following.*

Proposition 1.6. *Here is a new proposition.*

MAKING A NEW THEOREM ENVIRONMENT WITH AMSTHM

Here are the steps needed to make a new theorem command:

Setting theorem style: The definition will be made using the theorem style current when the newtheorem is made; use either

1. `\theoremstyle{plain}` for italic text; or
2. `\theoremstyle{definition}` for roman text.

Setting the theorem counter: The theorem counter we choose will be used on all the following theorem type environments.

(Notice that the theoremstyle given is surrounded with curly brackets, so that the style given does not confuse the style commands in the PUP-Book.cls file)

1. Chapter.Section.Thmnumber:
   `{\theoremstyle{plain} \newtheorem{theorem}{Theorem}[section]}}`
2. Only theorem number:
   `{\theoremstyle{plain} \newtheorem{theorem}{Theorem}}`
3. Chapter number.Theorem number
   (default and PUP preferred theorem numbering style)
   `{\theoremstyle{plain} \newtheorem{theorem}{Theorem}[chapter]}`

Using the current theorem definition to build a new kind of theorem:

```
\newtheorem{suppose}[theorem]{Supposition}
```

Using the new theorem environment:

```
\begin{suppose}
Here is a supposition.
\end{suppose}
```

Supposition 1.7. *Here is a supposition.*

1.4 MORE THEOREM EXAMPLES

Assumption 1.8. Here is the assumption.

Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width. Here is the normal page width.

Definition 1.9. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

Definition 1.10 (Title of Definition)**.** Here is definition with a title.

Example 1.11. This is an **example**. An integrable dynamical system with two centers and a saddle point: the standard Duffing-Holmes oscillator.

Remark 1.12. This is remarkable.

Remark 1.13 (Optional title of remark)**.** Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

Exercise 1.14. This is an exercise

1.5 MATH AND TABLES

The pairs (S, C) with

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

and

$$S = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

are examples of SNS-matrix pairs.

Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exercitation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Let $S = [s_{ij}]$ $(1 \leq i, j \leq n)$ be a $(0, 1, -1)$-matrix of order n. Then S is a *sign-nonsingular matrix* (SNS-matrix) provided that each real matrix with the same sign pattern as S is non-singular. There has been considerable recent interest in constructing and characterizing SNS-matrices. There has also been interest in strong forms of sign-nonsingularity. In this paper we

Table 1.1: An example of a generic table.

	Spanned Head		
	Column 1	Column 2	Column 3
Row 1	(x_1, y_1)	(x_2, y_1)	(x_3, y_1)
Row 2	(x_1, y_2)	(x_2, y_2)	(x_3, y_2)
Row 30	(x_1, y_3)	(x_2, y_3)	(x_3, y_3)

Table footnote Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi

give a new generalization of SNS-matrices and investigate some of their basic properties.

$$\int_a^b \Big(\sum_i E_i B_{i,k,x}(t)\Big)\Big(\sum_j F_j B_{j,l,y}(t)\Big) dt, \tag{1.1}$$

$$\int_a^b f(t)\Big(\sum_i E_i B_{i,k,x}(t)\Big) dt, \tag{1.2}$$

where $B_{i,k,x}$ is the ith B-spline of order k defined over the knots $x_i, x_{i+1}, \dots, x_{i+k}$. We will consider B-splines normalized so that their integral is one. The splines may be of different orders and defined on different knot sequences x and y. Often the limits of integration will be the entire real line, $-\infty$ to $+\infty$.

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$$\frac{x+1}{y-1} \tag{1.3}$$

Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi [3] enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. This implies that for every $\alpha > \beta$ there exists $\bar{T} > 0$ such that for $T \geq \bar{T}$ we have $V_T(x) \leq \rho_T G(x)$ with $\rho_T < 1$, and thus the theorem applies for $T \geq \bar{T}$. Duis autem vel eum facilisi. [4]

EXERCISES

1.1 Here is the first exercise, in the first set of exercises.

1.2 Here is the second exercise.

Another set of exercises:

EXERCISES

1.3 Here is the first exercise in the second set of exercises. Here is the first exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise.

- **(a)** Here is a sub-exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise.
- **(b)** Here is another sub-exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise. Here is the first exercise.

1.4 The final exercise in this set.

SELECTED ANSWERS TO EXERCISES

1.1 Here is the answer for exercise listed.

1.3 Here is the answer for the first exercise in the second exercise set.

- **(a)** Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.
- **(b)** Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.

Part II

Second Part Title

Chapter Two

The Second Chapter

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>
> —Author of Chapter Epigraph

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$$\begin{cases} \dfrac{d}{dt}x(t) = f(x(t), \quad u(t)) \quad \text{for} \quad t > 0, \\ x(0) = x_0, \quad u(t) \in U. \end{cases} \tag{2.1}$$

EXERCISES

2.1 Here is the first exercise in the second chapter.

2.2 Here is the second exercise.

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$$\min \int_{T_i}^{T_i+T} f^0(x(t), u(t))\, dt + G(x(T_i + T)) \tag{2.2}$$

molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan, suscipit lobortis nisl ut aliquip ex ea commodo consequat, et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait suscipit lobortis nisl ut aliquip ex ea commodo consequat, nulla facilisi.

2.1 LONGTABLE SAMPLE

An example Long Table follows.

Table 2.1: A sample long table.

First column	**Second column**	**Third column**
And	So	On
And	So	On
And	So	On
And	So	On
And	So	On
And	So	On
And	So	On
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And	So	On
And	So	On
And	So	On
And	So	On
And	So	On

(Continued on next page)

Table 2.1 – *Continued from previous page*

First column	***Second column***	***Third column***
And	So	On
And	So	On
And	So	On

2.2 ROTATED TABLE

On the next page you will see a rotated table. Tables should only be rotated if they are larger than 30pc. In other words, you wouldn't need to rotate a table unless it was too wide to fit in the normal text width.

Table 2.2: Effects of two types of $\alpha\beta \sum_B^A$ scaling proposed by Dennard and co-workers[a,b]

Parameter	$\varkappa$ Scaling	$\varkappa$, λ Scaling
Dimension	$\varkappa^{-1}$	λ^{-1}
Voltage	$\varkappa^{-1}$	$\varkappa^{-1}$
Currant	$\varkappa^{-1}$	$\lambda/\varkappa^2$
Dopant Concentration	$\varkappa$	$\lambda^2/\varkappa$

[a] Refs. 19 and 20.

[b] $\varkappa, \lambda > 1$.

if $i \geq maxval$ **then**
    $i \leftarrow 0$
else
    if $i + k \leq maxval$ **then**
        $i \leftarrow i + k$
    end if
end if

Algorithm 2.1: Here is an algorithm caption. Here is an algorithm caption. Here is an algorithm caption. Here is an algorithm caption. Here is an algorithm caption.

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. [2]

Box 2.1 / Here is a Boxed Text Title

Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat.

$$f(x,u) \cdot G_x(x) + f^0(x,u) \geq \delta \tag{2.3}$$

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By introducing the product topology on $R^{m\times m} \times R^{n\times n}$ with the induced inner product

$$\langle (A_1, B_1), (A_2, B_2) \rangle := \langle A_1, A_2 \rangle + \langle B_1, B_2 \rangle, \tag{2.4}$$

we calculate the Fréchet derivative of F as follows:

$$\begin{aligned} F'(U,V)(H,K) &= \langle R(U,V), H\Sigma V^T + U\Sigma K^T - \\ &\qquad P(H\Sigma V^T + U\Sigma K^T) \rangle \\ &= \langle R(U,V), H\Sigma V^T + U\Sigma K^T \rangle \\ &= \langle R(U,V)V\Sigma^T, H \rangle + \langle \Sigma^T U^T R(U,V), K^T \rangle. \end{aligned} \tag{2.5}$$

In the middle line of (2.5) we have used the fact that the range of R is always perpendicular to the range of P.

$$\nabla F(U,V) = (R(U,V)V\Sigma^T, R(U,V)^T U\Sigma) \in R^{m\times m} \times R^{n\times n}. \tag{2.6}$$

Because of the product topology, we know

$$\mathcal{T}_{(U,V)}(\mathcal{O}(m) \times \mathcal{O}(n)) = \mathcal{T}_U \mathcal{O}(m) \times \mathcal{T}_V \mathcal{O}(n), \tag{2.7}$$

where $\mathcal{T}_{(U,V)}(\mathcal{O}(m)\times\mathcal{O}(n))$ stands for the tangent space to the manifold $\mathcal{O}(m)\times\mathcal{O}(n)$ at $(U,V) \in \mathcal{O}(m)\times\mathcal{O}(n)$ and so on. The projection of $\nabla F(U,V)$ onto $\mathcal{T}_{(U,V)}(\mathcal{O}(m)\times\mathcal{O}(n))$, therefore, is the product of the projection of the first component of $\nabla F(U,V)$ onto $\mathcal{T}_U\mathcal{O}(m)$ and the projection of the second component of $\nabla F(U,V)$ onto $\mathcal{T}_V\mathcal{O}(n)$. In particular, we claim that the projection $g(U,V)$ of the gradient $\nabla F(U,V)$ onto $\mathcal{T}_{(U,V)}(\mathcal{O}(m)\times\mathcal{O}(n))$ is given by the pair of matrices:

$$g(U,V) = \left(\frac{R(U,V)V\Sigma^T U^T - U\Sigma V^T R(U,V)^T}{2}U, \frac{R(U,V)^T U\Sigma V^T - V\Sigma^T U^T R(U,V)}{2}V\right). \tag{2.8}$$

Thus, the vector field

$$\frac{d(U,V)}{dt} = -g(U,V) \tag{2.9}$$

defines a steepest descent flow on the manifold $\mathcal{O}(m)\times\mathcal{O}(n)$ for the objective function $F(U,V)$. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat.

$$\begin{cases} \dfrac{d}{dt}x(t) = f(x(t),u(t)), \quad t > 0, \\ x(0) = x_0, \end{cases} \tag{2.10}$$

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Box 2.2 / Frank Wilczek on Einstein and Gravitation

Einstein's general relativity, as a theory of gravitation, is so tight conceptually that it allows only two free parameters: Newton's constant and the cosmological term. It has passed every test that physicists and astronomers have devised. Yet there are reasons to remain dissatisfied.

2.3 FIRST

First, the strength of gravity is grossly disproportionate to the strength of other forces. If we believe in the unity of nature's operating system, how can that be?

2.3.1 Second

Second, the measured value of the mass density of space devoid of matter—the cosmological term, often called dark energy—is incommensurate with reasonable expectations. Why is it much smaller than theory suggests, yet not zero?

2.3.1.1 Third

Third, the equations that follow from straightforward quantization of general relativity break down in extreme conditions. What are the consequences? Those issues are important agenda items for the next 100 years of physics. In the boxes, I've indicated a promising way to approach the question of the weakness of gravity. Here I'll offer a few comments on the other issues. ... (Frank Wilczek, Physics Today, April 2016, scitation.aip.org/content/aip/magazine/physicstoday/article/69/4/10.1063/PT.3.3137)

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Appendix A

Derivation of Expression for the Melnikov Function, Being a Complete Explication of Mysteries of the World

Here is an optional subtitle

A.1 A SECTION HEAD IN THE APPENDIX

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$$\begin{cases} \inf \int_0^\infty (\ell(x(t)) + \frac{1}{2}|u(t)|^2)dt, \\ \dot{x}(t) = a(x(t)) + B(x(t))u(t), \quad x(0) = x_0, \end{cases} \tag{A.1}$$

if $i \geq maxval$ **then**
    $i \leftarrow 0$
else
    if $i + k \leq maxval$ **then**
        $i \leftarrow i + k$
    end if
end if

Algorithm A.1: Here is algorithm caption.

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$$Ax \cdot W_x(x) - \frac{1}{2}\,|b^T W_x(x)|^2 + \ell(x) - Ax \cdot U_x + \frac{1}{2}\,|b^T U_x|^2 = 0.$$

Figure A.1: Test figure caption in Appendix

Table A.1: Test table caption in Appendix

Here	is a	little tiny	table
Centering over 4 columns			

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Appendix B

B.1 A SECTION HEAD IN THE APPENDIX

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```
my_packages  c("tidyverse", "broom", "coefplot", "cowplot",
"here", "interplot", "margins", "maps", "mapproj",
        "mapdata", "MASS", "quantreg", "rlang", "scales",
"survey", "srvyr", "viridis", "viridisLite", "devtools")
```

Showing normal text in gray box. dolore te feugait nulla facilisi. Lorem ipsum dolor sit amet, consectetuer adipiscing elit, sed diam nonummy nibh euismod tincidunt ut laoreet dolore magna aliquam erat volutpat. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea .

```
Here is a
        code listing
```

```
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        code listing
             without gray background
```

List of Abbreviations

WHO World Health Organization, a global presence that puts countries at the centre of our work. From our ongstanding Geneva headquarters to our 6 regional offices, 150 country offices and other offices around the world, WHO plays an essential role improving local health systems and coordinating the global response to health threats. Discover how we work to support the efforts of governments and partners to ensure everyone, everywhere has an equal chance at a safe and healthy life. (from WHO website)

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

WTO World Trade Organization

GD Group Discussion

WHO World Health Organization

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GD Group Discussion

WHO World Health Organization

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GD	Group Discussion

List of Definitions

Poisson distribution In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event.

The Poisson distribution can also be used for the number of events in other specified interval types such as distance, area or volume.

Calculus derivative In mathematics, the derivative of a function of a real variable measures the sensitivity to change of the function value with respect to a change in its argument. Derivatives are a fundamental tool of calculus.

Third derivative In calculus, a branch of mathematics, the third derivative is the rate at which the second derivative, or the rate of change of the rate of change, is changing.

The third derivative of a function $y = f(x)y = f(x)$ can be denoted by

$$\frac{d^3y}{dx^3}, \quad f'''(x), \quad \text{or} \quad \frac{d^3}{dx^3}[f(x)].$$

Other notations can be used, but the above are the most common.

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Other notations can be used, but the above are the most common.

Glossary

NormGibbs Draw a sample from a posterior distribution of data with an unknown mean and variance using Gibbs sampling.

pNull Test a one sided hypothesis from a numberically specified posterior CDF or from a sample from the posterior

sintegral A numerical integration using Simpson's rule

long term here Sample of a term that might break over lines.

NormGibbs Draw a sample from a posterior distribution of data with an unknown mean and variance using Gibbs sampling.

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sintegral	A numerical integration using Simpson's rule
long term here	Sample of a term that might break over lines.

Making Answers to Exercises

Answers to exercises all through the book may be given at the end of the book; or at the end of the chapter, or both.

SELECTED ANSWERS TO EXERCISES

1.1 Here is the answer for exercise listed.

1.3 Here is the answer for the first exercise in the second exercise set.

- **(a)** Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.
- **(b)** Here is the answer for subexercise listed. Here is the answer for subexercise listed. Here is the answer for subexercise listed.

2.1 Here is the answer to the first exercise in Chapter 2.

2.2 Here is the answer to the second exercise in Chapter 2.

Notes

1. Sample for footnote text enim ad minim veniam, quis nostrud exerci tation ullamcorper
2. Here is another note from the first chapter.
3. Ut wisi enim ad minim veniam, quis nostrud exerci tation ullamcorper suscipit lobortis nisl ut aliquip ex ea commodo consequat. Duis autem vel eum iriure dolor in hendrerit in vulputate velit esse molestie consequat, vel illum dolore eu feugiat nulla facilisis at vero eros et accumsan et iusto odio dignissim qui blandit praesent luptatum zzril delenit augue duis dolore te feugait nulla facilisi.
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1. Text of the sample footnote. Text of the sample footnote. Text of the sample footnote. Text of the sample footnote.
2. Here is a sample footnote that will not become an endnote.

1. Here is a footnote or endnote in an appendix.

1. Here is an endnote in the second appendix. Here is an endnote in the second appendix. Here is an endnote in the second appendix. Here is an endnote in the second appendix.

Sample citations

These commands are found near the top of this file. Please add these commands to your .tex file if you want to use natbib commands.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Choose bibliography styles:
%% Annals of Math bibliography style:
   \bibliographystyhle{alpha}

%% PUP preferred bibliography style:
 \usepackage[round,sort]{natbib}
 \bibliographystyle{authordate1}
```

A list of natbib commands and their results is found in the PUP-docs.pdf file, under `/endmatter/bibliography/cites`.

Here are example citations:

```
\cite{einstein},
\cite[chap.~1]{einstein}, \cite{goosens93},
and \cite{knuthwebsite}.
```

And their results: Einstein (1905), (Einstein, 1905, chap. 1), Goossens *et al.* (1993), and Knuth (2000).

Bibliography

Einstein, Albert. 1905. Zur Elektrodynamik bewegter Körper. (German) [On the electrodynamics of moving bodies]. *Annalen der Physik*, **322**(10), 891–921.

Goossens, Michel, Mittelbach, Frank, & Samarin, Alexander. 1993. *The LaTeX Companion*. Reading, Massachusetts: Addison-Wesley.

Knuth, Donald. 2000. *Knuth: Computers and Typesetting*.

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